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A Dynamic Scaling Based Control Redesign Procedure for Uncertain Nonlinear Systems with Input Unmodeled Dynamics ⋆

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Abstract: The global asymptotic stabilization problem for a general class of uncertain nonlinear systems with dynamic input uncertainties (input unmodeled dynamics) is considered. The system structure addressed in this paper is comprised of a nominal system (which can have a general nonlinear dynamics structure and is assumed to have a nominal control law to stabilize the nominal system) and an appended input unmodeled dynamics. The proposed approach is based on a control redesign utilizing the nominal control law and incorporating a singular-perturbation-like dynamic system extension along with a dynamic scaling to address the dynamic input uncertainties. The control input uncertainty structure is modeled through an uncertain non-affine function of the control input and the unmeasured state of an uncertain input unmodeled dynamics subsystem. The proposed control redesign approach is applicable to a wide class of nonlinear systems including triangular as well as non-triangular system structures as long as a set of structural and inequality conditions on the system dynamics are satisfied and provides a global robust output-feedback stabilizing controller.

Keywords: Dynamic output feedback, Robust control, Uncertain dynamic systems, Nonlinear control systems, Lyapunov methods.

1. INTRODUCTION

The following class of systems with input unmodeled dynamics is considered:

$$\begin{aligned} \dot{x} &= f(x, \overline{u}) \quad ; \quad y = h(x) \\ \overline{u} &= \mu(x, \eta, u) \quad ; \quad \dot{\eta} = q_\eta(x, \eta, u) \end{aligned} \tag{1}$$

where [1] $x \in \mathcal{R}^n$ is the state of the nominal system, $u \in \mathcal{R}$ is the control input, $\overline{u} \in \mathcal{R}$ is the perturbed control input that takes the place of a nominal control input (i.e., the nominal unperturbed system is of form $\dot{x} = f(x, u)$), $y \in \mathcal{R}^{n_y}$ is the measured output, and $\eta \in \mathcal{R}^{n_\eta}$ is the state of an appended dynamics that represents a dynamic input perturbation (input unmodeled dynamics). f, μ, and q_η are uncertain continuous functions and h is a known continuous function. The system (1) is a perturbed form of a nominal system given by

$$\dot{x} = f(x, u) \quad ; \quad y = h(x) \tag{2}$$

wherein the control input u is replaced by the function μ that represents the input perturbation both in terms of static uncertainty (as the uncertain function μ) and the dynamic perturbation due to the appearance of η, the state of the input unmodeled dynamics subsystem.

The stabilization problem for various classes of nonlinear systems has been widely studied in the literature, including specific classes of system structures such as the feedforward (Kaliora and Astolfi [2001], Teel [1992], Mazenc and Praly [1996], Sepulchre et al. [1997]) and strict-feedback (Krstić et al. [1995]) triangular forms under various sets of assumptions (on, for instance, uncertainties in the system, appended dynamics, state and input time delays, etc). The control of systems with input nonlinearities and input unmodeled dynamics under various sets of assumptions has also been addressed (e.g., Praly and Wang [1996], Zhang and Ge [2006], Ren et al. [2008], etc., and references therein). A dual dynamic high-gain scaling approach has been addressed in Krishnamurthy et al. [2003], Krishnamurthy and Khorrami [2004a,b] providing a unified control design methodology applicable to both feedforward and strict-feedback triangular system structures. High-gain and low-gain approaches (e.g., Khalil and Saberi [1987], Ilchmann and Ryan [2003], Lin [2009] and references therein) are popular design techniques for the control of various types of systems. State-dependent scaling techniques for control of nonlinear systems are also addressed in Ito [2006]. A combination of a high-gain observer and a backstepping based controller was proposed in Praly [2003], Krishnamurthy et al. [2003].

