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Nordic Process Control Workshop 2009, Porsgrunn, Norway

” Model Reduction applied on Natural Gas Pipeline Systems ”

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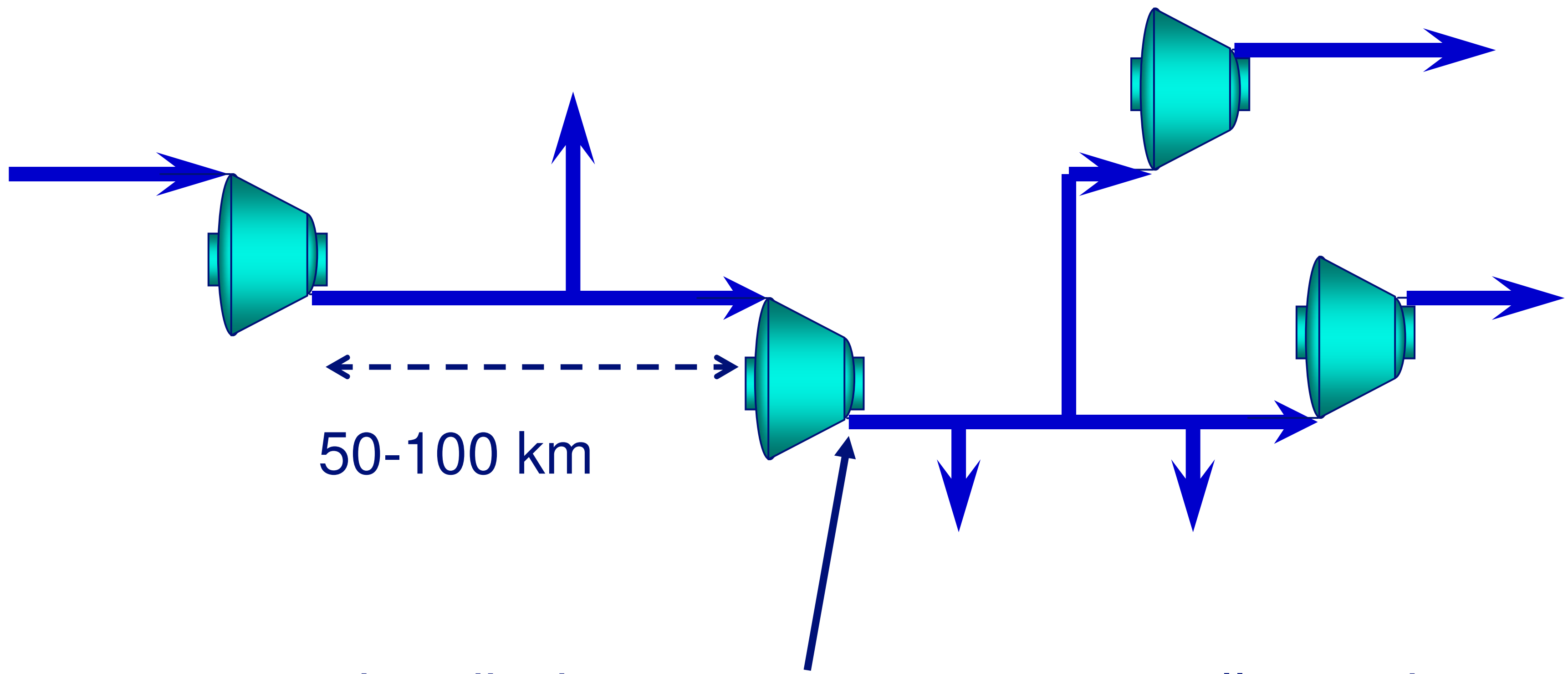
Main pipeline system components:

Compressor stations, pipeline segments and offtakes



Compressor station discharge pressures are usually used to operate the pipeline

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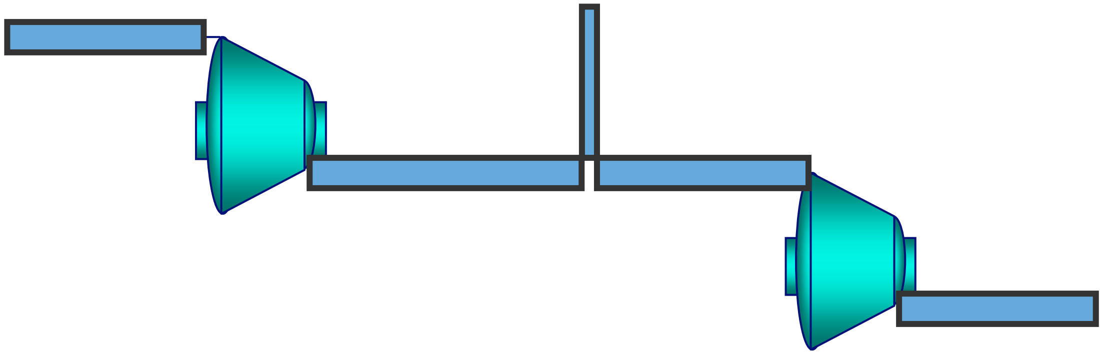
Start with the PDE for a (=each!) pipeline segment



