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Disassembly Line Balancing Problem with Fixed Number of Workstations under Uncertainty

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Abstract: This paper deals with the problem of disassembly line balancing where partial disassembly and uncertainty of task times are studied. Few papers have addressed the stochastic disassembly line balancing problem and most of existing work focused on complete disassembly and have not considered AND/OR graphs. In the present work, tasks of the best selected disassembly alternative are to be assigned to a fixed number of workstations while respecting precedence and cycle time constraints. Task times are assumed to be random variables with known probability distributions. An AND/OR graph is used to model the disassembly alternatives and the precedence relationships among tasks and subassemblies. The objective is to balance workstations' idle times, *i.e.* differences among stations' loads are as small as possible. A stochastic binary program is developed. To illustrate the applicability of the solution method proposed, it was performed on on a set of problem instances from the literature.

Keywords: Disassembly, Line balancing, Stochastic programming, Monte Carlo sampling.

1. INTRODUCTION

The main purpose of disassembly process is to revalorize End of Life (EOL) products by a systematic separation of their parts and materials for recycling, remanufacturing and reuse (Güngör and Gupta (1999b)). For higher productivity rate and automated disassembly, disassembly lines are more suitable to carry out disassembly operations (Güngör and Gupta (2002)). The disassembly process is more complex than assembly. Indeed, in a disassembly environment, a product is broken down into many parts and subassemblies whose qualities, quantities and reliabilities cannot be controlled as in an assembly environment. The assembly process has to be complete while the disassembly process does not have to be carried out completely due to technical and economic restrictions. For instance, irreversible connections of components of a product can be seen as a technical restriction and the disassembly cost being greater than the revenue obtained from retrieved parts as an economic restriction. Hence, disassembly is typically a partial process (Lambert (2002)). A comparison of operational and technical considerations of assembly and disassembly lines is provided in (Gupta and Gungor (2001)). These characteristics of disassembly make disassembly lines more challenging. Therefore, a particular attention should be reserved to their balancing phase and efficient tools are needed in order to optimize their performances and effectiveness. Such tools must take into account the uncertainty in the structure and the quality of the products to be disassembled.
The present paper deals specifically with the Disassembly Line Balancing Problem (DLBP) with a fixed number of workstations, which is here referred to as F–DLBP. The DLPB was introduced by (Güngör and Gupta (1999a)) where a heuristic approach minimizing the number of workstations was presented.

To deal with the deterministic DLBP, heuristic and meta-heuristic approaches were developed. Tang et al. (Tang et al. (2001)) developed a heuristic algorithm to facilitate disassembly line design and optimization. An iterative heuristic using branch and bound notion was developed in (Lambert and Gupta (2005)) to deal with the line balancing problem subjected to sequence dependent costs. A multiobjective heuristic for U–shaped DLBP was developed in (Avikal et al. (2013)). The authors considered several performance criteria with a lexicographic order: minimize the workstations idle time, maximize the priority of removing hazardous components and maximize the priority of removing high demand components. Two multiobjective metaheuristics, a distributed agent ant system and an uninformed deterministic search, for the design and balancing of disassembly lines were developed and compared in (McGovern and Gupta (2005)). Another multiobjective formulation of the DLBP was presented in (McGovern and Gupta (2006)) where the objectives are also ordered lexicographically. An ant colony optimization metaheuristic was developed for this problem. Ding et al. developed the same metaheuristic based approaches to deal with multiobjective disassembly line balancing (Ding et al. (2010)). The authors considered the different objectives separately and provided a scheme to determine the Pareto set. Tang and Zhou (Tang and Zhou (2006)) developed a Petri net approach where a heuristic was employed to maximize the line productivity. Qualitative and quantitative comparisons of different heuristics and metaheuristics for DLBP, including genetic algorithm, ant colony opti-

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mization method, greedy algorithm, greedy/hill-climbing, greedy/2-opt hybrid heuristics and hunter-killer heuristic, were undertaken in (McGovern and Gupta (2007)).
Mathematical programming formulations and exact solution approaches were proposed as well for the DLBP. Altekin et al. (Altekin et al. (2008)) developed an integer programming formulation for profit maximization for the case of partial disassembly. Koc et al. (Koc et al. (2009)) studied the DLBP with the objective to minimize the number of workstations. Two exact approaches were developed based on mixed integer and dynamic programs, respectively.