The dual observer/controller dynamic high-gain scaling technique was introduced in Krishnamurthy and Khorrami [2002, 2004b] and shown to be a flexible design technique capable of handling uncertain terms dependent on all states and uncertain Input-to-State Stable (ISS) appended dynamics with nonlinear gains from all the system states and the input. The dynamic high-gain scaling technique provides a unified framework for state-feedback and output-feedback control of both strict-feedback (Krishnamurthy and Khorrami [2004b], Kaliora et al. [2006], Krishnamurthy and Khorrami [2007b]) and feedforward (Krishnamurthy and Khorrami [2004a, 2008]) systems as well as state-feedback control of nontriangular polynomially-bounded systems (Krishnamurthy and Khorrami [2007a]). A three-time-scale based control design utilizing two dynamic scaling parameters (with the first being analogous to the scaling parameter utilized in our prior dual dynamic high-gain scaling based control designs as in Krishnamurthy and Khorrami [2004a] and the second scaling parameter specifically introduced to handle the nonlinear input uncertainties) was proposed in Krishnamurthy and Khorrami [2013] to address feedforward-like systems with input uncertainties (analogous to the uncertain function μ).

The dynamic scaling based controller proposed in this paper

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[1] $\mathcal{R}$, $\mathcal{R}^+$, and $\mathcal{R}^k$ denote the set of real numbers, the set of non-negative real numbers, and the set of real k-dimensional column vectors, respectively. The notation $\max(a_1, \ldots, a_n)$ indicates the largest value among the numbers $a_1, \ldots, a_n$; the notation $\min(a_1, \ldots, a_n)$ similarly indicates the smallest value among the given arguments. $|a|$ denotes the Euclidean norm of a column vector a.

provides a general control redesign procedure that takes a dynamic output-feedback control law for the nominal system and adds a singular-perturbation-like dynamic extension and dynamic scaling to achieve robust global stabilization of the system (1). The proposed dynamic control design is based on the dynamic scaling based output-feedback controller developed in our earlier work (Krishnamurthy and Khorrami [2013]) for feedforward systems with uncertain input unmodeled dynamics. While these previous works addressed the specific class of upper triangular (feedforward) system structures, it is shown in this paper that this dynamic scaling based approach can be applied to the general class of systems (1) to provide a robust control redesign procedure to address uncertain dynamic input perturbations $\mu(x,\eta,u)$. The assumptions introduced on the system (1) are given in Section 2 along with discussions on the system structure and illustrative examples. The control redesign comprised of a dynamic extension and a dynamic scaling is summarized in Section 3. The stability analysis and the design of the scaling parameter dynamics are presented in Section 4.

2. SYSTEM STRUCTURE AND ASSUMPTIONS

This paper considers the class of systems shown in (1) which forms a perturbed form of the nominal dynamics (2) with the nominal control input u in (2) replaced by $\overline{u} = \mu(x,\eta,u)$. It is assumed that sufficient conditions (e.g., local Lipschitz property) on f, q_η, and μ for local existence and uniqueness of solutions of (1) are satisfied. The control objective is to regulate x and η to zero (utilizing the measurement of the output y) starting from any initial conditions $x(0) \in \mathcal{R}^n$ and $\eta(0) \in S_\eta$ where S_η is some (not necessarily compact or bounded) known subset of $\mathcal{R}^{n_\eta}$. The proposed controller is based on a dynamic scaling based redesign procedure starting from an *a priori* known dynamic output-feedback control law (for the nominal system) of the following form:

$$\dot{\xi} = f_\xi(y,\xi) \quad ; \quad u = h_\xi(y,\xi) \tag{3}$$

where ξ is the n_ξ-dimensional state vector of the dynamic controller (that could, in general, include an observer state, adaptation parameters, dynamic scaling parameters, etc.). Based on the design of this control law for the nominal system, the set of possible initial conditions for ξ is denoted by $S_{\xi,0} \subset \mathcal{R}^{n_\xi}$, i.e., $\xi_0 \in S_{\xi,0}$, and the set of possible values of $\xi(t)$ for all time $t \geq 0$ is denoted by $S_\xi \subset \mathcal{R}^{n_\xi}$. The sets $S_{\xi,0}$ and S_ξ are not necessarily compact or bounded and could be the entire set $\mathcal{R}^{n_\xi}$, but are simply introduced to model the possibility that certain components of the nominal dynamic controller state vector are initialized in a specific region of the state space, e.g., adaptation parameters introduced to estimate the magnitude of uncertain parameters and are commonly required in adaptive control designs to be initialized to be non-negative and are furthermore non-decreasing due to the design of the adaptation parameter dynamics, dynamic scaling parameters (Krishnamurthy and Khorrami [2004b]) that are initialized to be greater than 1 and are non-decreasing due to the design of the scaling parameter dynamics, etc. The assumptions introduced on the system (1) and the nominal control law (3) are given as Assumptions A1-A5 below.