$$\frac{\partial P}{\partial t} + \frac{b}{A}\frac{\partial q}{\partial z} = 0$$

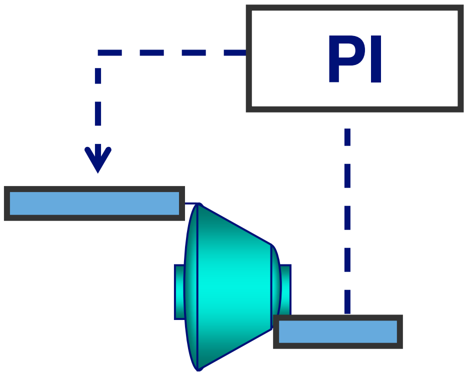
$$\frac{\partial q}{\partial t} + A\frac{\partial P}{\partial z} + f\frac{b}{DA}\frac{q^2}{P} = 0$$

This is the simplest isothermal PDE model for pipelines in the horizontal plane only and with small gas velocities!

Discretize w.r.t to the space co-ordinate z, using N elements (nodes) for *each segment* (!) i=1,2,….N

$$\frac{dP_i}{dt} = \frac{b_i}{A_i \Delta z_i}(q_{i-1} - q_i)$$

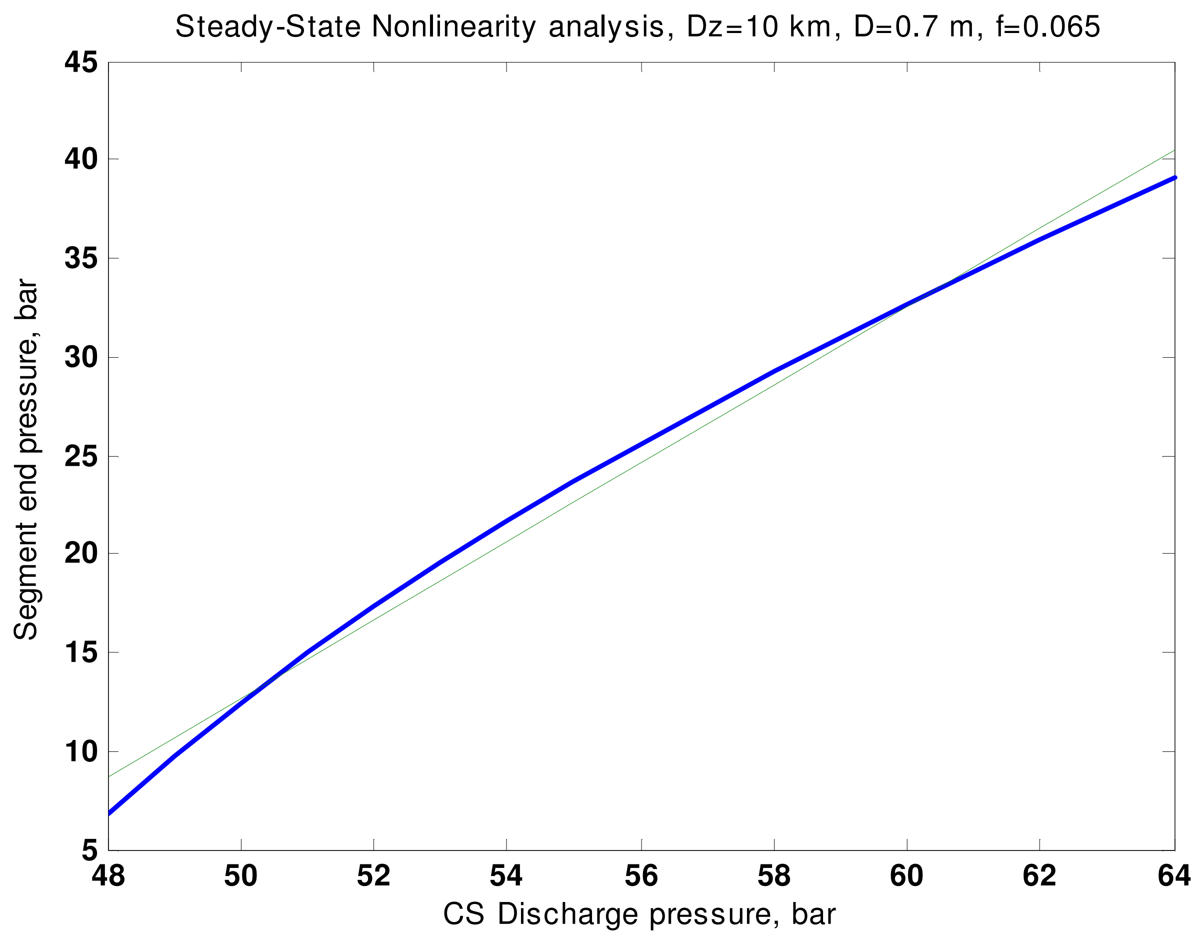
$$\frac{dq_i}{dt} = \frac{A_i}{\Delta z_i}(P_i - P_{i+1}) - f_i \frac{b_i}{D_i A_i} \frac{q_i^2}{P_i}$$



