OBSERVER BASED FDI/FTC

Raimo Ylinen

Laboratory of Process Control and Automation
P.O.Box 6100, 02015 HUT, Finland

Abstract: The paper presents a model-based design method for fault diagnosis (detection and isolation) and fault-tolerant control (FDI/FTC).The design is based on pole placement of input-output models. Furthermore, the design of an output feedback controller is divided into two independent design tasks: a generalized state and fault observer design and a fault feedforward and state feedback design.

Keywords: Fault-tolerant control, polynomial methods, control system design, observers

1. INTRODUCTION

The model-based analysis methods for fault diagnosis and fault-tolerant control (FDI/FTC) use architectural and structural models to analyse the propagation of the fault through the process, to test the fault detectability and to find the redundancies in the process that can be used to ensure fault tolerance. Design methods for diagnostic systems and fault-tolerant controllers are usually based on more detailed analytical models, discrete-event models or on quantised systems. (Blanke *et al.*, 2003). A typical FDI/FTC-system is described in Fig.1. In this paper, multivariable, time-invariant linear differential models are used for analysis and design. The models are described by polynomial matrices in the differential operator, i.e. by so-called *polynomial systems*.

First some basic concepts and definitions of polynomial systems and their interconnections as presented in (Blomberg and Ylinen, 1983) are given. Then the pole placement designs of observers and feedback controllers introduced in (Blomberg, 1974),(Blomberg and Ylinen, 1976), (Blomberg and Ylinen, 1978) are considered. The parametrization of controllers is shown to be closely related to the well-known Youla-Kučera parametrization (Youla *et al.*, 1976), (Kučera, 1979).

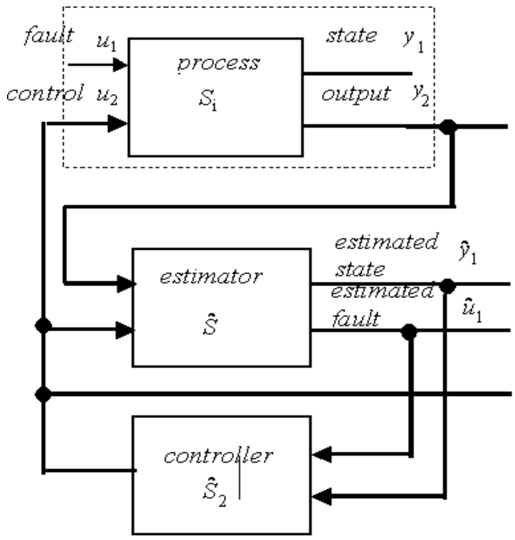

Fig. 1. Fault-tolerant control

The observer and state feedback system are integrated to a complicated feedback system and its dynamics is shown to consist of the dynamics of the estimation error and the dynamics of state feedback systems. Furthermore, these can be designed separately.

The methodology is then applied to state and fault estimation and to the design of fault feed-forward and state feedback controllers. Finally, a numerical example is presented.

2. INPUT-OUTPUT RELATIONS

A set of linear time-invariant differential equations can be written as a matrix equation

$$A(p)y = B(p)u \tag{1}$$

where $u \in \mathcal{X}^r, y \in \mathcal{X}^s$ and $A(p)$, $B(p)$ are polynomial matrices over $\mathbf{C}[p]$. Here $\mathcal{X} \triangleq$ the *signal space*, a space of complex-valued infinitely differentiable functions from $T \subset \mathbf{R}$ to $\mathbf{C}$ ($\mathbf{C}$ and $\mathbf{R}$ are the fields of complex and real numbers, respectively) and p is the *differentiation operator* on $\mathcal{X}$.

