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From Linear to Nonlinear Model Predictive Control of a Building

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Abstract: In the building climate control area, the linear model predictive control (LMPC)—nowadays considered a mature technique—benefits from the fact that the resulting optimization task is convex (thus easily and quickly solvable). On the other hand, while nonlinear model predictive control (NMPC) using a more detailed nonlinear model of a building takes advantage of its more accurate predictions and the fact that it attacks the optimization task more directly, it requires more involved ways of solving the non-convex optimization problem. In this paper, the gap between LMPC and NMPC is bridged by introducing several variants of linear time-varying model predictive controller (LTVMPC). Making use of linear time-varying model of the controlled building, LTVMPC obtains predictions which are closer to reality than those of linear time invariant model while still keeping the optimization task convex and less computationally demanding than in the case of NMPC. The concept of LTVMPC is verified on a set of numerical experiments performed using a high fidelity model created in a building simulation environment and compared to the previously mentioned alternatives (LMPC and NMPC) looking at both the control performance and the computational requirements.

Keywords: Predictive control; adaptive control; recursive identification.

1. INTRODUCTION

Energy savings in buildings and reduction of their energy consumption are some of the most emerging challenges for society today. The reason is simple and the numbers speak for themselves—up to 40 % of the total energy consumption can be owed to the building sector [1]. Out of this amount, more than half is consumed by various building heating/cooling systems. Therefore, the recent emphasis on the energy savings in this area is right on target. With the clearly evident need for savings in the area of the building climate control, improvements can be found when considering the latest control techniques.

Model Predictive Control (MPC) stands as one of the most promising candidates for the energetically efficient control strategy. This was demonstrated also within the framework of the Opticontrol project where one research team at ETH Zurich (Switzerland) showed on numerous simulations that using MPC instead of the classical control strategies, more than 16 % savings can be achieved [2,3] depending on the building type. Under real-operational conditions, these savings can be even higher than considering the simulation environment due to the software simplifications compared to the real building. Handling the real-life challenges properly, the improvement achieved by the MPC compared to the classical controller is usually more impressive. This was shown by teams from Prague [4,5] and UC Berkeley [6] where the actual cost savings were even better than the theoretical expectations.

However, MPC suffers from several drawbacks. Besides the need for a reliable mathematical model of the building which should be both simple enough (so that it can be handled effectively) and able to predict the building behavior with sufficient accuracy for several hours ahead, one very severe bottleneck is the complexity of the optimization routine. In order to be feasible and computable, simplified formulations are often considered. Moreover, linear models are usually assumed and exploited by the optimizer. Therefore, in the majority of the MPC applications, the overall task is formulated as a linear/convex optimization problem easily solvable by the commonly available solvers for quadratic or semidefinite programming [4,7]. Although being computationally favorable and able to find the global minimum in case of the convex formulation of the optimization task, their disadvantage is that they do not enable minimization of the nonlinear/nonconvex cost criteria and therefore, only certain approximation of the real cost paid for the control is optimized. Moreover, they resort to the optimization of either the setpoints or the energy delivered to the heating/cooling system while leaving all its distribution to the suboptimal low-level controllers which can lead to a significant loss of the optimality gained by the MPC.

In several recent works, the effort to introduce the nonlinearities (caused either by the dynamical behavior of the building or by the control requirements formulation) into the optimization task can be found [6,8]. In this paper, we discuss both possibilities for the zone temperature control (the linear and the nonlinear MPC) and moreover, we bridge the two banks of the gap between the nonlinear and the linear variant of the MPC by introducing linear models that change in time. Such models can describe the building dynamics in a more reliable and flexible way while they still keep the low complexity of the optimization task (since the linear model remains convex). Two ways of obtaining a time-varying model are described and the results of the modified controllers are compared with the results of the original (linear and nonlinear) MPCs.

The paper is organized as follows: Sec. 2 illustrates the problem of the building climate control on a simple example. Both the building and the heat delivery system description are provided. Furthermore, control performance criterion, comfort requirements and restrictions are introduced. In Sec. 3, the models supplying predictions to the model based controllers are described. The nonlinear model is derived based on the thermodynamics while for the linear model, the assumed simplifications are presented. For the linear time-varying models, two ways of obtaining them are explained. All four models are verified and their results are discussed. Sec. 4 brings a description of the controllers including the low level re-calculation (for the linear MPC) and the nonlinear optimization routine (for the nonlinear MPC). In Sec. 5, the control behavior of the LMPC, NMPC and LMPC with time-varying models (LTVMPC) is investigated and their results are presented and examined. Sec. 6 draws conclusion of the paper.