Compressor station between node “k-1” and “k” : PI-controller of discharge pressure manipulating gas flow:

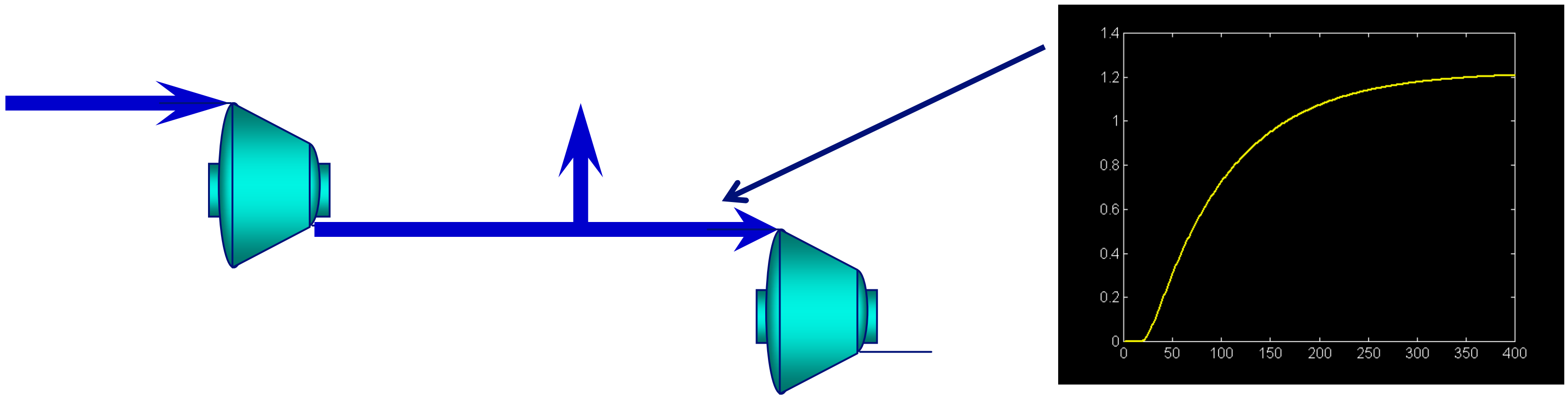
$$\frac{dq_{k-1}}{dt} = -K\beta_k (q_{k-1} - q_k) + \frac{K}{T_i}(P_{k,SET} - P_k)$$

Nonlinearity measure (example)



1 bar perturbation: @ 48 bar Gain=3.08 , Timeconst. 119 min.
@ 64 bar Gain=1.53 , Timeconst. 59 min.

Transfer functions from reduced models:



How would we obtain the <u>transfer function</u> between 2 variables of a given pipeline?

- Identify from true pipeline system data
- Identify from dynamic simulator data

“Direct method”: from design data to transfer functions!

… Linearize this large ODE model in a given steady state operating point

$$\frac{d\Delta P_i}{dt} = \alpha_i(\Delta q_{i-1} - \Delta q_i)$$

$$\frac{d\Delta q_i}{dt} = \beta_i(\Delta P_i - \Delta P_{i+1}) - 2\gamma_i \frac{\Delta q_i}{P_{i,SS}} + \gamma_i \frac{q_{i,SS}^2 \Delta P_i}{P_{i,SS}^2}$$

or:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t),$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$$

where $\mathbf{x} \triangleq [\Delta P_1\ \Delta q_1\ \Delta P_2\ \Delta q_2\ \ldots\ \Delta P_N\ \Delta q_N]^T$

Matrices **A** (2Nx2N) and **B** (2Nxm) depend on the geometry, physical parameters, node partition and steady state data = design (engineering) information

