Calibration of Parallel Hybrid Vehicles Based on Hybrid Optimal Control Theory

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Abstract: Most energy management systems for hybrid electric vehicles rely on information stored in lookup tables, to define the current mode of operation under certain circumstances. In this paper it is demonstrated how the theory of hybrid optimal control can be used to calculate an initial parameter set for the calibration of parallel hybrid electric vehicles. After solving a hybrid optimal control problem for the fuel optimal operation of the vehicle, taking into account continuous as well as discrete dynamics, the results can be used to automatically calculate lookup tables for optimal gear shifts, optimal torque-split between motor/generator and internal combustion engine and the determination of the drive mode (electric or hybrid mode). The algorithms proposed are easy in their application and can be used for other hybrid vehicle configurations as well and therefore constitute a valuable tool for the initial calibration.

Keywords: Optimal Control, Hybrid Nonlinear Systems, Automotive Systems

1. INTRODUCTION

Increasing prices for crude oil and growing environmental concerns give rise to the continuing development of hybrid vehicles. This type of vehicle adds additional degrees of freedom (DOF) to a conventional powertrain. In parallel configurations of hybrid electric vehicles (HEV), the addition of an electrical motor/generator (M/G) allows the internal combustion engine's (ICE) torque to be chosen within certain limits independently from the driver request. This degree of freedom can be used to improve the overall efficiency of the system and is controlled by a cascaded controller structure. The energy management of an HEV has the task to supply reference trajectories to this controller structure.

Many different strategies exist in the literature and in practical applications to calculate the reference trajectories. Those can be distinguished in rule-based and analytical approaches. A promising analytical approach is the use of optimal control theory. The fuel-optimal operation of an HEV over a representative drive cycle can be formulated as an optimal control problem (OCP) and this problem can then be solved during the operation of the vehicle, using the existing theory, e.g. Pontryagin's Minimum Principle. The solution of such OCPs is discussed in Kim et al. [2009], Stockar et al. [2011], Kim et al. [2011] and is also implicitly connected to the well developed theory of equivalent consumption minimization strategies (ECMS) proposed in Paganelli et al. [2002] and expanded by Chen and Salman [2005], Musardo et al. [2005] among others.

However, the existence of both, continuous and discrete controls, such as gear shifts and drive mode, make the OCP much harder to solve. The mathematical derivation of such a system can be classified as hybrid system and the corresponding control problem as hybrid optimal control problem (HOCP). A solution of this type of problem contains continuous controls $u(t)$ as well as a sequence of discrete decisions that can be expressed formally by a piecewise constant switching function $\sigma(t)$. Furthermore, the use of strategies based on online optimal control is yet prevented by the limited performance of the electronic control units (ECU) and the fact, that information on the future driving profile is required to solve the OCP.

As a consequence, most HEVs still rely on rule-based strategies. In this paper, it is demonstrated, how hybrid optimal control theory can be used to calculate an initial set of calibration parameters for a given HEV, considering continuous controls as well as discrete decisions. This parameter set is stored in the form of lookup tables (LUT), that are then evaluated during operation and the respective information is passed on to the lower layer controller structure. Because of the wide range of possible parameters, this analytical method of defining the paramters has big advantages in contrast to often cumbersome heuristic procedures.

An indirect variation of extremals algorithm is used to solve the HOCP efficiently. It is shown that the costate can be assumed to be constant without significant loss of accuracy. This assumption is also widely made in the literature (Kim et al. [2011]). In this paper we demonstrate how the results, especially the constant costate, can then be used to calculate LUTs for the optimal choice of

- torque-split between M/G and ICE
- gear choice
- drive mode.

The paper is structured as follows: In section 2, we outline the theory of hybrid optimal control problems and state necessary conditions for optimality. In section 3, a compact mathematical model for the underlying HEV is derived. An HOCP for the fuel-optimal operation of the HEV over a given cycle is defined in section 4. In section 5, the algorithm used for the solution of the HOCP is formulated and the results are analyzed. Section 6 shows the minor optimization error of an assumed constant costate. Finally, in section 7, it is demonstrated how the results can be used to calculate the LUTs listed above.