A multivariable differential *input-output (IO-) relation* generated by (1) is defined as the set

$$S = \{(u, y)|A(p)y = B(p)u\} \tag{2}$$

Equation (1) can be written also in the form

$$[A(p)\vdots - B(p)] \begin{bmatrix} y \\ u \end{bmatrix} = 0 \tag{3}$$

The matrix $[A(p)\vdots - B(p)]$ is called a *generator* for S. Generators for the same input-output relation are *input-output (IO-) equivalent.*

The IO-relation S (2) is not necessarily a mapping $\mathcal{X}^r \to \mathcal{X}^s$ but it should be *realizable* in the sense that given an input $u \in \mathcal{X}^r$ and a sufficient (finite) number of *initial values* at some time t_0, then the corresponding output $y \in \mathcal{X}^s$ is *unique* from t_0 onwards. Such kind of S as well as its generators $[A(p)\vdots - B(p)]$ are said to be *regular.* If $A(p)$ is square, a necessary and sufficient condition for that is $\det A(p) \neq 0$.

The rational matrix $\mathcal{G}(p) \triangleq A(p)^{-1}B(p)$ is a *transfer matrix* of S. A more strict realizability condition, the *causality* or *nonantipativeness* property requires that the transfer matrix $\mathcal{G}(p)$ has to be *proper.* This means that the degrees of the denominators of its entries are not lower than the degrees of the corresponding numerators.

A polynomial matrix $P(p)$ is *unimodular* if it is invertible as a polynomial matrix, i.e. if its determinant is a nonzero constant. Two polynomial matrices $A(p), B(p)$ are *row equivalent* if there is a unimodular matrix $P(p)$ such that $A(p) = P(p)B(p)$. A unimodular matrix is also invertible as a mapping. This means that a generator can be brought to arbitrary IO-equivalent forms by use of

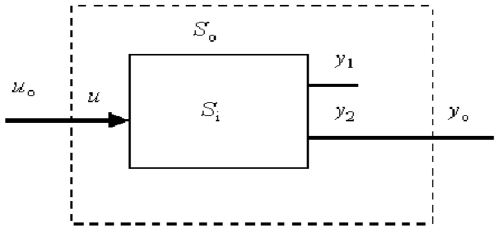

Fig. 2. General composition

unimodular *elementary row operations.* Then the *canonical forms* like *Canonical Upper Triangular* (CUT-) Form or *Canonical Row Proper* (or *Reduced*) (CRP-) Form for the row equivalence are also canonical forms for the IO-equivalence)(see (Blomberg and Ylinen, 1983)).

An IO-relation S generated by $[A(p)\vdots - B(p)]$ is said to be *(asymptotically) stable* if every solution y to $A(p)y = 0$ approaches 0 when the time t approaches the infinity. This is true if all the roots of the *characteristic polynomial* $\det A(p)$ have negative real parts. Then the matrix itself $A(p)$ is also called *stable*

Suppose that $L(p)$ is a Greatest Common Left Divisor (GCLD) of $A(p)$ and $B(p)$ i.e. $[A(p)\vdots - B(p)]$ can be factorized

$$[A(p)\vdots - B(p)] = L(p)[A_1(p)\vdots - B_1(p)] \tag{4}$$

with $A_1(p), B_1(p)$ left coprime, i.e. they have no common left divisors apart from unimodular ones. Now if $L(p)$ is not unimodular, S contains outputs related to $L(p)$ which cannot be affected by the input u. This means that S is not *controllable.* If S is not *controllable* but $L(p)$ is stable, then S is called *stabilizable.* The IO-relation generated by $[A_1(p)\vdots - B_1(p)]$ is the controllable part of S.

3. COMPOSITION OF IO-RELATIONS

A more general description for a system is a *composition* of input-output relations. It consists of a set of input-output relations ('subsystems') or their generators, and some kind of description of the interconnections between the (signals of the) subsystems. The interconnections can be given *graphically*, using different kind of *interconnection matrices* etc.

Every composition can be brought to the general form of Fig. 2, where S_i is the *internal IO-relation* and S_o the *overall IO-relation* generated by the composition. Conversely, the composition is then said to be a *decomposition* of S_o.