C (1x2N) is needed just to select which state variable is of interest

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t),$$

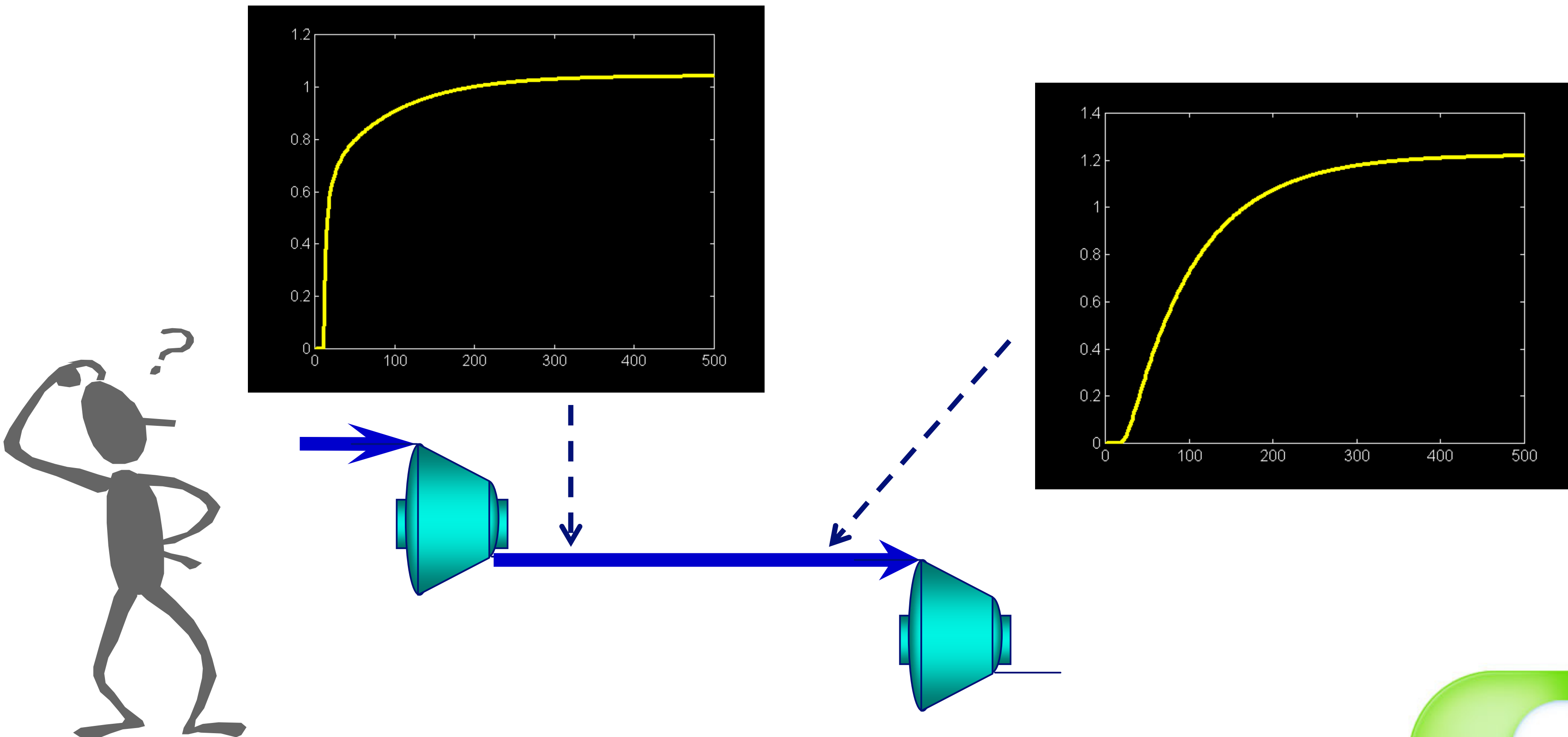
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$$

The rest is easy, obtain the transfer function from (A,B,C) using standard methods **???**

$$G(s) = \frac{K(T_a s+1)(T_b s+1)\ldots}{(T_1 s+1)(T_2 s+1)(T_3 s+1)\ldots}$$

eg. ss2tf of Matlab

NO! Transfer function from large system is difficult, even if dominating time constants may be obtained. In our case, numerator dynamics has relevance!



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=> Use Linear Model Reduction techniques!

Truncation: Solve **P** and **Q** from

$$\mathbf{AP} + \mathbf{PA}^T + \mathbf{BB}^T = 0$$

$$\mathbf{A}^T\mathbf{Q} + \mathbf{QA} + \mathbf{C}^T\mathbf{C} = 0$$

Compute Hankel singular values

$$\sigma_i = \sqrt{\lambda_i(\mathbf{PQ})}$$

Arrange eigenvectors of **PQ** into: a transformation matrix

$$\mathbf{T} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \ldots \mathbf{v}_{2N}]$$

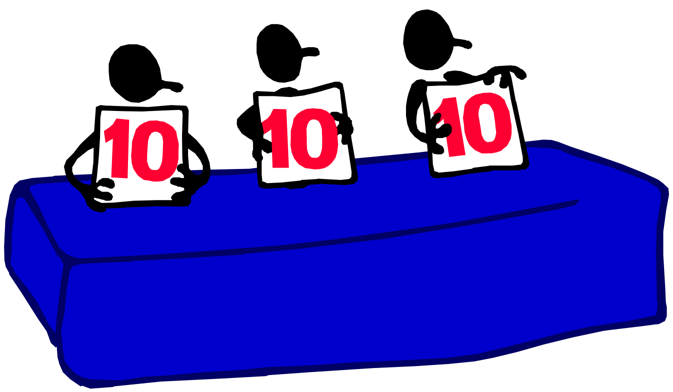
The upper N_r <<2N submatrices of the transformed matrices = a reduced linear state space system

$$\begin{cases} \tilde{\mathbf{A}} = \mathbf{T}^{-1}\mathbf{A}\mathbf{T} \triangleq \mathbf{W}^T\mathbf{A}\mathbf{V} \\ \tilde{\mathbf{B}} = \mathbf{T}^{-1}\mathbf{B} \triangleq \mathbf{W}^T\mathbf{B} \\ \tilde{\mathbf{C}} = \mathbf{C}\mathbf{T} \triangleq \mathbf{C}\mathbf{V} \end{cases}$$