2. HYBRID OPTIMAL CONTROL PROBLEMS

We follow the definition of a hybrid system in Sager [2005], where the vector field $f : \mathbb{R}^{n+m} \times \mathbb{Z} \times [t_0, t_f] \rightarrow \mathbb{R}^n$, governing the evolution of the system's state, besides the state $x(t) \in \mathbb{R}^n$ and the continuous control $u(t) \in \mathbb{R}^m$, also depends on the piecewise constant function $\sigma(t) \in \mathbb{Z}$:

$$\dot{x}(t) = f(x(t), u(t), \sigma(t), t) \quad (1)$$
$$x(t_0) = x_0. \quad (2)$$

In this case we assume a given initial state x_0. The time just before a change in the switching function $\sigma(t)$ occurs is defined by t_j^- and the time just after a change by t_j^+. In this paper, we regard systems with continuous states and controlled switchings, meaning that the vector field f changes discontinuously only in response to a commanded change in $\sigma(t)$ (Branicky and Mitter [1995]). The continuous control $u(t)$ as well as the discrete control $\sigma(t)$ are constrained by the functions

$$c_u(u(t), t) \leq 0, t \in [t_0, t_f] \quad (3)$$
$$c_\sigma(\sigma(t), t) \leq 0, t \in [t_0, t_f]. \quad (4)$$

The set of feasible continuous controls can then be defined as $\mathcal{U} = \{u | c_u(u(t), t) \leq 0\}$ and the set of feasible discrete controls as $\Theta = \{\sigma | c_\sigma(\sigma(t), t) \leq 0\}$.

With the cost-function $\phi : \mathbb{R}^n \rightarrow \mathbb{R}$, the HOCP can then be defined as

$$\min_{u(t) \in U, \sigma(t) \in \Theta} \phi(x(t_f)). \quad (5)$$

The functions $c_x : \mathbb{R}^n \times [t_0, t_f] \rightarrow \mathbb{R}^n$ and $\psi : \mathbb{R}^n \rightarrow \mathbb{R}^n$ impose general and final state constraints

$$c_x(x(t), t) \leq 0, t \in [t_0, t_f] \quad (6)$$
$$\psi(x(t_f)) = 0 \quad (7)$$

on the dynamical system.

To allow for a compact description of necessary conditions for optimality, the Hamiltonian function is defined as

$$H(x(t), u(t), \sigma(t), \lambda(t), t) := \lambda^T(t) f(x(t), u(t), \sigma(t), t) \quad (8)$$

where the time dependent multiplier $\lambda \in \mathbb{R}^n$ is called the costate. The following necessary conditions for optimality can be stated (Bryson and Ho [1975], Riedinger et al. [1999], Shaikh [2004]) for any $t \in [t_0, t_f]$ for an unconstrained arc ($c_x(x(t), t) \leq 0$):

- There is a costate $\lambda(t)$ governed by the differential equation
$$\dot{\lambda}(t) = -\nabla_x^T H(x(t), u(t), \sigma(t), \lambda(t), t) \quad (9)$$
- For almost all $t \in [t_0, t_f]$ the Hamiltonian function fulfills
$$H(t) = \min_{u \in \mathcal{U}, \sigma \in \Theta} H(x(t), u(t), \sigma(t), \lambda(t), t) \quad (10)$$
- The transversality conditions
$$\lambda_i(t_f) = \frac{\partial \phi(x(t_f))}{\partial x_i(t_f)} - \nu_i \frac{\partial \psi(x(t_f))}{\partial x_i(t_f)} \quad (11)$$
apply for all $i = 1, \ldots, n$, where ν_i are additional Lagrange-multipliers
- At a switching time t_j for a controlled switching
$$\lambda(t_{j+}) = \lambda(t_{j-}) \quad (12)$$
$$H(t_{j+}) = H(t_{j-}) \quad (13)$$
holds.

3. SYSTEM DESCRIPTION

The parallel hybrid powertrain configuration, for which the HOCP is to be solved, adds an additional electrical M/G to the conventional powertrain consisting of ICE, clutch, gearbox and differential. The M/G is installed between clutch and gearbox. Thus, pure electric driving and hybrid driving are possible. Figure 1 depicts the underlying powertrain.

