

Nonlinear event-triggered tracking control of a mobile robot: design, analysis and experimental results

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Abstract: We consider the scenario where a controller communicates with a mobile robot via a network. Our objective is to design the control law such that the robot tracks a given reference trajectory while reducing the usage of the communication channel. For that purpose, we design a nonlinear event-triggered feedback law. We follow an emulation-like approach in the sense that we first synthesize the controller while ignoring the communication constraints and we then derive an appropriate triggering condition. We prove that the states of the robot's model practically converge towards the states of the reference system which generates the desired trajectory to be tracked, using an invariance principle for hybrid systems. The existence of a dwell-time between any two transmissions is discussed. We have implemented the proposed strategy on a benchmark where the controller is located on a remote computer which communicates with the mobile robot via a IEEE 802.11g wireless network. The proposed event-triggering strategy is shown to significantly reduce the need for communication compared to a classical time-triggered setup while ensuring similar, if not better, tracking performances.

1. INTRODUCTION

Event-triggered control is currently attracting a lot of attention because of its abilities to significantly reduce the need for communication in control systems, which is extremely useful in *networked control systems*. The idea is to transmit data between the controller and the plant whenever a state-dependent criterion is satisfied and not periodically as in traditional setups. In that way, the transmissions are adapted to the state of the system and they only occur when it is needed. Various techniques have been developed, see Arzén [1999], Astrom and Bernhardsson [2002], Heemels et al. [2009], Henningsson et al. [2008], Tabuada [2007] to mention a few. Most of them address the stabilization of equilibrium points, while very few controllers have been synthesized to stabilize time-varying trajectories, see *e.g.* Tallapragada and Chopra [2011]. It appears that tracking control induces additional difficulties as only *approximate* tracking can usually be ensured under communication constraints because of the time-varying component of the control law, see for more detail Postoyan et al. [2012]. As a consequence, available results on the stabilization of equilibrium points are not directly applicable in this context.

In this paper, we investigate the event-triggered tracking control of a mobile robot whose dynamics is described

by a nonholonomic system. We follow an emulation-like approach (see *e.g.* Tabuada [2007], Nešić and Teel [2004]). Thus, we first design the controller while ignoring the communication constraints and we derive the triggering strategy afterwards. We have selected the state-feedback controller of Jiang and Nijmeijer [1997] among others, because the law is continuous and an explicit Lyapunov function is provided. The continuity of the feedback law helps to guarantee the existence of a minimum amount of time between any two transmissions, under some conditions on the reference trajectories. This point is essential in practice as the hardware cannot transmit infinitely fast. The Lyapunov function is used to design the event-triggering condition. It has to be noted that it is a *weak* Lyapunov function in the sense that it does not strictly decrease along the solutions of the closed-loop system (in the absence of communication constraints), see Jiang and Nijmeijer [1997]. This is an important difference with existing Lyapunov-based event-triggered controllers, which rely on *strong* Lyapunov functions (see Tabuada [2007], Postoyan et al. [2011], Seuret and Prieur [2011] and the references therein).

We model the problem using the hybrid formalism of Goebel et al. [2012], like in Donkers and Heemels [2012], Forni et al. [2010], Postoyan et al. [2011], Seuret and Prieur [2011]. The event-triggering condition is inspired

by the technique in Anta and Tabuada [2012], Forni et al. [2010] where the Lyapunov function is forced to decrease at a certain rate, except that we exclude a region of the state-space, which we call a *dead-zone*, to allow the existence of dwell-times. We prove that the solutions of the robot's model converge to a neighborhood of the reference trajectory under some conditions on these references. This neighborhood depends on a tuneable parameter ε which defines the dead-zone of the triggering condition. The parameter ε can be used to adjust the accuracy of the tracking error at the price of more transmissions. The analysis relies on an invariance principle for hybrid systems developed in Sanfelice et al. [2007]. The existence of a dwell-time is discussed and sufficient conditions are given to ensure it. Finally, the designed controller has been implemented on a benchmark where the controller is located on a remote controller and communicates with the robot via a IEEE 802.11g wireless network. The obtained results show that we can significantly reduce the amount of transmission compared to periodic sampling while ensuring similar, if not better, performances. Notice that we do not rely on a local linear approximation of the plant's dynamics as in other studies where experimental tests have been lead (see Heemels et al. [1999], Lehmann and Lunze [2011], Henningson and Cervin [2009], Téllez-Guzmán et al. [2012]) but on a global nonlinear model.