Assumption A1 (a control law of form (3) for the closed-loop nominal system with robustness in a Lyapunov sense to additive perturbations in the span of the nominal control input): Functions f_ξ, h_ξ, and $V_{x\xi}$ are known such that for all $x \in \mathcal{R}^n$, $\xi \in S_\xi$, and $\epsilon \in \mathcal{R}$, the following inequality is satisfied:

$$\begin{aligned}\Bigg[&\frac{\partial V_{x\xi}}{\partial x} f(x,h_\xi(y,\xi)) + \frac{\partial V_{x\xi}}{\partial \xi} f_\xi(y,\xi) \\ &+ \frac{\partial V_{x\xi}}{\partial x}[f(x,h_\xi(y,\xi)+\epsilon) - f(x,h_\xi(y,\xi))]\Bigg] \\ &\leq -\alpha_{x\xi}(x,\xi) + \gamma_1(y,\xi)\tilde{\gamma}(\epsilon^2)\end{aligned} \tag{4}$$

where

- $V_{x\xi}$ is a non-negative function such that [2] the boundedness of $V_{x\xi}$ along with the form of the dynamics (2) and (3) implies the boundedness of x and ξ, i.e., for any positive constant $\overline{V}$, a positive constant $c_{\overline{V}}$ can be found such that if it is known that $V_{x\xi} \leq \overline{V}$ at a time $t \geq 0$, then using the dynamics (2) and (3), it can be shown that $|x| + |\xi| \leq c_{\overline{V}}$.
- $\alpha_{x\xi}$ is a known non-negative function such that if the signal $\alpha_{x\xi}(x(t),\xi(t))$ seen as a function of time t goes to zero as $t \to \infty$, then it can be inferred by using the properties of $V_{x\xi}$ as defined above and the dynamics (2) and (3) that $x(t) \to 0$ as $t \to \infty$.
- $\gamma_1(y,\xi)$ is a known non-negative function
- $\tilde{\gamma}(\epsilon^2)$ being a known non-negative function that satisfies $\tilde{\gamma}(0) = 0$ and is polynomially bounded, i.e., $\tilde{\gamma}(\epsilon^2) \leq \sum_{i=1}^{n_\epsilon} p_{\epsilon,i}\epsilon^{2i}$ where $n_\epsilon \geq 1$ is an integer and $p_{\epsilon,i}, i = 1\ldots,n_\epsilon$ are non-negative real numbers.

Assumption A2 (conditions on uncertain input perturbation function μ): The function $\mu(x,\eta,u)$ is such that for any system trajectory starting from any initial values $x(0) \in \mathcal{R}^n$ and $\eta(0) \in S_\eta$ and given any input signal $u(t)$, the following inequalities are satisfied for the values of $(x(t),\eta(t),u(t))$ at any time $t \geq 0$ with $\overline{\mu}(y,u)$ being a known continuous function and $\underline{\mu}$ being a known constant: (a) $\frac{\partial}{\partial u}\mu(x,\eta,u) \geq \underline{\mu} > 0$; (b) $|\mu(x,\eta,u)| \leq \overline{\mu}(y,u)$. Furthermore, $\lim_{|u|\to\infty} |\mu(x,\eta,u)| = \infty$ for any $x \in \mathcal{R}^n$ and any $\eta \in \mathcal{R}^{n_\eta}$.