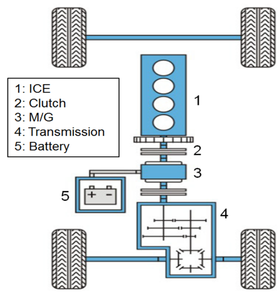

Fig. 1. Configuration of the parallel hybrid powertrain

A quasi-stationary model is sufficient for modeling the vehicle with appropriate accuracy. The active drive modes at given time t will be denoted by the binary variable

$$\zeta(t) = \begin{cases} 0 & \text{, pure electric mode} \\ 1 & \text{, hybrid mode.} \end{cases} \quad (14)$$

The vehicle has an automatic gearbox with six gears, whose gear-numbers are included in the set $K = \{1, 2, .., 6\}$. The active gear at time t is given by the discrete function $\kappa(t) \in K$. Consequently, the discrete decisions at time t can be identified by the function

$$\sigma(t) = 6 \cdot \zeta(t) + \kappa(t), \tag{15}$$

that assigns a unique value $\sigma(t) \in \Theta = \{1, 2, \ldots, 12\}$ to every possible combination of gear and drive mode. The required wheel torque T_{req} can be calculated with the help of a longitudinal vehicle dynamics model (see Mitschke and Wallentowitz [2004]). The input torque of the gearbox T_{clth} is obtained as follows:

$$T_{clth} = \frac{T_{req}(t)}{i_{gbx}(\kappa(t))} + T_{loss}(\kappa(t), T_{req}(t), n_{wh}(t)), \tag{16}$$

with the wheel speed n_{wh}, the gearbox ratio i_{gbx} and the powertrain friction losses T_{loss}. For the corresponding speed $n_{clth} = n_{wh} \cdot i_{gbx}(\kappa)$ applies. T_{clth} needs to be supplied in sum by ICE and M/G at any time:

$$T_{clth}(t) = T_{ice}(t) + T_{mg}(t). \tag{17}$$

Within its limits, the torque-split between ICE and M/G is variable. Hence, the control $u(t)$ is defined as

$$u(t) = T_{ice}(t). \tag{18}$$

During pure electric drive mode, the ICE is disconnected from the powertrain by a clutch and switched off. In this case

$$u(t) = T_{ice}(t) \overset{!}{=} 0 \tag{19}$$

applies. The corresponding speeds are

$$n_{mg}(t) = n_{clth}(t) \tag{20}$$

$$n_{ice}(t) = \begin{cases} 0 & , \zeta(t) = 0 \\ n_{clth} & , \zeta(t) = 1. \end{cases} \tag{21}$$

The electrical system of the hybrid vehicle can be modeled as follows: The lithium ion battery is modeled using a simple circuit consisting of an ideal voltage source and an internal resistance. The open circuit voltage V_{OC} is a function of the battery state of charge ($SoC(t)$). The internal battery resistance R_i is assumed to be constant in the allowed SoC-range. This assumption holds for the most modern battery types for hybrid vehicles. Considering the power losses caused by the battery's internal resistance R_i, one obtains

$$P_{batt} - R_i \cdot I^2 = V_{OC} \cdot I, \tag{22}$$

where I is the battery current and P_{batt} is the sum of the electrical power P_{mg} of the M/G and the power required to supply the electrical on board system, P_{on}

$$P_{batt} = -P_{mg}(T_{mg}, n_{mg}) - P_{on}. \tag{23}$$

$P_{mg}(T_{mg}(t), n_{mg}(t))$ is given by a smooth map. Solving this equation for the battery current I yields

$$I = \frac{-V_{OC} + \sqrt{V_{OC}^2 + 4R_i \cdot P_{batt}}}{2R_i}. \tag{24}$$

The differential equation for the SoC can then be written using (24) as

$$\dot{SoC} = \frac{100}{Q_{batt}} \cdot I(SoC(t), u(t), \sigma(t), t), \tag{25}$$

where Q_{batt} is the maximal battery capacity. The fuel consumption can be calculated by the differential equation

$$\dot{\beta} = \gamma \cdot bsfc(T_{ice}(t), n_{ice}(t)) \cdot T_{ice}(t) \cdot n_{ice}(t) \tag{26}$$

$$\beta(t_0) = 0, \tag{27}$$

with the brake specific fuel consumption $bsfc$ and γ being a product of natural constants. Both differential equations are concatenated as state $x(t)$, governed by the differential equation system

$$\dot{x} = \begin{bmatrix} \dot{SoC} \\ \dot{\beta} \end{bmatrix}, \qquad x(t_0) = \begin{bmatrix} SoC_0 \\ 0 \end{bmatrix}. \tag{28}$$

The initial state SoC_0 is predefined.