The paper is organized as follows. In Section 2, the controller of Jiang and Nijmeijer [1997] is recalled and the hybrid model is introduced. The event-triggering condition is designed in Section 3 where the analytical results are presented. Section 4 shows the experimental results. The proofs have been omitted for space reasons.

Notation. Let $\mathbb{R} = (-\infty, \infty)$, $\mathbb{R}_{\geq 0} = [0, \infty)$, $\mathbb{R}_{> 0} = (0, \infty)$, $\mathbb{Z}_{\geq 0} = \{0, 1, 2, \dots\}$, $\mathbb{Z}_{> 0} = \{1, 2, \dots\}$. For $(x, y) \in \mathbb{R}^{n+m}$, the notation (x, y) stands for $[x^T, y^T]^T$. For a piecewise continuous function $f : \mathbb{R} \rightarrow \mathbb{R}^n$, $f(t^+)$ stands for $\lim_{s \rightarrow t, s > t} f(s)$ for $t \in \mathbb{R}$. We recall that the function sinc is defined from \mathbb{R} to \mathbb{R} by $\text{sinc}(x) = \frac{\sin x}{x}$ when $x \neq 0$, and $\text{sinc}(0) = 1$, and it is continuously differentiable with the derivative $\text{sinc}'(x) = \frac{x \cos x - \sin x}{x^2}$ when $x \neq 0$, and $\text{sinc}'(0) = 0$.

2. SYSTEM MODEL

2.1 Jiang and Nijmeijer's controller

We consider a mobile robot for which the dynamics are defined by the following nonholonomic system

$$\dot{x} = v \cos(\theta), \quad \dot{y} = v \sin(\theta), \quad \dot{\theta} = w, \quad (1)$$

where (x, y) are the Cartesian coordinates and θ is the angle between the heading direction and the x -axis, (v, w) denotes the control input vector, see Figure 1. Our objective is to make the system (1) track a given trajectory (x_r, y_r, θ_r) . We restrict our attention to the case where (x_r, y_r, θ_r) satisfies the following dynamical equations

$$\dot{x}_r = v_r \cos(\theta_r), \quad \dot{y}_r = v_r \sin(\theta_r), \quad \dot{\theta}_r = w_r, \quad (2)$$

where v_r and w_r are the feedforward inputs to system (2). In practice, the term w_r is obtained by differentiating θ_r . The term v_r is deduced by using that $v_r^2 = (\dot{x}_r)^2 + (\dot{y}_r)^2$ and its sign is obtained by investigating the first equations

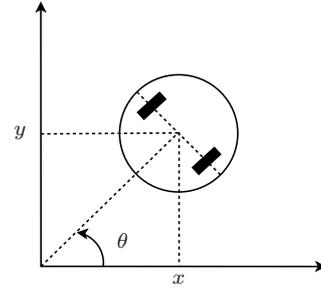


Fig. 1. Mobile robot.

of (2) for the given trajectory (x_r, y_r, θ_r) . We refer to (2) as the *reference system*. We use the following error coordinates (see Kanayama et al. [1990])

$$\begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{pmatrix} \quad (3)$$

which allow us to derive the error system

$$\begin{aligned} \dot{x}_e &= w y_e - v + v_r \cos(\theta_e) \\ \dot{y}_e &= -w x_e + v_r \sin(\theta_e) \\ \dot{\theta}_e &= w_r - w. \end{aligned} \quad (4)$$