Assumption A3 (ISS condition on input unmodeled dynamics η): The η subsystem is ISS with ISS Lyapunov function $V_\eta : \mathcal{R}^{n_\eta} \to [0,\infty)$ such that the following inequality is satisfied:

$$\frac{\partial V_\eta}{\partial \eta} q_\eta(x,\eta,u) \leq -\alpha_\eta(|\eta|) + \beta_\eta(x,u) \tag{5}$$

for all $x \in \mathcal{R}^n$, $u \in \mathcal{R}$, and $\eta \in \mathcal{R}^{n_\eta}$, with α_η being a known class $\mathcal{K}_\infty$ function and β_η being a non-negative function. Also, a positive constant $\underline{V}_\eta$ is known such that $\alpha_\eta(|\eta|) \geq \underline{V}_\eta V_\eta(|\eta|)$ for all $\eta \in \mathcal{R}^{n_\eta}$.

Assumption A4 (a known function $\tilde{h}(y)$ that is relative degree one with respect to the control input in the nominal system (2)): A function $\tilde{h}$ is known such that for all $x \in \mathcal{R}^n$ and $\overline{u} \in \mathcal{R}$,

$$\frac{\partial \tilde{h}(h(x))}{\partial x} f(x,\overline{u}) = f_{h1}(y) + f_{h2}(y)\overline{u} \tag{6}$$

with f_{h1} and f_{h2} being known functions of y. A positive constant $\underline{f}_{h2}$ exists such that $f_{h2}(y) \geq \underline{f}_{h2}$ for all $y \in \mathcal{R}^{n_y}$.

Assumption A5 (inequality conditions on perturbation functions in overall system dynamics and the function $\alpha_{x\xi}$ in nominal Lyapunov function): Non-negative constants k_{11}, k_{13}, k_{21}, k_{22}, and k_{31} and non-negative functions $\beta_{11}(y,\xi,u)$, $\beta_{13}(y,\xi,u)$, $\beta_{21}(y,\xi,u)$, $\beta_{22}(y,\xi,u)$, and $\beta_{31}(y,\xi,u)$ are known such that the following inequalities are satisfied for all $x \in \mathcal{R}^n$, $\xi \in S_\xi$, $u \in \mathcal{R}$, $\epsilon \in \mathcal{R}$, $\eta \in \mathcal{R}^{n_\eta}$:

$$\begin{aligned}&\left[\left(\frac{\partial \mu}{\partial x} - \frac{\partial h_\xi}{\partial y}\frac{\partial h}{\partial x}\right) f(x,h_\xi(y,\xi)+\epsilon)\right]^2 \\ &\leq k_{11}\alpha_{x\xi}(x,\xi)\beta_{11}(y,\xi,u) + k_{13}\tilde{\gamma}_1(\epsilon^2)\beta_{13}(y,\xi,u)\end{aligned} \tag{7}$$

$$\begin{aligned}\left[\frac{\partial \mu}{\partial \eta} q_\eta(x,\eta,u)\right]^2 &\leq k_{21}\alpha_{x\xi}(x,\xi)\beta_{21}(y,\xi,u) \\ &\quad + k_{22}V_\eta(\eta)\beta_{22}(y,\xi,u)\end{aligned} \tag{8}$$

$$\left[\frac{\partial h_\xi}{\partial \xi}\right]^2 \leq k_{31}\alpha_{x\xi}(x,\xi)\beta_{31}(y,\xi,u) \tag{9}$$

where $\tilde{\gamma}_1(\epsilon^2)$ is a non-negative function that satisfies $\tilde{\gamma}_1(0) = 0$

[2] The formulation of these requirements in Assumption A1 on $V_{x\xi}$ and $\alpha_{x\xi}$ is discussed further in Section 2.1.