4. PROBLEM FORMULATION

Given the dynamical system (28) with continuous and discrete dynamics, the optimization task is

$$\min_{u(t) \in \mathcal{U}, \sigma(t) \in \Theta} \beta(t_f) \tag{29}$$

in compliance with the final state constraint

$$\psi(SoC(t_f)) = SoC(t_f) - SoC(t_0) = 0, \tag{30}$$

the general state constraints

$$c_x(SoC(t)) = \begin{pmatrix} c_{x1}(t) \\ c_{x2}(t) \end{pmatrix} := \begin{pmatrix} SoC(t) - SoC_{max} \\ SoC_{min} - SoC(t) \end{pmatrix} \tag{31}$$

and the control restraints

$$c_u(u(t), t) := \begin{pmatrix} u(t) - u_{max}(t) \\ u_{min}(t) - u(t) \end{pmatrix}. \tag{32}$$

5. SOLVING THE HOCP

The algorithm used to solve the HOCP is a variation of extremals algorithm as in Oberle and Grimm [1989] expanded for HOCPs. The algorithm is explained in detail in Schori et al. [2013]. Evaluating condition (8) and the transversality conditions (11) for the given system, results in the Hamiltonian

$$H = \gamma \cdot bsfc \cdot T_{ice} \cdot n_{ice} + \lambda \cdot \frac{100}{Q_{batt}} I. \tag{33}$$

Evaluating the transversality conditions, the costate variable for the first term in the Hamiltonian function results to 1 and is therefore omitted. Applying the necessary conditions for optimality to the given system, a two-point boundary value problem (TBVP) results. With an initial guess λ_0 of $\lambda(t_0)$, the problem is reduced to an initial value problem (IVP) of the form

$$\dot{y} = \begin{bmatrix} \dot{SoC} \\ \dot{\lambda} \end{bmatrix}, \qquad y(t_0) = \begin{bmatrix} SoC_0 \\ \lambda_0 \end{bmatrix} \tag{34}$$

with the time-derivative $\dot{\lambda}$ given by (9). The IVP can then be solved numerically by an appropriate solver, e.g. Euler's method, on a time-grid. The controls $u(t_k)$ and $\sigma(t_k)$ at each step are calculated as follows: For each $\sigma \in \Theta$ the optimal continuous controls

$$u_\sigma(t_k) = \begin{cases} \arg\min_{u\in\mathcal{U}} H, & c_x(t_k) < 0 \\ \arg\min_{u\in\mathcal{U}, I\leq 0} H, & c_{x1}(t_k) \geq 0 \\ \arg\min_{u\in\mathcal{U}, I\geq 0} H, & c_{x2}(t_k) \geq 0 \end{cases} \quad (35)$$

and the corresponding values of the Hamiltonian function $H(SoC(t_k), u_\sigma(t_k), \sigma(t_k), t_k)$ are calculated. $\sigma(t_k)$ is then chosen such that H_σ takes on the lowest possible value and hence condition (10) is fulfilled. Once the IVP has been solved, the function

$$\psi(\lambda_0) = SoC(t_f) - SoC(t_0) = 0 \quad (36)$$

can be evaluated. An optimal $\lambda(t_0)$ is then found by iteratively improving the initial guess of λ_0 such that condition (36) is satisfied. This is done numerically by finding a sequence $\lambda_{0,k}, k = 1, 2, \ldots$ such that ψ approaches 0 up to a desired exactness. The scalar equation (36) can robustly be solved with methods of the regula-falsi class, i.e. the Pegasus method (Dowell and Jarrat [1972]).

Figures 2 and 3 show the results obtained for the New European Driving Cycle (NEDC). In particular, Fig. 2 depicts the operation points of the ICE in the brake specific fuel-consumption map. It can clearly be noted that the discrete DOFs, namely gear-choice and drive mode are used to completely avoid the operation of the ICE at low load and hence at low efficiency. Experiments, where only the continuous control $u(t)$ was optimized with a fixed switching function $\sigma(t)$, obtained from a measurement of a near mass-production car, have shown to be less effective.