It has to be noticed that the change of coordinates in (3) is invertible and that the transformation matrix and its inverse are bounded. As a consequence, if we can guarantee that x_e, y_e, θ_e converge to a neighborhood of the origin, then so do $x_r - x, y_r - y, \theta_r - \theta$. In Jiang and Nijmeijer [1997], the following controller is shown to ensure the global convergence of x_e, y_e, θ_e towards the origin (under some conditions on the feedforward terms v_r and w_r)

$$v = v_1 + c_4 \bar{x}_e, \quad w = w_r + \gamma y_e v_r \text{sinc} \theta_e + c_5 \gamma \theta_e, \quad (5)$$

where $c_4, c_5, \gamma > 0$ are design parameters and

$$\begin{aligned} \bar{x}_e &:= x_e - c_3 w y_e \\ v_1 &:= v_r \cos \theta_e - c_3 \dot{w} y_e + c_3 w (w x_e - v_r \sin \theta_e) \end{aligned} \quad (6)$$

with $c_3 > 0$. We do not necessarily require to differentiate online \dot{w} to compute the term v_1 in (6) as we can replace \dot{w} by its algebraic expression by using (1), (2) and (5).

2.2 Assumptions on the feedforward terms

In Section 3, we will use an invariance principle developed in Sanfelice et al. [2007] to prove some stability properties hold for the event-triggered controlled system. We will need for that to work with an autonomous system which generates solutions that have a compact range (see Sanfelice et al. [2007]) which loosely speaking means that it exhibits bounded solutions. We see that the system (4) is time-varying and thus non-autonomous in general when the terms v_r and w_r are time-varying. The assumption below allows us to derive an autonomous system from (4) under some conditions on the feedforward terms v_r and w_r .

Assumption 1. The signals v_r and w_r can be written as the output of a bounded autonomous dynamical system i.e. there exist a continuous vector field $f_\chi : \mathbb{R}^{n_\chi} \rightarrow \mathbb{R}^{n_\chi}$, a continuously differentiable vector field $u : \mathbb{R}^{n_\chi} \rightarrow \mathbb{R}^2$ (where $n_\chi \in \mathbb{Z}_{> 0}$) and $\chi_0 \in \mathbb{R}^{n_\chi}$ such that

$$(v_r(t), w_r(t)) = u(\chi(t)) \quad \forall t \geq 0 \quad (7)$$

where $\chi(t)$ is the solution of $\dot{\chi} = f_\chi(\chi)$ with initial condition $\chi(0) = \chi_0$ and such that the closure of the set $\{z \in \mathbb{R}^{n_\chi} : \exists t \geq 0 \ z = \chi(t)\}$ is compact. \square

Assumption 1 allows us to transform the system (4) into an autonomous system with the extended state $(x_e, y_e, \theta_e, \chi)$ and v_r, w_r being functions of χ . One might argue that it suffices to introduce the time as an extra variable to derive an autonomous system. However, the solution of the system will then not be bounded because of the time variable. That is the reason why we have imposed Assumption 1. It has to be emphasized that our results apply to a *class* of reference trajectories (which are solutions to (2) and of the χ -system of Assumption 1), rather than a single trajectory. A simple sufficient condition to verify the satisfaction of Assumption 1 is given below.

Lemma 2. If there exist $\alpha_1, \dots, \alpha_m, \beta_1, \dots, \beta_m \in \mathbb{R}$ with $m \in \mathbb{Z}_{>0}$ and a continuously differentiable vector field $u : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^2$ such that $(v_r(t), w_r(t)) = u(\sin(\alpha_1 t + \beta_1), \dots, \sin(\alpha_m t + \beta_m))$ for all $t \geq 0$, then Assumption 1 holds. \square

It is important to emphasize that we do not need to explicitly build up the χ -system of Assumption 1 in the remaining part of the paper but we only need to make sure that such a system does exist. Assumption 1 implies the following properties which correspond to the first conditions of Proposition 2 in Jiang and Nijmeijer [1997].

Lemma 3. Under Assumption 1, there exists an initial condition $\chi(0) \in \mathbb{R}^{n_x}$ for the χ -system of Assumption 1 such that the signals v_r, w_r, \dot{v}_r and \dot{w}_r are bounded on $[0, \infty)$ by $M > 0$. \square

We also make the following assumption on the feedforward terms which is satisfied by the reference trajectories studied in Section 4.