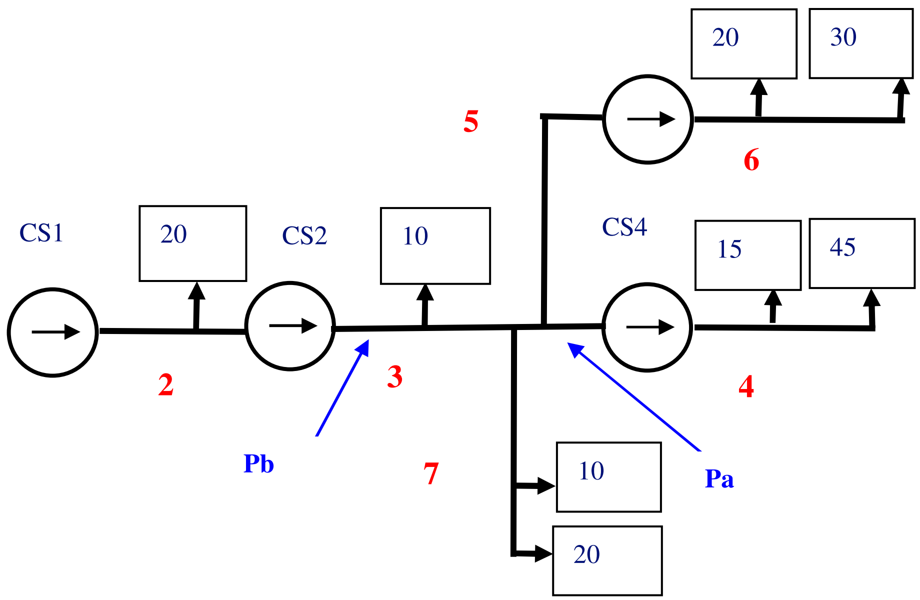
Balanced truncation: P and **Q** required to be diagonal

. . .

Transfer function from reduced model with N_r = 3…4 is easily obtained with standard methods!



Pipeline system w. 6 segments, 8 offtakes, 4 compressor stations and 70 nodes => 2N=140



Transfer function from CS2 discharge pressure to “Pa”, far downstream CS2   [time constant]=minutes!:

$$\frac{1.44(116.9s+1)}{(117.7s+1)(190.6s+1)} \sim \frac{1.44}{(190.6s+1)}$$

Dito for “Pb”, close to CS2:

$$\frac{1.16(83.5s+1)}{(11.2s+1)(183.6s+1)}$$

<u>Empirical</u> calculation of the Gramians P and Q

-Step (impulse) perturbations on the system or on a full-scale simulation model
- After obtaining **T**, define a transformed state and apply a <u>Galerkin projection</u> to get a reduced <u>non-linear</u> model:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \ , \quad \mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t))$$