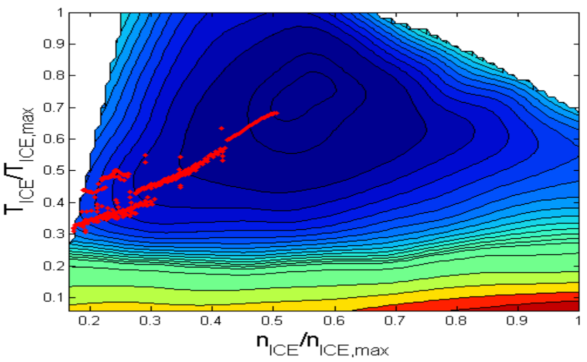

Fig. 2. Operation points of the ICE

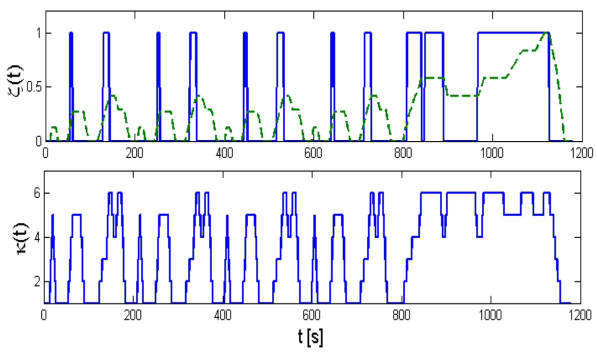

Fig. 3. Drive mode $\zeta(t)$ and gear choice $\kappa(t)$ over the NEDC (green dashed line)

6. CONSTANT COSTATE

In general, the costate is time-dependent and its time derivative is given by (9). For the HOCP stated in this paper, the time derivative on an unconstrained arc would yield

$$\dot{\lambda} = -\lambda \cdot \frac{100}{Q_{batt}} \cdot \frac{\partial I}{\partial SoC} \quad (37)$$

$$= -\lambda \cdot \frac{100}{Q_{batt}} \cdot \frac{\partial I}{\partial V_{OC}} \cdot \frac{dV_{OC}}{dSoC}. \quad (38)$$

For most battery types, V_{OC} changes only slightly in the allowed SoC range and hence the assumption

$$\frac{dV_{OC}}{dSoC} \approx 0 \quad (39)$$

holds. As a consequence the costate remains constant over the time-interval $[t_0, t_f]$. To demonstrate the negligible effect of this assumption, the HOCP was solved with and without the constant costate assumption for two different parallel HEV configurations and drive cycles. The resulting fuel consumptions can be seen in Table 1. However, by violating a necessary condition for optimality, the solution cannot be referred to as optimal but only as suboptimal.

HEV	drive cycle	fuel consumption [l] for continuous λ	fuel consumption[l] for constant λ
HEV1	NEDC	0.4055	0.4058
HEV1	FTP	0.3362	0.3366
HEV2	NEDC	0.3498	0.3498
HEV2	FTP	0.2914	0.2918

Table 1. Effect of the constant costate assumption

7. ENERGY MANAGEMENT PARAMETER CALCULATION

In Fig. 4, a sketch of the LUT-based energy management is depicted. The reference values κ, ζ and T_{mg}, supplied to the lower level controller structure will be determined by LUTs. In this section it is demonstrated, how the corresponding LUTs can be calculated automatically from the HOCP solution.

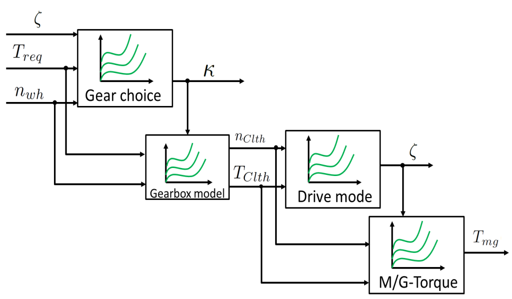

Fig. 4. Schematic of a lookup table based energy management

An important result of the optimization is the value of the constant costate λ. Once the costate is known, the value of the Hamiltonian function only depends on the clutch torque T_{clth}, the clutch speed n_{clth} and the control T_{ice}. Consequently, a LUT with suboptimal values of

$T_{ice}(n_{clth}, T_{clth})$ during hybrid mode can be generated by calculating

$$LUT_1 : \hat{T}_{ice} := \hat{u} = \arg\min_{u \in \mathcal{U}} H(n_{clth}, T_{clth}, \lambda, u) \quad (40)$$

on a grid of (n_{clth}, T_{clth}). In automotive practice, it is common to store the desired torques T_{mg} in a LUT instead of T_{ice}. This transformation can easily be performed with the help of equation (17). One LUT is exemplarily shown in Fig. 5.