Assumption 4. One of the following conditions needs to hold. (i) The signal w_r is not constantly equal to 0, i.e. $\exists t \geq 0$ such that $w_r(t) \neq 0$. (ii) The signal v_r is not constant, i.e. $\exists t_1, t_2 \geq 0$ such that $v_r(t_1) \neq v_r(t_2)$. \square

These assumptions are used to ensure the convergence of the states of the robot's model (1) towards the state of the reference system (2). These are different from the conditions of Proposition 2 in Jiang and Nijmeijer [1997] where v_r or w_r is required not to converge to zero. We do tolerate v_r and w_r to converge to zero but w_r cannot be constantly equal to zero or v_r cannot be constant. These conditions come from the stability analyzes which are different: Barbalat's lemma is used in Jiang and Nijmeijer [1997] while invariance principles will be invoked here.

2.3 Controller implementation & hybrid model

We consider the scenario where the controller (v, w) in (5) is digitally implemented and communicates with the robot (1) via a network, see Figure 2. We assume that the controller has access to the robot's measurements, the reference trajectory and the feedforward terms v_r and w_r . Our objective is to reduce the usage of the network from the controller to the robot while ensuring the convergence of the robot's states towards the reference trajectory. We will design for that purpose an event-triggering condition which will decide when the robot's input needs to be updated. We write the obtained model using the formalism of Goebel et al. [2012] like in Donkers and Heemels [2012], Forni et al. [2010], Postoyan et al. [2011], Seuret and Prieur [2011]

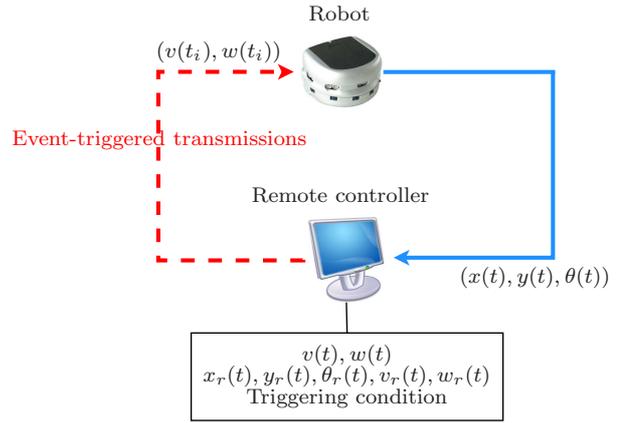


Fig. 2. Controller implementation.

$$\left. \begin{aligned} \dot{x}_e &= (w + e_w)y_e - v - e_v + v_r \cos(\theta_e) \\ \dot{y}_e &= -(w + e_w)x_e + v_r \sin(\theta_e) \\ \dot{\theta}_e &= w_r - w - e_w \\ \dot{e}_w &= f_w(q) \\ \dot{e}_v &= f_v(q) \\ \dot{\chi} &= f_\chi(\chi) \end{aligned} \right\} q \in C, \quad (8)$$

$$\left. \begin{aligned} x_e^+ &= x_e \\ y_e^+ &= y_e \\ \theta_e^+ &= \theta_e \\ e_w^+ &= 0 \\ e_v^+ &= 0 \\ \chi^+ &= \chi \end{aligned} \right\} q \in D$$

where $q := (x_e, y_e, \theta_e, e_w, e_v, \chi)$, $e_v := \hat{v} - v$ and $e_w := \hat{w} - w$ (where \hat{v}, \hat{w} are respectively the values of v, w at the last jump instant), χ comes from Assumption 1 and v, w are defined in (5). The sets C and D are closed and are defined by the triggering condition. Indeed, the set C corresponds to the region of the state-space where the triggering condition is not satisfied and D is defined as the region of the state-space where the triggering condition is verified. In that way, defining the triggering condition is equivalent to defining the sets C and D . We recall that when $q \in C \cap D$, the system can either jump or flow, the latter only if flowing keeps q in C . For more detail about hybrid systems of the form of (8), see Goebel et al. [2012]. The vector fields f_w and f_v are respectively obtained by computing the algebraic expressions of $-\dot{w}$ and $-\dot{v}$ using (5) and (8). We use the notation $\phi = (\phi_{x_e}, \phi_{y_e}, \phi_{\theta_e}, \phi_{e_w}, \phi_{e_v}, \phi_\chi)$ to denote a solution to (8) in the following in order to avoid any confusion with the state vector $q = (x_e, y_e, \theta_e, e_w, e_v, \chi)$. We consider the term v_r and w_r in (8) as functions of χ . In that way, the system (8) is autonomous. We recall the following definitions which will be used in the following.