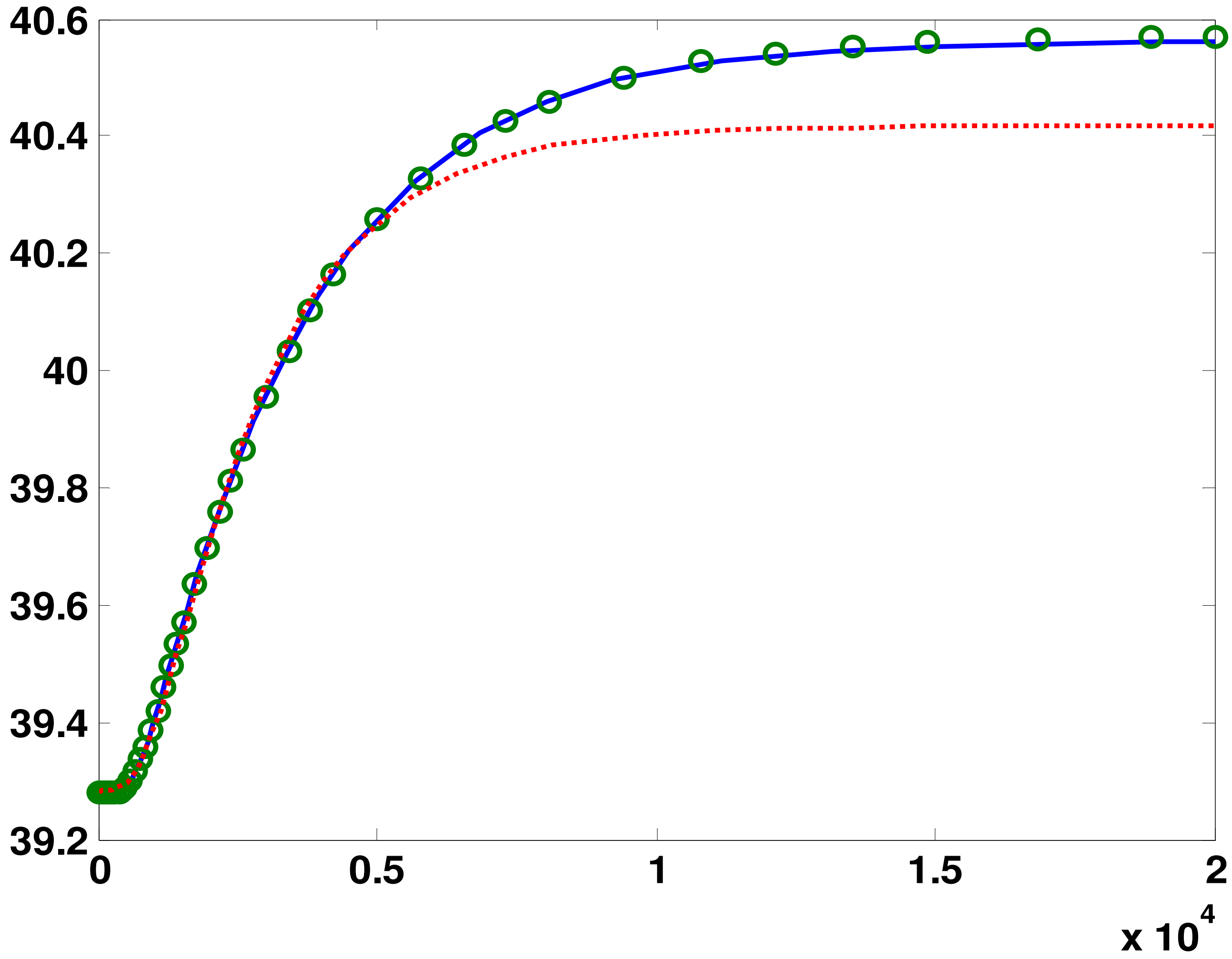
$$\dot{\mathbf{x}}_{\mathbf{r}}(t) = \mathbf{PTf}(\mathbf{T}^{-1}\overline{\mathbf{x}}(t), \mathbf{u}(t))$$

$$\dot{\mathbf{x}}_{N-\mathbf{r}}(t) = 0, \quad \mathbf{x}_{N-\mathbf{r}}(t) = \mathbf{x}_{N-\mathbf{r},SS}$$

$$\mathbf{y}(t) = \mathbf{g}(\mathbf{T}^{-1}\overline{\mathbf{x}}(t), \mathbf{u}(t))$$

$$\overline{\mathbf{x}}(t) \triangleq \begin{bmatrix} \mathbf{x}_r \\ \mathbf{x}_{N-r} \end{bmatrix} = \mathbf{Tx}(t) \quad , \mathbf{P} = \begin{bmatrix} \mathbf{I}_{\mathbf{r}} & \mathbf{0} \end{bmatrix}$$

Response of original ODE model for 92 km pipeline with 58 states, reduced nonlinear model Nr= 8 (circles) and dito with Nr=4.



General problem (not necessarily natural gas pipeline):
We need an EKF for a large non-linear system

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))$$

$$\mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t))$$

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{f}(\hat{\mathbf{x}}(t), \mathbf{u}(t)) + \mathbf{K}[\mathbf{y}_M(t) - \mathbf{g}(\hat{\mathbf{x}}(t))]$$

Steady state Riccati equation applies for linearized continuous time system, dim(**K**)=n x ny :

$$\mathbf{K} = \mathbf{P}\mathbf{C}^T\mathbf{R}^{-1}$$

$$\mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^T + \mathbf{Q} - \mathbf{P}\mathbf{C}^T\mathbf{R}^{-1}\mathbf{C}\mathbf{P} = 0$$

All “mappings” in the sequel from a smaller matrix $\mathbf{K_r}$ to a bigger $\mathbf{K}$, actually mean:

$$\mathbf{K} = \mathbf{f}(\mathbf{K_r}) \triangleq \mathbf{f}\left(\begin{bmatrix} \mathbf{K_r} & \mathbf{0} \\ \mathbf{0} & \varepsilon\mathbf{I} \end{bmatrix}\right), \ \varepsilon \to 0$$