2. PROBLEM FORMULATION

In this section, the description of the building, constraints and the evaluative performance criterion are formulated.

2.1 Building of interest

The building under our investigation is a simple medium weight one-zone building modeled in the TRNSYS16 [9] environment, which is a high fidelity simulation software package widely accepted by the civil engineering community as a reliable tool for simulating the building behavior. The Heating, Ventilation and Air Conditioning (HVAC) system used in the building is of the so called active layer type. The pipes in the ceiling distribute supply water which then performs thermal exchange with the concrete core of the building consequently heating the air in the room.
Fig. 1 shows a sketch of the considered building. We considered four directly measured outputs: zone temperature T_Z, ceiling temperature T_C, temperature of the return water T_R and temperature of the south-oriented wall T_S. The supply water temperature T_{SW} and the mass flow rate of the supply water $\dot{m}$ are the controlled inputs while predictions of disturbances (solar radiation $\dot{Q}_S$ and outside-air temperature T_O) are considered to be available.

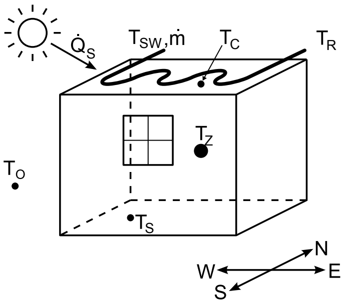

Fig. 1. A scheme of the modeled building

The last step is to describe the heat distribution system. In our application, we consider the configuration of the heating system as shown in Fig. 2. Clearly, the storage tank

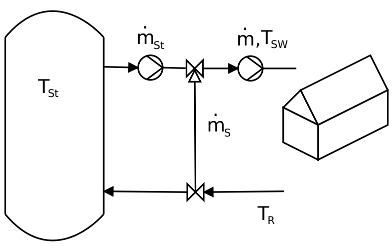

Fig. 2. A scheme of heat distribution system

plays a key role as the sole heat supplier in this system. In fact, having obtained the requirements for the supply water temperature T_{SW} and the supply water mass flow rate $\dot{m}$, these two values are "mixed" using the return water with the temperature T_R flowing into the building inlet pipe through the side-pipe at the mass flow rate $\dot{m}_s$ and the water from the storage tank which is kept at certain constant value T_{St} (in this paper, $T_{St} = 60^\circ$ C is considered) and can be withdrawn from the tank at mass flow rate $\dot{m}_{St}$. Based on this, the following set of equations can be written for the upper three-way valve:

$$\dot{m}T_{SW} = \dot{m}_{St}T_{St} + \dot{m}_S T_R$$

$$\dot{m} = \dot{m}_{St} + \dot{m}_S. \tag{1}$$

which can be further rewritten into an expression for the calculation of the storage water mass flow rate, $\dot{m}_{St} = \dot{m}(T_{SW} - T_R)/(T_{St} - T_R)$. Having the return water temperature measurement at our disposal and extracting the storage water with the temperature of T_{St} at this mass flow rate, both the supply water temperature and supply water mass flow rate related to the heating requirements can be achieved. Last of all, let us note a situation which requires a value of T_{SW} to be lower than the return water temperature T_R would mean negative storage water mass flow rate $\dot{m}_{St}$, which is practically unrealizable. On the other hand, it is also obvious that such T_{SW} requirement really can not be satisfied as only the hot water storage is considered in this configuration. With no cold water storage provided, the temperature of the supply water can not be decreased below the return water temperature which means that the active cooling mode is neither allowed nor realizable.

2.2 Control performance requirements

Besides the building description, it is important to specify the performance requirements, constraints and the criterion according to which the control strategy is evaluated. Considering the building climate control, one of the most important tasks is to ensure the required thermal comfort which is specified by a pre-defined admissible range of temperatures related to the way of use of the building (office building, factory, residential building, ...). Under the weather conditions of middle Europe with quite low average temperatures where heating is required for more than half of year, the thermal comfort satisfaction requirement can be further simplified such that the zone temperature is bounded only from below. As we consider an office building with regular time schedule, the lowest admissible zone temperature $T_Z^{min}(t)$ whose violation will be penalized is defined as a function of working hours as

$$T_Z^{min}(t) = \begin{cases} 22^\circ\text{ C from 8 a.m. to 6 p.m.,} \\ 20^\circ\text{ C otherwise.} \end{cases} \tag{2}$$