Definition 5. (Goebel et al. [2008]). We say that a solution ϕ to (8) is:

- **complete** if its domain¹ $\text{dom } \phi$ is unbounded.
- a **dwell-time solution** if there exists $\tau \in \mathbb{R}_{>0}$ such that for any $j \in \mathbb{Z}_{\geq 0}$, $t_{j+1} - t_j \geq \tau$ where $\text{dom } \phi = \bigcup ([t_j, t_{j+1}], j)$. \square

For the sake of convenience, we denote

$$\dot{q} = f(q) \quad q \in C, \quad q^+ = g(q) \quad q \in D. \quad (9)$$

¹ The domain of ϕ is the subset of $\mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ where ϕ is defined, see Goebel et al. [2012] for more detail.

We will now design the triggering condition, i.e. the sets C and D in (8), such that the errors x_e, y_e, θ_e practically converge to the origin and such that the usage of the communication channel, i.e. the number of jumps for system (8), is minimized. The tuneable parameter will be given by the triggering condition.

3. EVENT-TRIGGERED TRACKING CONTROL

3.1 Design of the triggering condition

We follow the main steps of the proof of Proposition 2 in Jiang and Nijmeijer [1997] to build up the event-triggering condition. The Lyapunov-like candidate function below is considered in Jiang and Nijmeijer [1997]

$$V(q) = \frac{1}{2}\bar{x}_e^2 + \frac{1}{2}y_e^2 + \frac{1}{2\gamma}\theta_e^2, \quad (10)$$

where \bar{x}_e is defined in (6) and γ comes from (5). By differentiating the function V along the vector fields of the system (9) on the set C , we obtain, by following similar lines as in Jiang and Nijmeijer [1997],

$$\langle \nabla V(q), f(q) \rangle = -\Sigma(q) + \Lambda(q) \quad (11)$$

where $\Sigma(q) := c_3 w^2 y_e^2 + c_4 \bar{x}_e^2 + c_5 \theta_e^2$ and $\Lambda(q) := \bar{x}_e (e_w y_e - e_v - c_3 e_w \rho(x_e, y_e, \theta_e, v_r) y_e + c_3 w e_w x_e) - y_e e_w x_e - \frac{1}{\gamma} \theta_e e_w$, with $\rho(x_e, y_e, \theta_e, v_r) := -\gamma x_e v_r \text{sinc} \theta_e - (\gamma y_e v_r \text{sinc}'(\theta_e) + c_5 \gamma)$. We note that we recover (41) in Jiang and Nijmeijer [1997] when $e_v = e_w = 0$ (as we have in this case $\Lambda(q) = 0$).

We use (11) to design the triggering condition. We define the flow and the jump sets as follows

$$\begin{aligned} C &:= \left\{ q : \langle \nabla V(q), f(q) \rangle \leq -\sigma \Sigma(q) \text{ or } V(q) \leq \varepsilon^2 \right\} \\ D &:= \left\{ q : \langle \nabla V(q), f(q) \rangle \geq -\sigma \Sigma(q) \text{ and } V(q) \geq \varepsilon^2 \right\} \end{aligned} \quad (12)$$

where $\sigma \in (0, 1)$ and $\varepsilon > 0$ are design parameters. The idea is to force V to decrease with the rate $\sigma \Sigma(q)$ like in Tabuada [2007], where $\sigma \in (0, 1)$. We take $\sigma \in (0, 1)$ in order to allow some flow between two jumps, see e.g. Tabuada [2007] or Section III in Anta and Tabuada [2012]. In addition, we impose that condition only when $V(q) \geq \varepsilon^2$ as we cannot otherwise guarantee the existence of dwell-time solutions to (8) in general, as discussed in more detail in Section 3.3. It has to be noticed that $C \cup D = \mathbb{R}^{n_a}$.