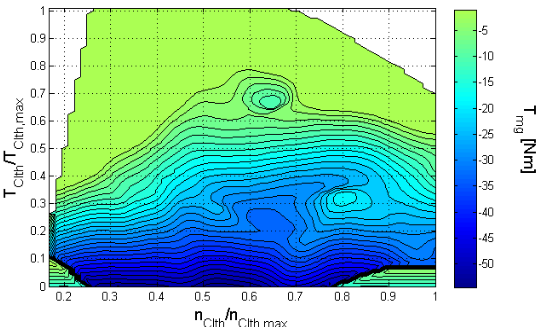

Fig. 5. LUT with suboptimal reference values for M/G-torque

In the next step, the LUT for the suboptimal choice of drive mode $\zeta(n_{clth}, T_{clth})$ can be calculated by

$$LUT_2 : \hat{\zeta} = \arg\min_{\zeta \in \{0,1\}} H(n_{clth}, T_{clth}, \lambda, T_{ice}), \quad (41)$$

on the same grid. Since now the drive mode ζ is not regarded as fixed, T_{ice} has the arguments $(n_{clth}, T_{clth}, \zeta)$. For $\zeta = 1$, $T_{ice}(n_{clth}, T_{clth})$ is determined from LUT_1, otherwise $T_{ice} = 0$ applies. The results can be seen in Fig. 6.

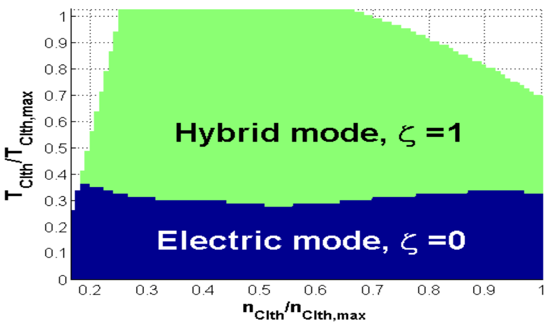

Fig. 6. LUT with suboptimal values for drive mode

For both driving modes, recommended gears $\kappa(T_{req}, n_{wh})$ can be calculated over a grid of (T_{req}, n_{wh}) as follows:

$$LUT_{3,4} : \hat{\kappa} = \arg\min_{\kappa \in K} H(n_{clth}, T_{clth}, \lambda, T_{ice}), \quad (42)$$

where n_{clth} depends on the wheel speed and the chosen gear and hence has the arguments (n_{wh}, κ). T_{clth} has the arguments (T_{req}, κ). $T_{ice}(n_{clth}, T_{clth})$ for $\zeta = 1$ is again determined from LUT_1 (40). Figures 7 and 8 depict LUTs of recommended gears over the grid of T_{req} and the vehicle velocity v in electric and hybrid drive mode.

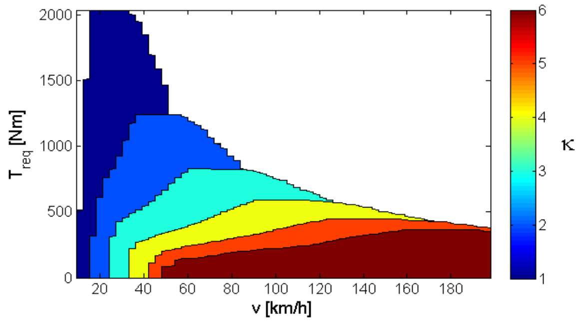

Fig. 7. LUT with suboptimal gear recommendation (hybrid mode)

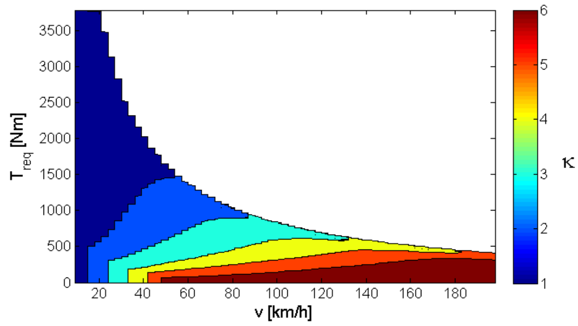

Fig. 8. LUT with suboptimal gear recommendation (electric mode)