Then, the thermal comfort violation is expressed as

$$CV(t) = \max(0, T_Z^{min}(t) - T_Z(t)). \tag{3}$$

Besides the comfort violation $CV(t)$, the price paid for the operation of the building is penalized in the cost criterion as well. Coming out of the considered structure of the building and its energy supply system, the monetary cost includes the price for the consumed hot water and the electricity needed to operate the two water pumps. While the hot water price P_W is considered constant (see Tab. 1), the electricity price $P_E(t)$ which applies to the operation of the supply and storage water pumps is piece-wise constant and similarly to the lowest admissible zone temperature profile, it depends on the working hours as follows:

$$P_E(t) = \begin{cases} HT \text{ from 8 a.m. to 6 p.m.,} \\ LT \text{ otherwise.} \end{cases} \tag{4}$$

In order to bring our case study closer to reality, the values of high and low tariff (HT and LT) have been chosen in accordance with the real prices approved by the Regulatory Office for Network Industries of Slovak Republic [10].The exact values of HT and LT in €/kWh are listed in Tab. 1. Thus, the overall performance criterion over a time interval $\langle t_1, t_2\rangle$ is formulated as

$$J = \int_{t_1}^{t_2} \omega CV \,\mathrm{d}t + \int_{t_1}^{t_2} (P_E(t)(P_C(\dot{m}) + P_C(\dot{m}_{St})) + P_W \dot{m}_{St})\,\mathrm{d}t. \tag{5}$$

Here, ω is the virtual price for the comfort violation $CV(t)$ which is defined by (3) and $P_W\dot{m}_{St}$ represents the cost paid for the consumed hot water. Time-varying electricity price is expressed as a function of time by (4) and the power consumptions of the water pumps corresponding to $\dot{m}$ and $\dot{m}_{St}$ can be calculated as a quadratic function of the particular mass flow rate, $P_C(\dot{m}) = \alpha_0 + \alpha_1\dot{m} + \alpha_2\dot{m}^2$, $P_C(\dot{m}_{St}) = \alpha_0 + \alpha_1\dot{m}_{St} + \alpha_2\dot{m}_{St}^2$. The parameters $\alpha_{0,1,2}$ are listed in Tab. 1.

Table 1. List of the specific parameters

T_Z^{min} (°C)	22/20	P_W (–)	2.6199
HT (€/kWh)	0.1168	α_0 (–)	9
LT (€/kWh)	0.0502	α_1 (–)	9.25×10^{-3}
T_{St} (°C)	60	α_2 (–)	1.875×10^{-6}

Let us note that as the criterion (5) specifies the satisfaction of the control requirements for the control of a building in a very compact form, all the considered controllers will be evaluated and compared according to the reached values of this criterion.

3. MODELING AND IDENTIFICATION

In this section, the derivation of models for the particular variants of the MPC is described and explained.

3.1 Nonlinear model (NM)

Based on the data gathered from the controlled building and following the methodology provided by the available literature [11], a nonlinear mathematical model has been derived serving as the predictor for the optimization routine within the NMPC. From the control engineering point of view, the most crucial phenomena effecting the dynamical behavior of the zone temperature are *1*) convection from both the heated and unheated wall, *2*) effects of ambient environment (solar radiation and ambient temperature), *3*) mutual interaction of the walls and *4*) heat supplied by the supply water. These phenomena are captured in the following set of differential equations:

$$\begin{aligned}
\dot{x}_1 &= p_1|x_2-x_1|(x_2-x_1)^{\frac{1}{3}}+p_2|x_3-x_1|(x_3-x_1)^{\frac{1}{3}}+p_3(d_1-x_1)\\
&\quad +p_4d_2\\
\dot{x}_2 &= -p_5|x_2-x_1|(x_2-x_1)^{\frac{1}{3}}+p_6(x_3-x_2)+p_7(x_4-x_2)\\
\dot{x}_3 &= -p_8|x_3-x_1|(x_3-x_1)^{\frac{1}{3}}-p_9(x_3-x_2)+p_{10}(d_1-x_3)+p_{11}d_2\\
\dot{x}_4 &= -p_{12}(x_4-x_2)+p_{13}u_2(u_1-x_4),
\end{aligned} \tag{6}$$

where $x = [T_Z, T_C, T_S, T_R]$ represent state variables, $u = [T_{SW}, \dot{m}]$ stand for the control inputs and $d = [T_O, \dot{Q}]$ correspond to the predictable disturbances.

Parameters p of the model have been estimated by prediction error method implemented in MATLAB environment [12]. More details on the theoretical background of the identification procedure can be found in [13].