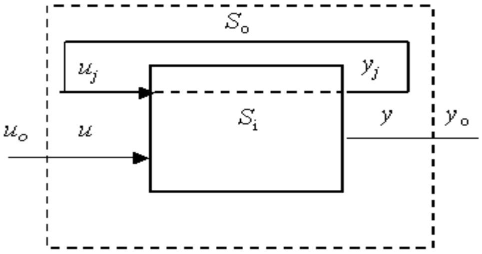

Fig. 3. General composition with free internal inputs

For the internal IO-relation S_i it is always possible to construct a generator from the generators of the subsystems and the interconnections

$$\begin{bmatrix} A_1(p) & A_2(p) & \vdots & -B_1(p) \\ A_3(p) & A_4(p) & \vdots & -B_2(p) \end{bmatrix} \tag{5}$$

Instead, for the overall IO-relation

$$S_o = \{(u_o, y_o) | \exists y_1 [(u_o, (y_1, y_o)) \in S_i]\} \tag{6}$$

the construction of a generator is a more complicated task.

Compositions determining the same overall IO-relations are *input-output (IO-) equivalent.* A composition is *regular* if the corresponding internal IO-relation is regular. Furthermore, a composition is *controllable* if the internal IO-relation is controllable.

Note that also the interconnections containing 'free' internal inputs (i.e. inputs which are not any overall inputs) u_j can be brought to this form by adding an 'identity' subsystem described by $y_j = u_j$ and an interconnection $u_j = y_j$. However, then the composition cannot be regular. Consider the composition of 2 and suppose that the composition is regular. The generator (5) can be brought to upper triangular form

$$\begin{bmatrix} \widetilde{A}_1(p) & \widetilde{A}_2(p) & \vdots & -\widetilde{B}_1(p) \\ 0 & \widetilde{A}_4(p) & \vdots & -\widetilde{B}_2(p) \end{bmatrix} \tag{7}$$

Now if for each (u_o, y_o) satisfying the equation

$$\tilde{A}_4(p) y_o = \tilde{B}_2(p) u_o \tag{8}$$

there exists a y_1 such that $(u_o, (y_1, y_o))$ satisfies the equation

$$\tilde{A}_1(p) y_1 = -\tilde{A}_2(p) y_o + \tilde{B}_1(p) u_o \tag{9}$$

then the overall IO-relation S_o is generated by the equation (8) or by the generator $[\tilde{A}_4(p) \vdots - \tilde{B}_2(p)]$. Especially, if $\det \tilde{A}_1(p) \neq 0$, then the condition above is satisfied.

If $\tilde{A}_1(p)$ is unimodular, then the y_1 satisfying (9) must be unique. In this case the composition is $(y_1(u_o, y_o))$*-observable.* $\tilde{A}_1(p)$ is unimodular if and only if $A_1(p)$ and $A_3(p)$ are right coprime. In this case it is always possible to take $\tilde{A}_1(p) = I$. If $\tilde{A}_1(p)$ is not unimodular but stable, the composition is $(y_1(u_o, y_o))$*-detectable.*

In a nonregular case it may happen that $[\tilde{A}_4(p) \vdots - \tilde{B}_2(p)]$ is empty or the equation (9) does not have a solution. In the latter case the output y_1 cannot be eliminated.

Consider again the composition of Fig.2. If the generator of S_i can be brought to the form

$$\begin{bmatrix} \widehat{A}_1(p) & 0 & \vdots & -\widehat{B}_1(p) \\ \widehat{A}_3(p) & I & \vdots & -\widehat{B}_2(p) \end{bmatrix} \tag{10}$$

the composition is called a *generalized state space decomposition* of S_o, and y_1 is the corresponding *generalized state.* This means that the composition, where y_1 instead of y_o is taken as an overall output, is $(y_o(u_o, y_1))-$*observable.*

4. OBSERVER DESIGN

Consider the composition of Fig. 2 and suppose that only the overall input $u_o = u$ and output $y_o = y_2$ are measured. The problem is to design a dynamic system, a so-called *observer* for continuous estimation of the internal output y_1, so that the estimation error $\tilde{y}_1 = y_1 - \widehat{y}_1$ behaves in a satisfactory way.