However, only few studies in the literature have considered the DLBP under uncertainty. Güngör and Gupta (Güngör and Gupta (2001)) proposed a heuristic to deal with task failures caused by defective parts of the EOL product. The objective is the assignment of disassembly tasks to workstations minimizing the cost of defective parts. A MIP based predictive–reactive approach to deal with task failures was also developed in (Altekin and Akkan (2011)) for DLBP aiming to maximize the profit generated by a disassembly line. A collaborative ant colony algorithm for stochastic mixed–model U–shaped disassembly line balancing was developed by Agrawal and Tiwari (Agrawal and Tiwari (2006)). Task times were assumed stochastic with known normal probability distributions. The objective was to minimize the number of workstations and the probability of line stoppage. A self–guided ants metaheuristic was proposed in (Tripathi et al. (2009)) for the disassembly line sequencing problem. A fuzzy disassembly optimization model was developed with the objective of maximizing the net revenue of the disassembly process under uncertainty for quality of EOL products. Tuncel et al. (Tuncel et al. (2012)) used a Monte Carlo based reinforcement learning technique to solve the multiobjective DLBP under demand variations of the EOL products. A nonlinear binary biobjective program was developed in (Aydemir-Karadag and Turkbey (2013)) for disassembly line design and balancing under uncertainty of the task times. Disassembly task times were assumed independent random variables with known normal probability distributions. Complete disassembly was considered and a genetic algorithm was designed to solve the problem.

The literature review provides evidence that there are few studies that have dealt with stochastic DLBP and were either restricted to the study of demand fluctuations, condition of the EOL products or stochastic task times as normal random variables with only heuristic/metaheuristic solution methods without assessment of the solution quality. Many of these papers have not studied the case of partial disassembly and have not used an AND/OR graph to model the precedence relationships among tasks. Koc et al. (Koc et al. (2009)) showed that the integration of the AND/OR graphs in the DLBP formulation allowed obtaining better solutions in comparison with the use of AND precedence diagrams. To bridge the gap, the problem studied in this paper seeks an assignment of a given set I, of disassembly tasks to a fixed number of workstations J while satisfying precedence and cycle time constraints under uncertainty. An AND/OR graph is used to model the precedence relations among tasks. The case of partial disassembly is considered and the objective is to balance the workload of the line. Task times are assumed to be random variables with known probability distributions. Any probability distribution can be used and can be different from one task to another. A mathematical formulation for the stochastic (F–DLBP) is developed and an exact solution method to solve the problem efficiently is proposed. The algorithm combines Monte Carlo sampling technique with the MIP optimizer of CPLEX.

The remainder of the paper is organized as follows. A formulation of the stochastic F-DLBP is provided in section 2. Section 3 follows with the proposed solution method. Section 4 is dedicated to the analysis of numerical experiments and section 5 concludes the paper with a further research direction.

2. PROBLEM MODELING

The F-DLBP consists to assign the disassembly tasks I to a known number $|J|$ of stations respecting precedence and cycle time constraints under uncertainty of task times. The objective is to balance the line by making the stations' workloads as smooth as possible. The following assumptions are considered: a single type discarded product is to be partially or completely disassembled on a straight paced line. The EOL products are sufficiently available. All received EOL products contain all parts with no addition or removing of components. Task times are assumed to be random variables with known probability distributions. A disassembly task can be performed by any but only one workstation. Each part of a product has a certain resale value.
The AND/OR graph utilized here represents explicitly all the possible alternatives to disassemble an EOL product and models the precedence relationships among tasks and subassemblies. An example of such a graph is illustrated in Figure 1 which is an adaptation of the ball point pen graph in (Lambert (1999)). An AND/OR graph is constructed from an EOL product as follows: each subassembly is represented by a node labeled $A_k, k \in K$, and each node labeled $B_i, i \in I$, models a disassembly task. Two types of arcs define the precedence relations among subassemblies and disassembly tasks: AND and OR. As an example, if a disassembly task generates two subassemblies, or more, then, it is related to these subassemblies by AND–type arcs, in bold in Figure 1. If several concurrent tasks may be performed on a subassembly, this latter is related to these tasks by OR–type arcs. For simplicity of the precedence graph, subassemblies with one component are not represented. A sink node S is introduced and linked with dashed arcs to all disassembly tasks. The use of the dummy task S allows a partial disassembly; if S is assigned to a