3.2 Linear model (LM)

In order to simplify the model (6), let us adopt the assumption that the cubic roots of the temperature differences related to the heat convection are constant over the whole range of the operating points of the building. This simplifies the nonlinear terms as follows:

$$p|x_i - x_j|(x_i - x_j)^{\frac{1}{3}} \approx \bar{p}(x_i - x_j). \tag{7}$$

Furthermore, $\dot{Q} = c\dot{m}(T_{SW}-T_R)$ is assumed to be the control input instead of the pair $\dot{m}$ and T_{SW}. Based on these assumptions, the linear version of the model (6) can be summarized as a discrete-time state space model as follows:

$$x_{k+1} = Ax_k + Bu_k + B_d d_k \tag{8}$$

with the state matrices having the following structure:

$$A = \begin{bmatrix} a_1 & a_2 & a_3 & 0 \\ a_5 & a_6 & a_8 & a_7 \\ a_9 & a_{10} & a_{11} & 0 \\ 0 & a_{12} & 0 & a_{13} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b \end{bmatrix}, B_d = \begin{bmatrix} b_{d1} & b_{d2} \\ 0 & 0 \\ b_{d3} & b_{d4} \\ 0 & 0 \end{bmatrix}. \tag{9}$$

In this model, state and disturbance variables correspond to the previously mentioned ones and $u = \dot{Q}$ refers to the optimized input. The sampling period of the system has been chosen as $t_s = 15\,\text{min}$. The model parameters a, b, b_d have been estimated by a multistep prediction error minimization procedure (MRI). For further details on this method, the readers are referred to [5].

3.3 Recursively identified linear model (RIL)

Although considering the same linear model structure as the off-line identified time invariant model described in the previous section, the recursively identified linear model makes use of the recursive identification technique. At the beginning of the operation, an off-line identified model is used. However, as the new data are gathered during the operation of the controller, they are used to update the model with the latest information about the building behavior and the model is therefore re-identified after certain pre-defined amount of time. In our case, the linear model with the structure corresponding to (8) is re-identified once per 5 days using the last five-day-long data set. For the initialization of the identification procedure estimating the parameters of the k-th model, the parameters of the $(k-1)$-st one are used. This is done in order to provide the best available initial guess for the non-convex MRI identification task [5] which depends strongly on the (sub)optimality of the initial values. Moreover, initializing the model identification with the previous parameters, recursive nature of this approach is ensured as the information from the previous data (captured in the previous parameter values) is incorporated. In this way, part of the problems of the linear models can be solved. With the original off-line identified linear model, part of the dynamics of the building is neglected and under these simplifying assumptions, the linear model is identified at certain operating point. In case that the building is shifted away from that operating point (which can be caused either by change of the weather or by excitation of the neglected dynamics), the one-shot identified models can fail to provide accurate predictions for the controller which can lead to the performance degradation. The recursively identified model, on the other hand, can adapt to these changes and therefore it is still able to provide accurate predictions over much larger range of the operating conditions than the one-shot identified linear model.

3.4 Switched linearly approximated models (SLAM)

The main idea of this approach is that for a combination of inputs u, disturbances d and state variables x, a linear time-varying approximation of model (6) can be found by replacing particular nonlinearities with time-varying terms. In case of a building, this approach is even more natural and expected as the nonlinear mathematical description of the building contains terms depending on the differences between two state variables, namely $p|x_i - x_j|(x_i - x_j)^{\frac{1}{3}}$ which are likely to vary much less than the temperatures themselves. As an opposite to the linear models described earlier where the nonlinear terms are linearized “before the identification” and having the gathered data at disposal, parameters of linear time invariant model are estimated considering the purely linear character of the model, in this case, the nonlinear model is identified off-line and using its parameters, the nonlinearity is continuously approximated on-line depending on the actual values of the chosen auxiliary variables which leads to a time-varying linear model. In order to get rid of the nonlinear terms coupling the states, let us propose an approximation procedure based on the auxiliary variables as follows.

Let us introduce two auxiliary variables, $\delta_{x_{1,2},t}$ and $\delta_{x_{1,3},t}$ defined such that

$$\begin{aligned}
\delta_{x_{2,1},t} &= \sqrt[3]{|x_{2,t_m} - x_{1,t_m}|}\\
\delta_{x_{3,1},t} &= \sqrt[3]{|x_{3,t_m} - x_{1,t_m}|},
\end{aligned} \tag{10}$$

where $t \geq t_m$ refers to continuous time and t_m indicates the time instant when the last measurements of the state variables arrived. The derived model shall predict the behavior of the building over certain prediction horizon during which no current measurements of the state variables are available. Therefore, at each “measurement” time instant, the values of $\delta_{x_{1,2},t}$ and $\delta_{x_{1,3},t}$ are calculated and they are used by the optimizer over the whole prediction horizon. The necessity of realizing the difference between the *real-life*