Let the internal IO-relation S_i be generated by the generator (7) of an upper triangular form and the observer $\widehat{S}$ to be designed by the generator

$$\begin{bmatrix} C & \vdots & -D_1 & -D_2 \end{bmatrix} \tag{11}$$

Here and in what follows (p) is omitted in order to shorten the notations.

It can be shown that the observer has to satisfy

$$\begin{bmatrix} C & -D_1 & -D_2 \\ 0 & \widetilde{A}_4 & -\widetilde{B}_2 \end{bmatrix} = \begin{bmatrix} T_1 & T_2 \\ 0 & I \end{bmatrix} \begin{bmatrix} \widetilde{A}_1 & \widetilde{A}_2 & -\widetilde{B}_1 \\ 0 & \widetilde{A}_4 & -\widetilde{B}_2 \end{bmatrix} \tag{12}$$

for some T_1, T_2 (see (Blomberg and Ylinen, 1983)). Thus the design problem has been changed to the construction of the matrices T_1, T_2. The matrix T_1 affects the stability of the estimation error, because the error is generated by

$$T_1 \tilde{A}_1 \tilde{y}_1 = 0 \tag{13}$$

After T_1 of order high enough has been chosen the matrix T_2 is used to achieve a proper or a strictly

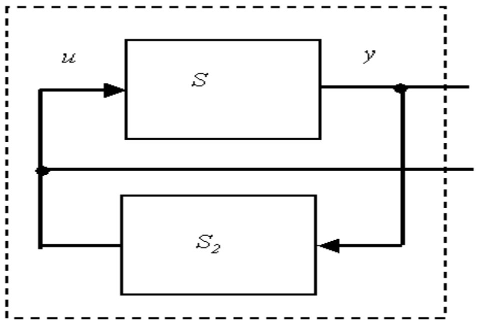

Fig. 4. General feedback control composition

proper observer. Both matrices can be constructed sequentially using the elementary row operations.

5. FEEDBACK CONTROLLER DESIGN

Consider an IO-relation S generated by $[A \vdots - B]$. The basic feedback control problem is to design a system S_2, a *feedback controller* generated by $[E \vdots - F]$ such that the overall system depicted by Fig.4 behaves satisfactorily, is stable, robust etc. Note that the internal output of the composition is (y, u) and the input is empty. It can be easily shown (Blomberg, 1974),(Blomberg and Ylinen, 1983) that every feedback composition can be written as

$$\begin{bmatrix} A & -B \\ -F & E \end{bmatrix} = \begin{bmatrix} L & 0 \\ 0 & I \end{bmatrix} \underbrace{\begin{bmatrix} I & 0 \\ T_3 & T_4 \end{bmatrix}}_{T} \underbrace{\begin{bmatrix} A_1 & -B_1 \\ Q_3 & Q_4 \end{bmatrix}}_{Q} \quad (14)$$

for some $[T_3 \quad T_4]$. Here $[A_1 \vdots - B_1]$ represents the controllable part of S and the *first candidate* Q for the generator of a feedback composition is unimodular and the inverse of P bringing $[A \vdots - B]$ to the form $[L \vdots 0]$.

The closed loop dynamics of the overall system is determined by T_4 and by the uncontrollable part L. In order to guarantee the robustness the controller should be at least proper but preferably strictly proper. This can be achieved by choosing T_4 of high order and using T_3 for decreasing the degrees in F. Analogously to the condition (12) also the condition (14) can be used repeatedly. The construction above is closely related to the *Youla-Kučera parametrization* (Youla *et al.*, 1976),(Kučera, 1979). The difference is that our parametrization (14) is based on polynomial matrices instead of proper stable rational matrices.