and is polynomially bounded, i.e., $\tilde{\gamma}_1(\epsilon^2) \leq \sum_{i=1}^{n_{\epsilon,1}} p_{\epsilon,i,1}\epsilon^{2i}$ where $n_{\epsilon,1} \geq 1$ is an integer and $p_{\epsilon,i,1}, i = 1 \ldots, n_{\epsilon,1}$ are non-negative real numbers. Also, non-negative constants $k_{\eta 1}$ and $k_{\eta 2}$ and functions $\varphi(\xi)$ and $\beta_{\eta 1}(y,\xi)$ are known such that the following inequalities are satisfied:

$$\frac{\beta_\eta(x,u)}{\varphi(\xi)} \leq k_{\eta 1}\alpha_{x\xi}(x,\xi) + \beta_{\eta 1}(y,\xi)|\mu(x,\eta,u)|^2 \quad (10)$$

$$\beta_{\eta 1}(y,\xi)[h_\xi(y,\xi)]^2 \leq k_{\eta 2}\alpha_{x\xi}(x,\xi) \quad (11)$$

with $\varphi(\xi)$ being a function that is lower bounded by a positive constant (i.e., a constant φ_0 exists such that $\varphi(\xi) \geq \varphi_0$ for all ξ) and such that the signal $\varphi(\xi(t))$ is a non-decreasing function of time (i.e., $\frac{\partial \varphi(\xi)}{\partial \xi} f_\xi(y,\xi) \geq 0$ for all y and ξ).

2.1 Remarks on Assumptions

Assumption A1 asserts that a dynamic output-feedback control law and corresponding Lyapunov function are known for the nominal system (wherein $\overline{u} = u$) with a robustness (in the sense of the Lyapunov inequality (4)) on an additive perturbation ϵ in the span of the nominal control input. The Lyapunov-like inequality requirement (4) in Assumption A1 is weaker than requiring that the nominal control law (3) should render the nominal closed-loop system (comprised of (2) and (3)) ISS with respect to an additive perturbation ϵ in the span of the nominal control input since (4) allows a function $\gamma_1(y,\xi)$ appearing multiplicatively with $\tilde{\gamma}(\epsilon^2)$ in (4). Several control design techniques (e.g., backstepping for strict-feedback systems, dynamic high-gain scaling based control design for feedforward and strict-feedback systems) can, in fact, make the nominal closed-loop system ISS with respect to an additive perturbation ϵ and hence can easily satisfy Assumption A1, making this assumption, in general, satisfiable for a very wide class of systems including lower triangular and upper triangular systems as well as various classes of non-triangular systems for which control designs have been studied in the literature. Also, the polynomial boundedness condition on the functions $\tilde{\gamma}$ and $\tilde{\gamma}_1$ in Assumptions A1 and A5 are trivially satisfied in the special, but highly common, case wherein the nominal system (2) is affine in the nominal control input, i.e., $f(x,u)$ is of the form $f_1(x) + g_1(x)u$, and is also, in general, satisfiable if the function $f(x,u)$ includes, for instance, higher-order powers of u, terms such as $u\sin(u)$, etc.

The requirement on the Lyapunov-like function $V_{x\xi}$ in Assumption A1 is written in terms of the boundedness of the state components x and ξ being inferrable from the boundedness of $V_{x\xi}$ and the dynamics (2) and (3) rather than simply from boundedness of $V_{x\xi}$ to also address control design procedures that result in "weak" Lyapunov functions wherein a part of the state component (such as the dynamic scaling parameter r in Krishnamurthy and Khorrami [2004b]) is inferred as bounded by also utilizing the dynamics of that state component rather than directly only from boundedness of $V_{x\xi}$. The condition on $\alpha_{x\xi}$ in terms of its convergence to zero implying convergence of x to zero and the requirement on $V_{x\xi}$ as outlined above essentially characterizes a wide class of nominal nonlinear systems (2) for which control laws (3) can be designed and closed-loop stability properties proved using LaSalle's Invariance Principle. While it is only required in the properties of $\alpha_{x\xi}$ that convergence of x (and not necessarily ξ) to zero be inferrable from convergence of $\alpha_{x\xi}$ to zero, some subset of the controller state component ξ might also, in general, converge to zero depending on the nominal controller design (3), in which case the proposed control redesign procedure will also achieve convergence of that subset of ξ to zero in the perturbed system (1).