Similar ideas are followed in Forni et al. [2010], Anta and Tabuada [2012], Postoyan et al. [2011], Seuret and Prieur [2011] to design the triggering condition. Nevertheless, none of the strategies proposed in these papers directly applies to our problem. Indeed, the triggering condition is designed based on a weak Lyapunov function V (as opposed to strong Lyapunov functions) which induces non-trivial difficulties to prove stability: we need to resort to invariance principles for hybrid systems (see Sanfelice et al. [2007]) and the existence of dwell-times is a priori not ensured for every solution. We therefore have to modify the policy and to derive a different stability analysis.

3.2 Stability results

We are ready to state the main results of this section.

Theorem 6. Consider the system (8) and suppose Assumptions 1 and 4 are satisfied, then the following holds.

- (i) Any solution to (8) is complete.
- (ii) For any $\Delta \geq 0$ there exists $\bar{\Delta} \geq 0$ such that for any solution ϕ to (8) with $|\phi(0,0)| \leq \Delta$, it holds that $|\phi(t,j)| \leq \bar{\Delta}$ for all $(t,j) \in \text{dom } \phi$.
- (iii) Any dwell-time solution to (8) with $\phi_{e_w}(0,0) = \phi_{e_v}(0,0) = 0$ approaches³ the set $\{q : \max\{|x_e|, |y_e|, |\theta_e|\} \leq \bar{M}\varepsilon\}$ with $\bar{M} = \sqrt{2} \max\{1, \sqrt{\gamma}, M + \gamma\sqrt{2}M + c_5\gamma^{\frac{3}{2}}\sqrt{2}\}$ where γ, c_5 and M come from (5) and Lemma 3. \square

Item (ii) of Theorem 6 ensures that the states of system (8) are bounded. Item (iii) of Theorem 6 guarantees that any dwell-time solution to (8) converges to a neighborhood of the origin whose ‘size’ depends on the design parameter ε . Therefore, by taking ε small, we ensure the convergence of the states of the robot closer to the reference trajectory. Nevertheless, this may give rise to more transmissions as discussed in Section 4. The fact that the solution needs to start from zero for the e_v and e_w variables in item (iii) of Theorem 6 adds no conservatism. Indeed, it suffices to transmit the control input to the robot once at the beginning of the experiment and to take this time to be the initial time to guarantee it (note that the system (8) satisfies the semi-group property, see Sanfelice et al. [2007]). It has to be noticed that Theorem 6 does not ensure the existence of dwell-time solutions to (8): this is discussed in the following.

3.3 Existence of dwell-time solutions

First, note that the system (8) may generate Zeno solutions (i.e. solutions which constantly jump) since $g(D) \cap D \neq \emptyset$. Indeed, take $q \in \mathcal{A}$ where $\mathcal{A} := \{q : \Sigma(q) = 0 \text{ and } V(q) \geq \varepsilon^2\}$. Then $q \in D$ and $G(q) \in D$. As a consequence, we cannot guarantee that every solution to (8) is a dwell-time solution in general. The following proposition provides sufficient conditions to ensure that there always exist dwell-time solutions to (8).

Proposition 7. Consider system (8) and suppose the following holds for a solution ϕ to (8).

- (i) Assumptions 1 and 4 hold.
- (ii) There exists $\delta > 0$ such that $|w_r(t,j)| \geq \delta$ for all $(t,j) \in \text{dom } \phi$.
- (iii) The initial condition $\phi(0,0)$ is such that $|(\phi_{x_e}(0,0), \phi_{y_e}(0,0), \phi_{\theta_e}(0,0))| \leq \Delta$ where $\Delta \geq 0$ and $\gamma(M + c_5) \max\{\Delta(2 + c_3(M + \gamma\Delta M + c_5\gamma\Delta)), \sqrt{\underline{\gamma}^{-1}\bar{\gamma}}, \varepsilon\sqrt{\underline{\gamma}^{-1}}\} < \delta$, with $\underline{\gamma} = \frac{1}{2} \min\{1, \gamma\}$ and $\bar{\gamma} = \frac{1}{2} \max\{1, \gamma\}$ and where γ, c_5, c_3, M come from (5), (6) and Lemma 3.