It was shown in (Ylinen, 2000) that for a generalized state decomposition (10) the feedback composition using both the state and the output as controller inputs can be brought to IO-equivalent compositions which use either the output or the state alone as controller inputs. The latter composition is depicted by Fig. 5. Thus for each output feedback there is an IO-equivalent state feedback and *vice versa*.

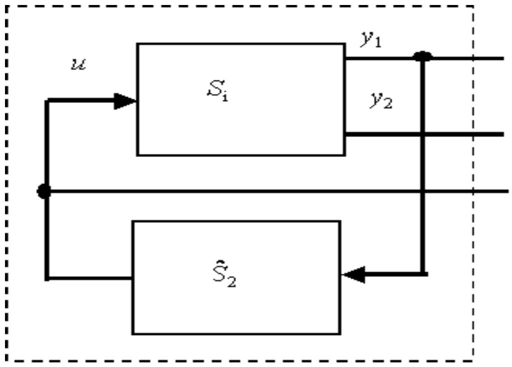

Fig. 5. State feedback control

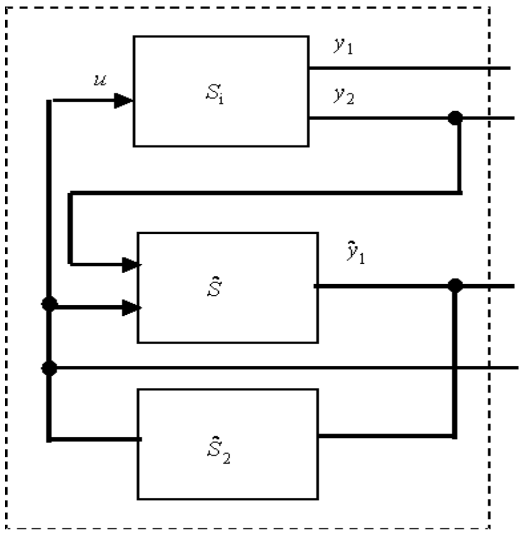

Fig. 6. State feedback with observer

If the original feedback controller is generated by $[E \vdots - F_2 \quad - F_2]$, the feedback controller using the output y_2 is generated by

$$\left[\widetilde{E} \vdots - \widetilde{F}_2\right] = \left[E - F_1\widetilde{B}_1 \vdots - F_2 + F_1\widetilde{A}_2\right] \quad (15)$$

The situation is symmetric with respect to outputs $y_1$and y_2 so that the feedback can also be based on the generalized state y_1 only. In this case, the feedback controller is generated by

$$\left[\widehat{E} \vdots - \widehat{F}_1\right] = \left[E - F_2\widehat{B}_2 \vdots - F_1 + F_2\widehat{A}_3\right] \quad (16)$$

Suppose then that instead of the output y_1 the corresponding estimate $\widehat{y}_1$ determined by the observer (12) is used for feedback control. The situation is depicted by Fig.6. Now the closed loop system with $(\widehat{y}_1, y_1, y_2, u)$ as output is generated by

$$A_{cl} \triangleq \begin{bmatrix} C & 0 & -D_1 & -D_2 \\ 0 & I & \widetilde{A}_2 & -\widetilde{B}_1 \\ 0 & 0 & \widetilde{A}_4 & -\widetilde{B}_2 \\ -\widehat{F}_1 & 0 & 0 & \widehat{E} \end{bmatrix} \quad (17)$$

If the observer is constructed according to (12), and state feedback according to (14), the characteristic polynomial of (17) satisfies

$$\det A_{cl} = k \det T_1 \det T_4, k = constant \neq 0 \quad (18)$$

(Ylinen, 2000). Thus the characteristic polynomial consists of two factors, the characteristic polynomial of the observer $\det T_1$ and the characteristic polynomial of the state feedback composition $\det T_4$. The important result is that they both can be designed *independently* of each other.