Kalman filter for the reduced model also obeys Riccati:

$$\mathbf{K_r} = \mathbf{P_r C_r}^{\mathbf{T}} \mathbf{R}^{-1}$$

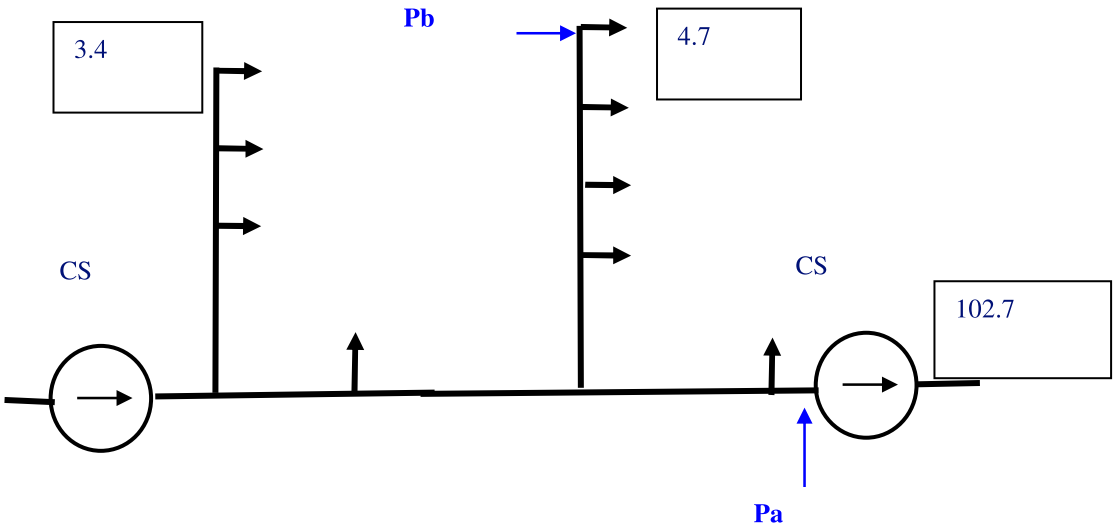
$$\mathbf{A_r P_r} + \mathbf{P_r A_r}^{\mathbf{T}} + \mathbf{Q_r} - \mathbf{P_r C_r}^{\mathbf{T}} \mathbf{R}^{-1} \mathbf{C_r P_r} = \mathbf{0}$$

Then, if: $\mathbf{K} = \mathbf{V K_r}, \mathbf{Q} = \mathbf{W Q_r W}^{\mathbf{T}}$

The Riccati equation for the full system holds, and:

$\mathbf{P} = \mathbf{W P_r W^T}$ **True also for discrete-time system**

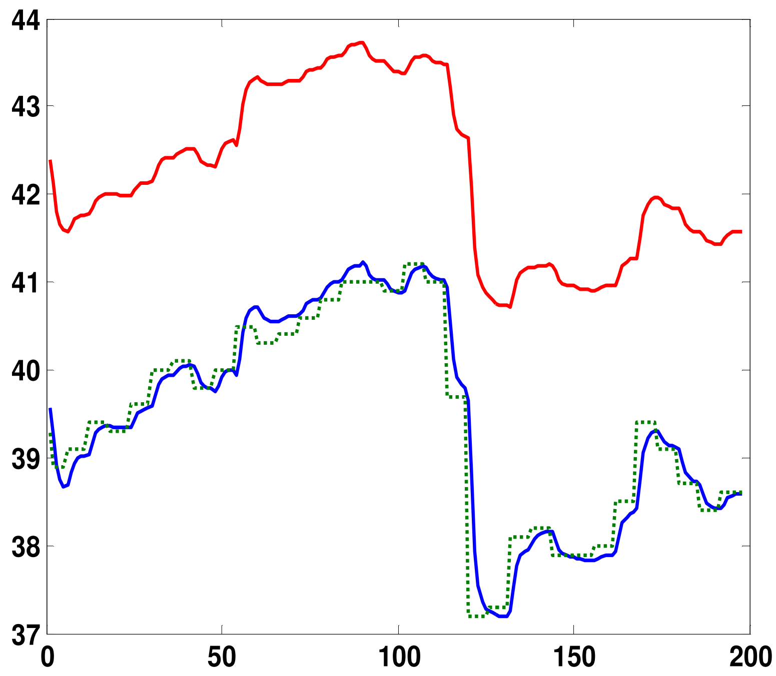
Exmple: 90-km long true pipeline segment:



Δz=1667 m => 82 elements = 164 states

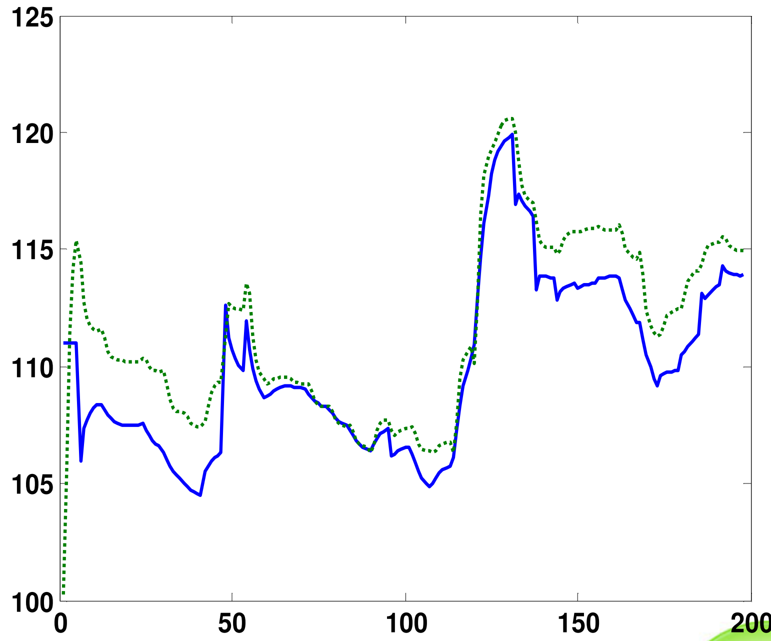
Design Kalman filter for n_r=4 and then scale up **K** = **V*K**$_r$ to obtain state estimator for 164 states

… **Results:**



Estimated upstream pressure

Estimated P_a
Measured P_a



Estimated gas flow after CS
Simulated gas flow

What if we want to do the EKF exercise but do not have access to the full scale linear n-dimensional system (**A**,**B**,**C**) ?

Recall:
- Empirical Gramians would give us **P**,**Q**,**V** and **W**
- Low dimensional model ($\mathbf{A}_r$, $\mathbf{B}_r$, $\mathbf{C}_r$) could be obtained by identification

=> Do the matrices match, can we do "scale up"
$\mathbf{K} = \mathbf{V}^*\mathbf{K}_r$?

Let us borrow some results from discrete-time subspace identification (The state space model realisation part of it)

Use the system impulse response to form a Hankel matrix:

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & & \mathbf{h}_{N+1} \\ \mathbf{h}_2 & \mathbf{h}_3 & & \mathbf{h}_{N+2} \\ \vdots & & \cdots & \\ \mathbf{h}_N & \mathbf{h}_{N+1} & & \mathbf{h}_{2N+2} \end{bmatrix} \qquad \mathbf{h}_i = \mathbf{C}\mathbf{A}^{i-1}\mathbf{B}$$

Svd of **H**, which is actually an estimate from data, **H^**

$$\mathbf{H} = \mathbf{Q}\mathbf{S}\mathbf{V}^T$$

Choose a model order ”r” and partition:

$$\mathbf{Q} = [\mathbf{Q}_r \ \mathbf{Q}_{N-r}] \qquad \mathbf{V} = [\mathbf{V}_r \ \mathbf{V}_{N-r}]$$

S_r is a diagonal matrix with r principal singular values.
Observability and Controllability matrix estimates:

$$\Gamma_N = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{N-1} \end{bmatrix} = Q_r S_r^{1/2} \qquad \Omega_N = \begin{bmatrix} B & AB & \cdots & AB^{N-1} \end{bmatrix} = S_r^{1/2} V_r^T$$

Read **C**, actually $\mathbf{C_r}$ from $\mathbf{\Gamma_N}$ and $\mathbf{B_r}$ from $\mathbf{\Omega_N}$

For **A**, actually $\mathbf{A_r}$, apply the ”shift invariance” $\Gamma_N = \Gamma_{N-1} A$

Solve **A** using pseudo-inverse

Actually we have done a **balanced truncation**!

-Using **H**, make a full state dimension model with r -> n < N:

- $\mathbf{P} = \mathbf{\Omega_N \Omega_N^T}$
- $\mathbf{Q} = \mathbf{\Gamma_N^T \Gamma_N}$
- Calculate $\mathbf{A_n}$, $\mathbf{B_n}$ and $\mathbf{C_n}$ as above = **linearization!**
- Calculate **T** (as above), call $\mathbf{W} = \mathbf{T^{-T}}$ and $\mathbf{V} = \mathbf{T}$

  Use **W** and **V** for a lower-dimensional model r<n:

$$\tilde{\mathbf{A}} = \mathbf{W}^{\mathbf{T}}\mathbf{A_n}\mathbf{V}\,,\ \mathbf{A_r} = \tilde{\mathbf{A}}(1:n_r, 1:n_r) \quad \text{etc.}$$

r:th order model can also be obtained by repeating the realisation procedure.

NOTE, that **V** is needed to do the ”scale up” for the Kalman filter

Impulse response data from a simulation model = no noise problem = should work, but non-linearity may harm

Fix: Discrete-time EKF innovations part to be combined with continuous-time non-linear model

Find weak points of ”scale up” procedure, some horrible counter-example etc.

Subspace realisation for a large full order linear (=n) system may be tough; almost redundant states etc.

Thank You!