Assumption A2 prescribes conditions on the unknown function $\mu(x,\eta,u)$ in terms of upper bounds on the function μ and its partial derivatives and comprises of the physically meaningful requirements that a lower bound on the variation of μ with respect to u should be known which is greater than 0 (e.g., $\frac{\partial}{\partial u}\mu$ cannot vanish), that μ does not saturate the available control authority (if it did saturate the control magnitude, then global stabilization would typically not be attainable, in general, by any control design), and that an upper bound (as a function of y and u) on the uncertain function should be known. Assumption A3 is a general ISS condition on the input unmodeled dynamics $\dot{\eta} = q_\eta(x,\eta,u)$; while an ISS condition is utilized for simplicity, it can be shown that this condition can be relaxed to an integral ISS condition (with α_η relaxed to be a continuous function which is positive for positive arguments). Assumption A4 requires that a measured signal $\tilde{y}$ (i.e., a function $\tilde{y} = \tilde{h}(y)$) should exist which is relative degree one with respect to the control input in the nominal system (2) and also requires that u appear linearly in the dynamics of this measured signal $\tilde{y}$ with the coefficient function being only output-dependent. In the output-feedback context, Assumption A4 is structurally the strongest of the imposed assumptions. For triangular systems (both for lower triangular and upper triangular systems) with state $x = [x_1,\ldots,x_n]^T$ wherein the control input appears in the dynamics of x_n (e.g., $\dot{x}_1 = f_1(x_1,x_2), \dot{x}_2 = f_2(x_1,x_2,x_3),\ldots,\dot{x}_n = f_n(x_1,\ldots,x_n,u)$ for lower triangular systems), this assumption effectively requires that x_n (or some combination of x_i's involving x_n) should be part of the measured output y.

Assumption A5 prescribes a set of inequality bounds that need to be satisfied. The inequalities (7), (8), (10), and (11) form essentially requirements on the size of $\alpha_{x\xi}$ (i.e., that $\alpha_{x\xi}$ is big enough in a nonlinear function sense). The inequality (9) is essentially only a local order condition, i.e., $\left[\frac{\partial h_\xi}{\partial \xi}\right]^2 = O[\alpha_{x\xi}(x,\xi)]$ around the origin, since $\frac{\partial h_\xi}{\partial \xi}$ is a function of only y and ξ and the right hand side of (9) includes the arbitrary function $\beta_{31}(y,\xi,u)$). Also, the inequalities (7) and (8) are essentially requirements on the size of $\alpha_{x\xi}$ in terms of unmeasured state variables (hence, trivially satisfied in the special case that $y = x$) since the arbitrary functions $\beta_{11}(y,\xi,u)$ and $\beta_{21}(y,\xi,u)$ appear in the right hand sides of (7) and (8). The set of inequality bounds in Assumption A5 forms essentially a nonlinear stability margin requirement (in a Lyapunov function sense) of the nominal closed-loop control system or equivalently a requirement that the nominal control law can be constructed to provide such a nonlinear stability margin (in practice, given some knowledge of the forms of the functions appearing in the system, the nominal control design would be customized to provide the appropriate stability margins as, for instance, in the example discussed in Remark 1). The function $\varphi(\xi)$ is introduced in Assumption A5 to address control design procedures such as the dynamic high-gain scaling design technique for strict-feedback and feedforward systems, which introduces a scaling parameter r that would appear in the terms within $\alpha_{x\xi}$ necessitating scaling of the Lyapunov function V_η and therefore β_η by an appropriate power of r (e.g., see Krishnamurthy and Khorrami [2004b], Krishnamurthy and Khorrami [2013]). In cases wherein the nominal control design does not involve a dynamic scaling or other mechanism (e.g., an adaptation parameter) that provides a relevant function $\varphi(\xi)$ that is monotonically non-decreasing as a function of time, then $\varphi(\xi)$ can simply be defined to be 1 in Assumption A5.