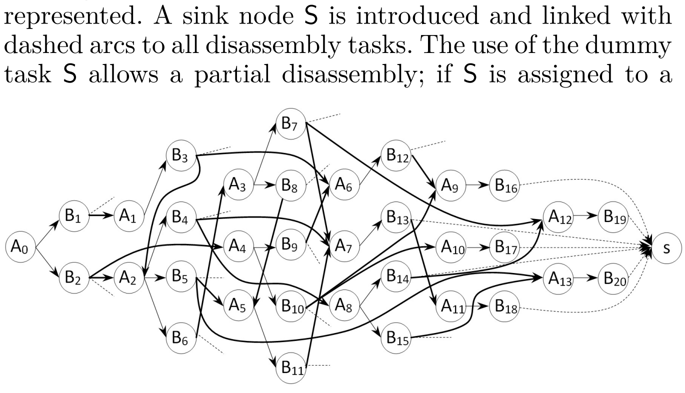

Fig. 1. And/Or precedence graph

workstation, the disassembly process is finished (partial or complete disassembly).

As mentioned earlier, task times $\tilde{t}_i, i \in I$, are assumed to be random variables with known probability distributions. Let $\tilde{t}_i = t_i(\tilde{\xi}), i \in I$, where $\tilde{\xi} = (\tilde{t}_1, \ldots, \tilde{t}_{|I|}) \in \Xi \subset \mathbb{R}_+^{|I|}$ is a random vector of the task times and Ξ is a set of a given probability space $(\Xi, \mathcal{F}, P)$ introduced by $\tilde{\xi}$.The following stochastic program (F–SBP) has been developed for the stochastic F–DLBP.

Stochastic Binary Program

Parameters

CT: Cycle time, $CT > 0$;
P_k: Predecessors index set of A_k, $k \in K$, *i.e.* $P_k = \{i |\ B_i \text{ precedes } A_k\}$;
S_k: Successors index set of A_k, $k \in K$, $S_k = \{i |\ A_k \text{ precedes } B_i\}$.

Decision Variables

$$x_{ij} = \begin{cases} 1, & \text{if disassembly task } B_i \text{ is assigned} \\ & \text{to workstation } j; \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{\mathsf{S}j} = \begin{cases} 1, & \text{if dummy task } \mathsf{S} \text{ is assigned} \\ & \text{to workstation } j; \\ 0, & \text{otherwise.} \end{cases}$$

Recall that the objective is to make the differences of workstations' workloads as small as possible under uncertainty of task times. Let $ST_j(\tilde{\xi}) = \sum_{i \in I} t_i(\tilde{\xi}) \cdot x_{ij}$, be the station workload of workstation $j, \forall j \in J$.

F–SBP

$$\min \max_{\forall j,j' \in J, j \neq j'} \left| \mathbb{E}_{\tilde{\xi}}\Big(ST_j(\tilde{\xi})\Big) - \mathbb{E}_{\tilde{\xi}}\Big(ST_{j'}(\tilde{\xi})\Big) \right| \qquad (\textbf{F–SBP})$$

s.t.