Practical experience has shown that the LUTs 1 and 2 can usually be implemented in a HEV without further modification and have yielded significant reductions in fuel consumption. To determine the recommended gears however, additional constraints such as limitations due to driving comfort apply. These factors are hard to account for in a mathematical model. As a consequence the suboptimal recommendations cannot always be followed. In this case, it has shown to be helpful to evaluate the effect of deviating from the recommended LUT. If the values from the calculated LUTs are used, in general, equation (13) holds and the Hamiltonian is continuous during a change in the piecewise constant switching function σ, that is a transition from one drive mode to another or at gear changes. When deviating from the recommended transitions, a difference in the Hamiltonian

$$\Delta H = H(t_j^+) - H(t_j^-) \quad (43)$$

occurs. The meaning of this difference is twofold: On the one hand, it constitutes a deviation from the optimality conditions. On the other hand, with the interpretation of the Hamiltonian as weighted sum of battery current and fuel mass flow, it is indicated that a control with lower value of this weighted sum exists, but cannot be used. This is illustrated in Fig. 9, where the minimal values of the Hamiltonian in hybrid mode for gears 2 and 3 are depicted. The best point of switching from gear 2 to gear 3 would be at t_1 such that at any time $H(t)$ has the lowest value. However, because of some unknown constraint, a switching cannot occur until t_2. Therefore, a difference ΔH in the Hamiltonians exists at the switching time.

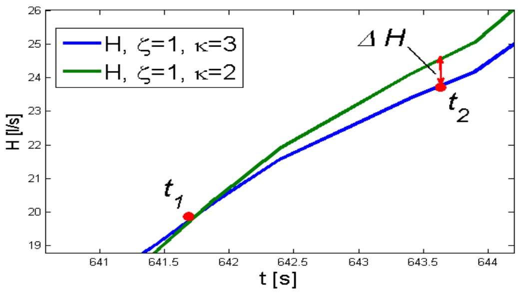

Fig. 9. Deviance from suboptimal switching

Since, with the constant costate assumption, the value of the Hamiltonian does not explicitly depend on time, but on the current driving situation, this view can be transferred to depictions of torque and speed. Figure 10 shows the absolute value $|\Delta H|$ depending on the wheel torque T_{req} and wheel speed n_{wh} for a change from gear 2 to gear 3. The recommended switching is, where the difference vanishes. If this recommended switching cannot be followed, the Figure allows for evaluation of the effects. A deviation from the recommended switching is more acceptable, when the value of $|\Delta H|$ is low.

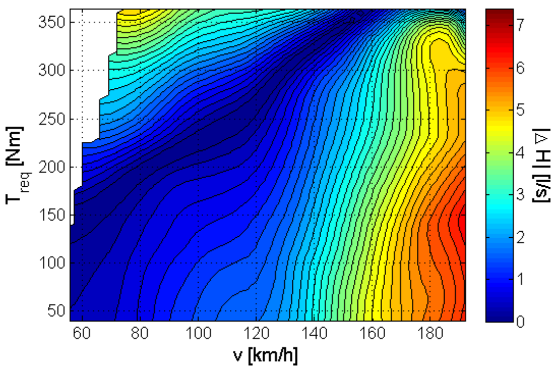

Fig. 10. $|\Delta H|$ between gears 2 and 3

8. CONCLUSION

This paper proposes a calculation method for lookup tables, needed to define the energy management of a HEV by first solving a HOCP to find the costate of the Hamiltonian function. The costate is assumed to be constant and hence the solution becomes suboptimal but this hardly affects the quality of the solution obtained. With the Hamiltonian only depending on the continuous controls u, and the discrete controls σ, for any driving situation suboptimal values for these controls can be stored in LUTs. Using the calculations proposed in this paper, a toolbox for the initial calibration of HEVs has been developed and successfully tested on several projects. Compared to iterative approaches, the analytical approach presented in this paper has shown the capability of dramatically reducing the time needed to obtain an initial calibration. Additional constraints, such as warm-up phases of the ICE can easily be implemented as fixed constraints.

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