time (in which the model is time-varying) and the *internal time of the optimizer* (in which the model stays constant over the prediction horizon) is obvious.
Then, the nonlinear terms appearing in the model (6) can be approximated as $|x_2 - x_1| \sqrt[3]{(x_2 - x_1)} \approx \delta_{x_{1,2},t}(x(t_m))(x_2 - x_1)$, $|x_3 - x_1| \sqrt[3]{(x_3 - x_1)} \approx \delta_{x_{1,3},t}(x(t_m))(x_3 - x_1)$ for all $t \geq t_m$. Here, the expressions $\delta_{x_{1,2},t}(x(t_m))$, $\delta_{x_{1,3},t}(x(t_m))$ are used to emphasize the fact that the values of auxiliary variables depend only on the last available state measurements.
The bilinear term in the last differential equation is (similarly to the previous approaches) considered as the new controlled input Q while the vector of disturbances d remains unchanged. The linearized differential equations can be now summarized as:

$$\dot{x} = A_{app}(t)x + B_{app}u + B_d d, \tag{11}$$

where

$$A_{app}(t) = \begin{bmatrix} -(\tilde{p}_1+\tilde{p}_2+p_3) & \tilde{p}_1 & \tilde{p}_2 & 0 \\ \tilde{p}_5 & -(\tilde{p}_5+p_6+p_7) & p_6 & p_7 \\ \tilde{p}_8 & p_9 & -(\tilde{p}_8+p_9+p_{10}) & 0 \\ 0 & p_{12} & 0 & -p_{12} \end{bmatrix}, \tag{12}$$

with

$$\begin{aligned} \tilde{p}_1 &= p_1 \delta_{x_{1,2},t}, & \tilde{p}_2 &= p_2 \delta_{x_{1,3},t}, \\ \tilde{p}_5 &= p_5 \delta_{x_{1,2},t}, & \tilde{p}_8 &= p_8 \delta_{x_{1,3},t} \end{aligned} \tag{13}$$

and

$$B_{app} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ p_{13} \end{bmatrix}, \quad B_d = \begin{bmatrix} p_3 & p_4 \\ 0 & 0 \\ p_{10} & p_{11} \\ 0 & 0 \end{bmatrix}. \tag{14}$$

At this point, the whole algorithm of obtaining the linear approximated model of the building can be summarized. At each discrete sample $k = t_m$, the values of the state variables x are measured and the auxiliary variables $\delta_{x_{1,2},t}$, $\delta_{x_{1,3},t}$, are evaluated according to (10). Making use of the calculated auxiliary variables, a linear continuous model (11) of the building is created with the corresponding matrices. In order to be used with the linear MPC, the model needs to be discretized [14] and then, a discrete-time model is obtained as:

$$x_{k+1} = A_{app}(k)x_k + B_{app}u_k + B_d d_k. \tag{15}$$

This approximated model is used until the new measurements arrive, which means that at each discrete time sample, a new model is approximated and used over the following prediction horizon.

3.5 Comparison

As long as the models are intended to be used with the MPC, their most important feature is to provide reasonable predictions over the whole prediction horizon. In this paper, we consider the prediction horizon $T_P = 16\,\text{h}$ which corresponds to 64 samples.
Fig. 3 shows 8 weeks of comparison of the models which are used for the building behavior predictions with the linear time invariant, linear time varying and nonlinear MPC, respectively. At each discrete time sample ($t_s = 15\,\text{min}$), 16-hours predictions are calculated based on the provided measurement. All the predictions of the models are plotted together with the real data.
Looking at Fig. 3, it is clear that the two "limit" cases (linear time invariant and nonlinear model) constrain the prediction behavior of the models from above and below. The performance of the time-varying models is somewhere in the middle between these two while for the recursively identified model, slightly higher prediction error can be observed than for the model obtained by piece-wise approximation of the nonlinear one. The most obvious are the differences in the behavior when looking at the 4-th and the 5-th week of the comparison. While the absolute value of prediction errors for the off-line identified linear model reach up to 4° C, the error obviously decreases through the recursively identified time-varying and switched linearly approximated time-varying model down to the nonlinear

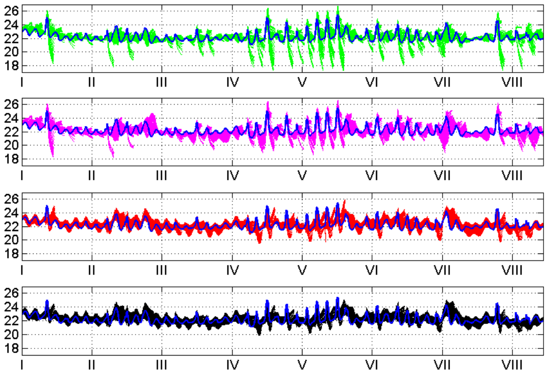

Fig. 3. Comparison of T_Z predictions ($°C$) of the LM (green), RIL (magenta), SLAM (red) and NM (black) with the real data (blue).