$$\sum_{i \in S_0} \sum_{j \in J} x_{ij} = 1 \qquad (1)$$

$$\sum_{j \in J} x_{ij} \leqslant 1, \forall i \in I \qquad (2)$$

$$\sum_{i \in S_k} \sum_{j \in J} x_{ij} \leqslant \sum_{i \in P_k} \sum_{j \in J} x_{ij}, \forall k \in K \backslash \{0\} \qquad (3)$$

$$\sum_{i \in S_k} x_{iv} \leqslant \sum_{i \in P_k} \sum_{j=1}^{v} x_{ij}, \forall k \in K \backslash \{0\}, \forall v \in J \qquad (4)$$

$$\sum_{j \in J} x_{\mathsf{S}j} = 1 \qquad (5)$$

$$\sum_{j \in J} j \cdot x_{ij} \leqslant \sum_{j \in J} j \cdot x_{\mathsf{S}j}, \forall i \in I \qquad (6)$$

$$\mathbb{E}_{\tilde{\xi}}\Big(ST_j(\tilde{\xi})\Big) \leqslant CT, \forall j \in J \qquad (7)$$

$$x_{ij}, x_{\mathsf{S}j} \in \{0, 1\}, \forall i \in I, \forall j \in J \qquad (8)$$

Constraint (1) imposes the selection of only one disassembly task to begin the disassembly process. Constraint set (2) indicates that a task is to be assigned to at most one workstation. Constraints (3) ensure that only one OR-successor is selected. Constraint set (4) defines the precedence relationships among tasks. Constraint (5) imposes the assignment of the dummy task S to one station. Constraints (6) ensure the assignment of all disassembly tasks to lower or equal-indexed workstations than the one to which S is assigned. Constraints (7) represent the cycle time ones. Constraints (8) defines the possible values of the decision variables.

3. SOLUTION METHOD

Let x be a vector of decision variables $x_{ij}, x_{\mathsf{S}j}, (i, j) \in I \times J$, $X = \{x |$ constraints (1)–(8), are satisfied$\}$ and

$$\Omega_{jj'} = \mathbb{E}_{\tilde{\xi}}\Big(ST_j(\tilde{\xi})\Big) - \mathbb{E}_{\tilde{\xi}}\Big(ST_{j'}(\tilde{\xi})\Big), \forall j, j' \in J, j \neq j'$$

The program (**DF–SBP**) given below represents an equivalent version of program (**F–SBP**).

$$\min Z \qquad (\textbf{DF–SBP})$$

s.t.

$$x \in X$$
$$-Z \leqslant \Omega_{jj'} \leqslant Z, \forall j, j' \in J, j \neq j' \qquad (9)$$
$$Z \geqslant 0$$

Constraint (9) can be replaced with its equivalent

$$e_{jj'} + \Omega_{jj'} = Z, \forall j, j' \in J, j \neq j'$$
$$0 \leqslant e_{jj'} \leqslant 2Z, \forall j, j' \in J, j \neq j'$$

Consider the random variable

$$ST_j^{\lambda}(\tilde{\xi}) = \frac{1}{\lambda} \cdot \sum_{\ell=1}^{\ell=\lambda} ST_j(\tilde{\xi}_\ell), j \in J$$

then $ST_j^{\lambda}(\tilde{\xi})$ is an unbiased estimator of $\mathbb{E}_{\tilde{\xi}}\Big(ST_j(\tilde{\xi})\Big), j \in J$:

$$\mathbb{E}_{\tilde{\xi}}\Big(ST_j^{\lambda}(\tilde{\xi})\Big) = \mathbb{E}_{\tilde{\xi}}\Big(\frac{1}{\lambda} \sum_{\ell=1}^{\ell=\lambda} ST_j(\tilde{\xi}_\ell)\Big) = \frac{1}{\lambda} \cdot \sum_{\ell=1}^{\ell=\lambda} \mathbb{E}_{\tilde{\xi}}\Big(ST_j(\tilde{\xi}_\ell)\Big)$$
$$= \mathbb{E}_{\tilde{\xi}}\Big(ST_j(\tilde{\xi})\Big), j \in J$$