Then ϕ is a dwell-time solution. \square

Item (ii) of Proposition 7 means that the feedforward term w_r has the same sign for all time and does not converge to the origin. It is satisfied in scenarios where the reference trajectory follows an ellipsoidal path or a

² Throughout the paper, the initial condition $\phi(0,0)$ is always assumed to be such that $\phi_\chi(0,0) = \chi_0$ where χ_0 comes from Assumption 1.

³ A solution ϕ to (8) approaches the set $\mathcal{A} \subset \mathbb{R}^n$ if for all $\varepsilon > 0$, there exists $(\bar{t}, \bar{j}) \in \text{dom } \phi$ such that for all (t,j) satisfying $(t,j) \succ (\bar{t}, \bar{j})$, $(t,j) \in \text{dom } \phi$, $\phi(t,j) \in \mathcal{A} + \varepsilon \mathbb{B}$ where \mathbb{B} is the unitary ball centered at the origin.

circular path. The constant δ can be computed based on the available expression of w_r . We argue that item (iii) of Proposition 7 can always be verified by selecting γ sufficiently small.

4. EXPERIMENTAL VALIDATION

We start by briefly describing the benchmark depicted in Figure 2. The robot is a Khepera⁴ III provided by k-team, which is equipped with an onboard computing platform with wireless capabilities. Its coordinates are obtained by odometry. The communication is ensured via a IEEE 802.11g wireless network. As a consequence, the event-triggered controller, which is located on a remote computer, verifies the transmission condition periodically every $T_{\text{eval}} = 20$ ms, and not continuously⁵ as in Section 2-3. This is justified by the fact that the time-interval between each position update varies from 10 to 15 ms. Moreover, the computation time at each step is less than 1 ms. In addition, the time for delivering a new control input is less than 5 ms. We have verified in experiments that the value of T_{eval} is sufficiently small to maintain the properties ensured by Theorem 6. The reduction of the usage of the communication channel only concerns the transmissions from the controller to the robot, which corresponds to the dashed channel in Figure 2 (and not from the robot to the controller where periodic sampling is applied). The following parameters have been selected for the control law in (5): $c_3 = 0.7$, $c_4 = 0.5$, $c_5 = 0.8$, $\gamma = 1$.

Two trajectories have been considered for which the paths are respectively an ellipse and a lemniscate. Assumptions 1 and 4 are guaranteed by using Lemma 2. The triggering condition (12) has been implemented with $\sigma = 0.5$ and we have studied the influence of ε . Simulations have been run over $T_{\text{simulation}} = 60$ s. We have quantified the usage of the communication network with the percentage:

$$r := 100 \times \text{number of control updates} \times \frac{T_{\text{eval}}}{T_{\text{simulation}}}.$$

The performance of the controller is evaluated by the constant e which is the average value of $|(x - x_r, y - y_r, \theta - \theta_r)|$ over the second half of the experiment, i.e.

$$e := \frac{2}{T_{\text{simulation}}} \int_{\frac{1}{2}T_{\text{simulation}}}^{T_{\text{simulation}}} \left| \begin{pmatrix} x(t) - x_r(s) \\ y(t) - y_r(s) \\ \theta(t) - \theta_r(s) \end{pmatrix} \right| ds.$$

In that way, e quantifies the tracking accuracy of the robot after the transient period. Table 1 shows that the controller only uses the communication channel 13.59% of the time, in the worst case, and that this quantity can be significantly decreased by varying ε . Thus, we see that to increase ε reduces the usage of the communication channel at the price of degraded tracking performances. It would be interesting to compare the event-triggered controller with a time-triggered one. Unfortunately, we are not aware of any analytical result which allows to derive an upper bound on the sampling period at which the controller (5) should be *periodically* emulated (this is again due to the fact that the Lyapunov function V in (10) is weak). Nevertheless, we have implemented the same

⁴ <http://www.k-team.com/>

⁵ We actually have a periodic event-triggered controller, see Heemels et al. [2013].