model which provides the predictions with the least prediction error out of the four compared models.
In order to compare the models in a more complete way, the statistical comparison of the models is provided in Tab. 2. Here, LM specifies the linear model, RIL corresponds to the recursively identified linear model, SLAM stands for the switched linearly approximated model and NM represents the nonlinear model. For each model, ε_{av} being the average prediction error over the whole 16-hours prediction horizon and the maximum prediction error ε_{max} over the prediction horizon are inspected.

Table 2. Statistical comparison of the models.

	LM	RIL	SLAM	NM
$\varepsilon_{av}(°C)$	0.81	0.75	0.49	0.40
$\varepsilon_{max}(°C)$	4.76	3.84	1.72	1.36

The table clearly demonstrates that the most reliable predictions are provided by the NM model. However, this is not a surprise as that model takes the whole dynamics of the building into account including the nonlinearities. On the other hand, it can be seen that considering the linear time-varying models, the quality of the predictions fairly improves compared to the linear time invariant model. Considering RIL model, 8% reduction of ε_{av} and 20% reduction of ε_{max} is achieved while with SLAM model, the reduction of ε_{av} is almost 40% and the reduction of ε_{max} is nearly 74%.

4. MODEL PREDICTIVE CONTROL

This section briefly describes the compared MPC variants.

4.1 Linear MPC

The control requirements which have been chosen for the linear MPC to be satisfied (minimization of both the thermal comfort violation and the energy consumption) can be mathematically summarized as follows:

$$J_{MPC,k} = \sum_{i=1}^{P} W_{1,\mathbf{p}}(k+i)\|\dot{Q}_{k+i}\|_{\mathbf{p}} + \sum_{i=1}^{P} W_{2,\mathbf{p}}\|CV_{k+i}\|_{\mathbf{p}} \tag{16}$$

$$\begin{aligned} \text{s.t.}: \quad & \text{linear dynamics (8)} \\ & 0 \leq \dot{Q}_{k+i} \leq \dot{Q}_{k+i}^{max}, \quad i = 1, \ldots, P \\ & \hat{T}_{Z,k+i|k} \geq T_{Z,k+i}^{min} - CV_{k+i}. \end{aligned}$$

This formulation considers a combination of linear and quadratic penalization indicated by the index $\mathbf{p} \in \{1, 2\}$ which enables to shape the penalization criterion conveniently. Time varying weighting matrices W reflecting the time dependence of the electricity tariffs and prediction horizon P stand for the tuning parameters of the controller. Comfort violation is calculated as the difference between the zone temperature prediction $\hat{T}_Z$ and its lowest acceptable bound T_Z^{min} and the hard constraints are relaxed

employing an auxiliary variable cV. Exact values of the optimization problem settings can be found in Tab. 3.
As the linear version of MPC optimizes supplied heat $\dot{Q}$, a postprocessing procedure is needed to obtain the particular values of T_{SW} and $\dot{m}$ which correspond to the true control inputs of the thermally activated building system (TABS). This straightforward postprocessing holds the mass flow rate fixed $\dot{m} = \dot{m}_{pp}$ and it calculates the supply water command as $T_{SW} = \dot{Q}/\dot{m}c_w + T_R$. If the heating effort is lower than a threshold value $\dot{Q}_{tr}$, the TABS manipulated variables are set to $T_{SW} = T_R$ and $\dot{m} = 0$. The settings of the postprocessing procedure are listed in Tab. 3.

Table 3. Table of settings

$W_{1,1}$ (high tariff)	0.01	$W_{1,2}$ (high tariff)	1.6
$W_{1,1}$ (low tariff)	0.005	$W_{1,2}$ (low tariff)	0.8
$W_{2,1}$	2×10^6	$\dot{Q}^{max}$	90×10^4
$W_{2,2}$	10^4	$\dot{Q}_{tr}$	700
T_{SW}^{min}	20	$\dot{m}^{min}$	0
T_{SW}^{max}	50	$\dot{m}^{max}$	2150
P	48	$\dot{m}_{pp}$	1250