6. FAULT DETECTION

Return now back to the original FDI/FTC system depicted by Fig. 1 and suppose that the system is generated by

$$\begin{bmatrix} \widetilde{A}_1 & \widetilde{A}_2 & \vdots & -\widetilde{B}_1 & -\widetilde{B}_2 \\ 0 & \widetilde{A}_4 & \vdots & -\widetilde{B}_3 & -\widetilde{B}_4 \end{bmatrix} \quad (19)$$

Considering the internal input u_1 as an output, reordering the columns and using the elementary row operations bring the generator further to the following upper triangular form

$$\begin{bmatrix} \widetilde{A}_1 & -\widetilde{B}_1 & \widetilde{A}_2 & \vdots & -\widetilde{B}_2 \\ 0 & -\widetilde{\widetilde{B}}_{31} & \widetilde{\widetilde{A}}_{41} & \vdots & -\widetilde{\widetilde{B}}_{41} \\ 0 & 0 & \widetilde{\widetilde{A}}_{42} & \vdots & -\widetilde{\widetilde{B}}_{42} \end{bmatrix} \quad (20)$$

Because the generator is not regular, it is possible that $[\widetilde{\widetilde{A}}_{42} \vdots -\widetilde{\widetilde{B}}_{42}]$ becomes empty. In this case the whole generator (20) must be used as an observer and it is not possible to improve its properties. This situation can be avoided, if the behavior of the internal input u_1 can be modelled e.g. by a model

$$\widetilde{B}_5 u_1 = 0 \quad (21)$$

which can be added as a new row to the equation (19).

In the nonempty case the new candidates for observer can be constructed using (12) repeatedly

$$\begin{bmatrix} C_1 & C_2 & -D_{11} & -D_{21} \\ C_3 & C_4 & -D_{12} & -D_{22} \\ 0 & 0 & \widetilde{\widetilde{A}}_{42} & -\widetilde{\widetilde{B}}_{42} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{21} \\ T_{13} & T_{14} & T_{22} \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \widetilde{A}_1 & -\widetilde{B}_1 & \widetilde{A}_2 & -\widetilde{B}_2 \\ 0 & -\widetilde{\widetilde{B}}_{31} & \widetilde{\widetilde{A}}_{41} & -\widetilde{\widetilde{B}}_{41} \\ 0 & 0 & \widetilde{\widetilde{A}}_{42} & -\widetilde{\widetilde{B}}_{42} \end{bmatrix} \quad (22)$$

Because the internal system is not regular and possibly not observable, a satisfactory observer, however, cannot always be obtained.

If T_1 is chosen to be upper triangular resulting in an upper triangular matrix C, the observer can be decomposed to a series composition of two observers.

Example 1. Consider a state decomposition of a two-input-two-output system generated by

$$\begin{bmatrix} (5p+1)(p+1) & 0 & 0 & \vdots & -(p+1) & -1 \\ -1 & 1 & 0 & \vdots & 0 & 0 \\ -p & 0 & 1 & \vdots & 0 & 0 \end{bmatrix} \quad (23)$$

Reordering the columns and bringing the matrix to the upper triangular form gives

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & p+1 & -1 & -(5p+6) & 1 \\ 0 & 0 & p & -1 & 0 \end{bmatrix} \quad (24)$$

A stable and proper observer candidate with error dynamics $p+1$ is obtained from

$$\begin{bmatrix} p+1 & 0 & \vdots & -1 & -1 & 0 \\ 0 & p+1 & \vdots & -1 & -(5p+6) & 1 \\ 0 & 0 & \vdots & p & -1 & 0 \end{bmatrix} \quad (25)$$

using transformation

$$\begin{bmatrix} T_{11} & T_{12} & T_{21} \\ T_{13} & T_{14} & T_{22} \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} p+1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (26)$$

to the first candidate.