Using the strong law of large numbers (DeGroot and Schervish (2012)), it follows that

$$P\Big(\lim_{\lambda \to +\infty} ST_j^{\lambda}(\tilde{\xi}) = \mathbb{E}_{\tilde{\xi}}\Big(ST_j(\tilde{\xi})\Big), j \in J\Big) = 1$$

This law states that $ST_j^{\lambda}(\tilde{\xi}), j \in J$, converges almost surely to the expected value $\mathbb{E}_{\tilde{\xi}}\Big(ST_j(\tilde{\xi})\Big), j \in J$. Using a λ–sample $(\xi_1, \ldots, \xi_{|j|})$ of the random vector $\tilde{\xi}$, the expectation $\mathbb{E}_{\tilde{\xi}}\Big(ST_j(\tilde{\xi})\Big), j \in J$, is then approximated with its Monte Carlo estimate $\frac{1}{\lambda} \cdot \sum_{\ell=1}^{\ell=\lambda} ST_j(\xi_\ell), j \in J$, (Mak et al. (1999)).

4. NUMERICAL EXPERIMENTS

The developed (**DF–SBP**) was implemented in MS VC++ 2008 and CPLEX 12.5 was used to solve it on a PC with Pentium(R) Dual–Core CPU T4500, 2.30 GHz and 3GB RAM. This optimization problem has been applied to 6 instances available in the literature which contain process alternatives for disassembly. The names of the problem instances were respectively composed of the first letters of authors' names and year of publication, i.e. BBD13 (Bentaha et al. (2013b)), BBD13a (Bentaha et al. (2013a)), KSE09 (Koc et al. (2009)), L99a and L99b from (Lambert (1999)) and MJKL11 from (Ma et al. (2011)).

Table 1. Problem instances.

	$\|I\|$	$\|K\|$	arcs	AND–relations			$\|J\|$	CT
				0	1	2		
MJKL11	37	22	76	4	27	6	3	35
L99a	30	18	60	2	26	2	3	30
BBD13a	25	11	49	4	18	3	2	91
KSE09	23	13	47	4	14	5	2	20
L99b	20	13	41	5	9	6	3	5.5
BBD13	10	5	18	3	6	1	2	0.51

The input data for each problem instance are given in Table 1. The columns 'AND–relations' report the number of disassembly tasks with no successor in subcolumn 0, with one AND–type arc in subcolumn 1 and with two AND–type arcs in subcolumn 2. The column 'arcs' gives the total number of AND–type and OR–type arcs.

Table 2 reports the results obtained for the processed

Table 2. Results of the optimization.

	$\mathrm{Max_{IT}}$	$\mathrm{Min_{IT}}$	o–tasks	s–tasks	CPU time
MJKL11	5.08	4.83	7	5	0.05
L99a	5.12	4.98	9	6	0.02
BBD13b	10.96	10.40	4	2	0.02
KSE09	4.960	4.957	6	5	0.03
L99b	1.53	1.48	8	5	0.05
BBD12	0.299	0.299	3	2	0.02

instances. Columns '$\mathrm{Max_{IT}}$', '$\mathrm{Min_{IT}}$', 'o–tasks', 's–task' and 'CPU–time report the maximum idle time of all workstations, the minimum idle time, the original number of tasks of the selected disassembly alternative, the number of selected tasks of the selected alternative and the resolution time in seconds, respectively. The value of λ was fixed to 1000.

As it can be noted, results of Table 2 show that all selected disassembly alternatives lead to a partial disassembly of EOL products. All instances were solved to optimality in less than 1 second.

5. CONCLUSION

In the present work, the problem of disassembly line balancing with a fixed number of workstations was studied. The cases of partial disassembly and uncertainty of task times are taken into account. Disassembly task times were assumed to be random variables with known probability distributions. To model the addressed stochastic F-DLBP, a stochastic binary program (F–SBP) was proposed. The developed mathematical model was evaluated on a set of instances from the literature. The obtained results were promising and have shown its applicability. All instances were solved to optimality in less than 1 second.

For further research, a direction to investigate is the implementation of the developed model for real life cases in order to assess its performance in practice.

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