4.2 Nonlinear MPC

Using the model (6) and being able to perform a nonlinear optimization, the nonlinear MPC minimizes directly the performance criterion (5). To find the solution, gradient optimization method with enhanced choice of the search step is used. The main idea of this approach is that the cost functional $J(x, u, t)$ can be minimized iteratively starting from the initial guess of the optimal input profile u_0 following the direction of the gradient of the corresponding hamiltonian $\mathcal{H}$ constructed as follows:

$$\mathcal{H} = L + \lambda^{\mathrm{T}} f. \tag{17}$$

The information about the integral term of the criterion J is incorporated through L being the integrand of the integral term. The terminal term ϕ of the criterion J is related to the so-called costate vector λ through the terminal condition $\lambda(T_P) = \partial\phi/\partial x|_{t=T_P}$, T_P refers to the optimization horizon. To be consistent with the other variants of MPC, $T_P = 16\,\mathrm{h}$ has been chosen. In this particular case, $\lambda(T_P) = [0, 0, 0, 0]^{\mathrm{T}}$. The dynamics of the costate vector is defined by the differential equation $\dot{\lambda} = -\partial\mathcal{H}/\partial x$. In (17), f corresponds to the vector field describing the dynamics of the controlled system given by (6).
The iterative optimization procedure is expressed as

$$u_{i,l+1} = u_{i,l} - \alpha_{i,l} \frac{\partial H}{\partial u_i}\Big|_l, \tag{18}$$

where $i \in \{1, 2\}$ characterizes the particular input, l stands for the number of iteration and α_i is the corresponding search step. Instead of constant search steps, we use steps $\alpha_{i,l}$ which change during the search and reflect the curvature of the hamiltonian $\mathcal{H}$ as follows:

$$\alpha_{i,l} = \beta_i \frac{|u_{i,l} - u_{i,l-1}|}{|\mathcal{H}_{u_i,l} - \mathcal{H}_{u_i,l-1}|} \tag{19}$$

where $\mathcal{H}_{u_i,l} = \frac{\partial\mathcal{H}}{\partial u_i}|_l$ refers to the gradient of $\mathcal{H}$ calculated at l-th iteration. β_i are adjustable parameters fixed over the whole iterative procedure. In order to prevent numerical problems, $\alpha_{i,l}$ are bounded from above and below.
The hamiltonian $\mathcal{H}$ and inputs $u_{1,2}$ are sampled with the sampling period $t_s = 15\,\mathrm{min}$. Constraints are handled by projecting the calculated input sequence on the admissible input intervals defined by T_{SW}^{min}, T_{SW}^{max} and $\dot{m}^{min}$, $\dot{m}^{max}$. The corresponding values are listed in Tab. 3.

5. RESULTS

First of all, visual comparison of the performance of the considered controllers is presented in the Fig. 4.
Based on Fig. 4 which shows the zone temperature evolution over a 6-day period for different controllers, it can be seen that all controllers are tuned well enough so that they

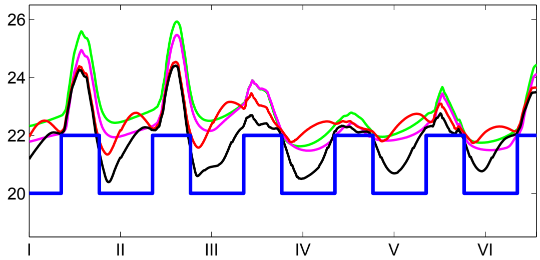

Fig. 4. Comparison of T_Z ($°C$) with the controller using LM (green), RIL (magenta), SLAM (red) and NM (black).

are able to satisfy the room temperature requirements. This is very crucial since a controller that significantly violates the room temperature requirements is basically useless. Besides the superiority of the NMPC resulting from the combination of the model with the best predictions and the direct minimization of the performance criterion (5), the improvement of the two LTVMPC variants compared to the LMPC should be noted. Controllers with time-varying models are much less likely to unnecessarily overheat the room thanks to more accurate predictions.
Next comparison is brought by Fig. 5 where the supply water temperatures applied by the predictive controllers using particular models are shown. The blue areas correspond to the hours when the high tariff is applied.

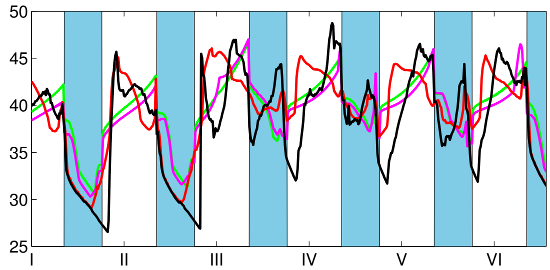

Fig. 5. Comparison of T_{SW} ($°C$) with the controller using LM (green), RIL (magenta), SLAM (red) and NM (black).