7. FAULT-TOLERANT CONTROL

Suppose next that the system in Fig.1 is described by a generalized state decomposition generated by

$$\begin{bmatrix} \widehat{A}_1 & 0 & \vdots & -\widehat{B}_1 & -\widehat{B}_2 \\ \widehat{A}_3 & I & \vdots & -\widehat{B}_3 & -\widehat{B}_4 \end{bmatrix} \quad (27)$$

For simplicity, the composition is assumed to be $y_1(y_2, u_2))-$observable and the state system generated by $[\widehat{A}_1 \vdots -\widehat{B}_2]$ controllable.

The problem is to construct either an output or a state feedback controller such that it takes into account also the estimated fault. In what follows, only the state feedback controller will be considered but as shown in (Ylinen, 2000), it is equivalent to an output controller.

So start from the generator $[\widehat{A}_1 \vdots -\widehat{B}_1 \;\; -\widehat{B}_2]$. Let the generator of the controller be

$$\begin{bmatrix} E_1 & E_2 & \vdots & -F_1 \\ E_3 & E_4 & \vdots & -F_2 \end{bmatrix} \tag{28}$$

where $E_4 = 0$ without loss of generality. Furthermore, because u_1 (the fault) must be kept free also the matrix F_2 must be zero. Thus the output u_1 is restricted only by equation

$$E_3 u_1 = 0 \tag{29}$$

which describes the probable behavior of the fault. The first candidate for the internal system can be chosen as

$$\begin{bmatrix} \widehat{A}_1 & -\widehat{B}_1 & -\widehat{B}_2 \\ Q_3 & 0 & Q_4 \\ 0 & I & 0 \end{bmatrix} \tag{30}$$

where

$$\begin{bmatrix} \widehat{A}_1 & -\widehat{B}_2 \\ Q_3 & Q_4 \end{bmatrix} \tag{31}$$

is unimodular (c.f.(14)). The other candidates can be constructed using

$$\begin{bmatrix} \widehat{A}_1 & -\widehat{B}_1 & -\widehat{B}_2 \\ -F_1 & E_1 & E_2 \\ 0 & E_3 & 0 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ T_{31} & T_{41} & T_{42} \\ 0 & 0 & T_{44} \end{bmatrix} \begin{bmatrix} \widehat{A}_1 & -\widehat{B}_1 & -\widehat{B}_2 \\ Q_3 & 0 & Q_4 \\ 0 & I & 0 \end{bmatrix} \tag{32}$$

repeatedly in order to achieve a stable, robust and proper controller.

The final controller is described by

$$E_2 u_2 = F_1 y_1 - E_1 u_1 \tag{33}$$

where the estimated fault and possibly also the estimated state must be used instead of the real ones.

Example 2. Consider the system in Example 1. The first candidate can be easily chosen

$$\begin{bmatrix} (5p+1)(p+1) & -(p+1) & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{34}$$

The use of the transformation

$$\begin{bmatrix} 1 & 0 & 0 \\ -(40p+12) & 25(2p+1)^3 & -p(40p+52) \\ 0 & 0 & p \end{bmatrix} \tag{35}$$

gives the closed loop generator

$$\begin{bmatrix} (5p+1)(p+1) & -(p+1) & -1 \\ 38p+13 & 12 & 40p+12 \\ 0 & p & 0 \end{bmatrix} \tag{36}$$

and the controller

$$(40p+12)u_2 = -(38p+13)y_1 - 12u_1 \tag{37}$$

8. CONCLUDING REMARKS

The polynomial systems theory gives tools for the analysis and design of linear estimators and feedback controllers. In this paper the methodology has been applied to design of fault detection and estimation systems as well as fault tolerant controllers. The main problems in the design are related to the lack of observability of the unknown inputs like faults. However, the theory gives tools to analyze the situation and to find the form of the needed additional information.

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