ε	Ellipse		Lemniscate	
	r	e	r	e
0.01	6.38 %	0.0122	13.59 %	0.0159
0.02	7.46 %	0.0129	9.51 %	0.026
0.05	0.98 %	0.048	2.9 %	0.0542
0.1	0.5 %	0.0731	1.37 %	0.1064
0.2	0.74 %	0.1564	0.87 %	0.1947
0.5	0.67 %	0.5256	0.7 %	0.6371

Table 1. Values of r and e for different ε

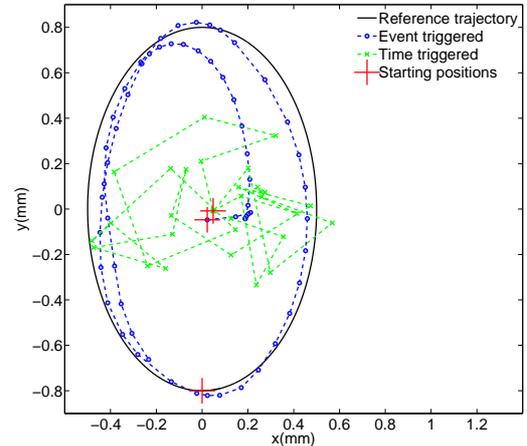


Fig. 3. Experimental results for the closed-loop trajectories when the reference describes an ellipse.

controller and we have transmitted the control input to the robot every T_{sampling} seconds. For a fair comparison, we have selected T_{sampling} to be the average inter-transmission interval of the corresponding event-triggered controller. We have obtained $T_{\text{sampling}} = 1.904$ s for the ellipse and $T_{\text{sampling}} = 1.234$ s for the lemniscate. In that way, both controllers do generate a similar amount of transmissions over an experiment. Figures 3 and 4 show the trajectories for each controller with $\varepsilon = 0.01$ for the ellipse and the lemniscate reference trajectories respectively. Note that ε impacts the average inter-transmission interval and thus the sampling period of the time-triggered controller. We see that the time-triggered controller is not able to ensure the tracking objective as opposed to the event-triggered controller. This is due to the fact that the latter adapts the frequency of control input updates when it is needed. The experiments have shown that we need to take sampling periods which are approximately four times smaller than the ones above for the time-triggered closed-loop system to ensure the tracking objective.

5. CONCLUSION

An event-triggered feedback law for the tracking control of a mobile robot has been presented. The problem has been modeled as an hybrid system using the formalism of Goebel et al. [2012]. The designed controller consists in the law of Jiang and Nijmeijer [1997] together with an event-triggering condition inspired by Forni et al. [2010], Anta and Tabuada [2012]. We needed to resort to the an invariance principle of Sanfelice et al. [2007] to prove that the robot practically tracks the reference trajectories. Sufficient conditions for the existence of a uniform minimum inter-transmission time have been provided. The controller has been implemented on a benchmark and we have no-

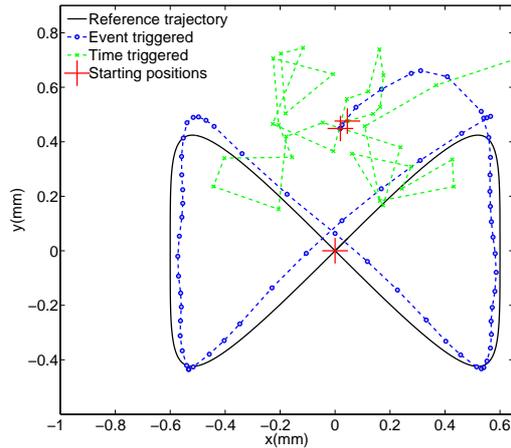


Fig. 4. Experimental results for the closed-loop trajectories when the reference describes a lemniscate.

ticed that it is able to reduce the amount of transmissions compared to periodic setups for similar performances.

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