It can be observed that the NMPC exploits the low tariff and heats the room during the night while during the working hours, it only gently maintains the temperature within the required zone. As the relation of the used models to the nonlinear model "fades away", the input profiles applied by particular controllers are less and less economical (higher supply water temperatures are applied also during the working hours). The statistical comparison of the energy consumption can be found in Tab. 4. TP expresses the overall price paid for zone temperature control. Moreover, the particular energy consumptions related to the consumption of the linear MPC using the off-line identified model are expressed.

Table 4. Comparison of the energy consumption and computational complexity.

	LM	RIL	SLAM	NM
TP	583.37	566.09	542.19	475.67
TP/TP_{LM} (%)	100	97	93	81
$T_{av}(s)$	0.83	0.89	0.93	13.7
$T_{max}(s)$	1.22	32.4	1.52	22.1

The superiority of the NMPC is demonstrated once again. Moreover, it can be seen that although the comparison of the identified models was very optimistic in the case of linear time-varying models versus the linear time-invariant one, the resulting effect of the good models on the overall energy consumption is not so attractive. This can be simply explained by the fact that although the good predictor is crucial for the proper functioning of the MPC (either linear or nonlinear), so is the properly chosen optimization criterion. Based on this observation, in the building climate control, the need for the use of nonlinear MPCs which are able to attack task of the real-life price minimization in a direct way instead of using certain approximation is obvious. However, one more aspect needs to be taken into account when choosing the controller type—its computational complexity. Tab. 4 shows two factors related to the computational demands of the particular control strategy: T_{av} being the average computational time needed for the calculation of the optimal input per one sampling instant of the input profile and T_{max} corresponding to the maximum calculation time needed. Let us mention that this calculation time includes also the time needed to obtain the model which (as will be shown) might contribute significantly to the overall calculation time. The comparison is evaluated depending on the type of the model which is used by the optimizer. The simplest controller being the LMPC with LM needs the shortest time to calculate the optimal input. As this variant does not consume any time to obtain the model and the same optimizer is used by the rest of the family of the linear MPCs (including also controller with RIL model and SLAM model), one can get a very good insight into how long does it take to obtain the model for the predictions. As the SLAM variant performs the approximation of the nonlinear model at each sampling instant, the increase of the average computational time is understandable. Although in case of the LMPC with SLAM, the average calculation time is the longest, this is compensated by the best control performance out of the linear approaches. On the other hand, the RIL variant needs slightly less time in average but evaluating the T_{max}, the RIL variant can be regarded as the most cumbersome. The reason is quite simple—as the optimization task which needs to be solved when performing the identification routine is a non-convex one, the results and also the computational time depend heavily on the initial estimate and in case of inaccurate initial estimate the computational time can be increased significantly.

Let us summarize the performance of the particular variants. Regarding the control performance and the energy consumption, the NMPC is the best candidate for the real-life application. On the other hand, the LMPC with the simplest off-line identified model is able to provide the fastest calculation of the optimal input sequence. Looking for a trade-off between the optimality and the time complexity, the newly presented time-varying approaches are able to bridge the gap between these two and therefore, they stand for promising candidates for the real-life application especially in case of large buildings complexes where it can be expected that the nonlinear optimization task can take too long to be solved.

6. CONCLUSION

In this paper, several variants of MPC for the building climate control have been presented and inspected. Two ways of obtaining a linear time-varying model of the controlled building have been proposed. The first is based on the recursive identification with the original linear structure and the second one performs a linear approximation of the nonlinear model according to the values of the chosen auxiliary variables. The models have been compared also with the off-line identified linear and nonlinear models showing that quite impressive improvement of the quality of the predictions can be achieved with the linear time-varying models. Then, the predictive controllers employing these models have been compared with respect to a predefined evaluation criterion based on the real-life requirements and costs. The results show that although being the most effective in minimizing those real-life costs, NMPC is the most time consuming variant and the newly presented time-varying alternatives (RIL and SLAM) can be more advantageous in case of huge building complexes where the complexity of the optimization task can be very high. The linear time-varying approaches combine the simplicity and time effectiveness resulting from the use of the convex optimizer and fair accuracy of the linear models which are updated according to the available data.

Regarding the future work, it would be interesting to examine the effect of incorporation of the persistent excitation condition into the predictive controller procedure. Based on the available literature, if the persistent excitation condition is included, more informative data are obtained which then turns into a better ability to estimate the model parameters accurately. The suggested procedure should be compared with the advanced Kalman filtering algorithms such as Extended or Unscented Kalman filtering. Last but not least, a procedure for the model parameter update should be designed for the nonlinear model. Moreover, based on the performed numerical experiments, we suggest the strategies be tested on a building in real-operation.

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