Remark 1: To illustrate the structure of the Assumptions A1-A5, consider the simple example system given by

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1^2 \quad ; \quad \dot{x}_2 = x_3 + x_1x_2 \\ \dot{x}_3 &= \mu(x,\eta,u) = u + (1+\cos(\eta))u + x_1^2u^3 \\ \dot{\eta} &= -\eta + x_1^2 \quad ; \quad y = [x_1,x_3]^T. \end{aligned} \quad (12)$$

The nominal system with state $x = [x_1,x_2,x_3]^T$ is of a strict-feedback form that can be easily addressed using observer backstepping (Krstić et al. [1995]), i.e., designing a full-order or reduced-order observer to estimate the unmeasured state component x_2, and performing backstepping from x_1 through observer estimate $\hat{x}_2$ to x_3 or an observer estimate $\hat{x}_3$, and then to u. It is easily seen that with this nominal control design, this system satisfies Assumption A1. Assumption A2 is easily seen to be satisfied. Assumption A3 is seen to be satisfied with, for instance, $V_\eta = \frac{1}{2}\eta^2$, which yields $\alpha_\eta(|\eta|) = \frac{1}{2}\eta^2$, $\beta_\eta(x,u) = x_1^4$. Assumption A4 is trivially satisfied since x_3 is measured where $\dot{x}_3 = \overline{u}$. It can be seen that Assumption A5 is also satisfied by an appropriate construction of the virtual control inputs and control Lyapunov functions in the backstepping based control design for the nominal x subsystem. To illustrate this backstepping based nominal control design, note that if the backstepping procedure is started with $V_1 = \frac{1}{2}x_1^2$ in the first step, then by defining $z_2 = \hat{x}_2 + x_1^2 + x_1 + x_1^3$, we would get $\dot{V}_1 = -x_1^2 - x_1^4 + x_1 z_2 + x_1 e_2$ where $e_2 = x_2 - \hat{x}_2$ is the observer error. The x_1^3 term in z_2 and the resulting x_1^4 term in $\dot{V}_1$ are introduced specifically to ensure inequality (10) since $\beta_\eta(x,u) = x_1^4$. Taking $\varphi(\xi) = 1$, the function $\beta_{\eta 1}$ for this example reduces to 0, thus trivially satisfying (11). Completing the backstepping procedure along the standard lines (Krstić et al. [1995]), it can be seen that the assumptions listed above can be satisfied. Hence, the control redesign procedure proposed in this paper is applicable to this example system.

Also, in the context of upper triangular (feedforward) systems, it was shown in our prior works (Krishnamurthy and Khorrami [2013]) that a singular-perturbation-like dynamic scaling based controller analogous to the control redesign procedure proposed in this paper can be applied to address dynamic input uncertainties. As an example, the control redesign procedure can be applied to the following feedforward system starting from a nominal control design based on the dual dynamic high-gain scaling approach (Krishnamurthy and Khorrami [2004a]) and utilizing the dynamic scaling based control redesign procedure proposed in this paper:

$$\begin{aligned}
\dot{x}_1 &= (1 + x_1^2 + x_4^2)x_2 + x_3 x_1^2 + x_1 x_4^2 \\
\dot{x}_2 &= (1 + x_1 x_4 + x_1^2 + x_4^2)x_3 + x_1 x_4^5 u \\
\dot{x}_3 &= (2 + x_1^2 + x_4^2 + 0.5x_4^2 \sin(x_1))x_4 + x_1 x_4 u \\
\dot{x}_4 &= \mu(x,\eta,u) \quad ; \quad y = [x_1, x_4]^T \\
\dot{\eta} &= -\eta + x_1 x_4 + (1 + x_4^2)u \frac{x_1}{1 + x_1^2}
\end{aligned} \tag{13}$$

where $\mu(x,\eta,u) = a_1(2 + x_1^2 + \cos(\eta))u + a_2(x_1^2 + x_4^2)u^3$ with known $\underline{a}_1, \overline{a}_1, \overline{a}_2, \underline{a}_2$ such that $0 < \underline{a}_i \leq a_i \leq \overline{a}_i, i = 1, 2$.

3. DYNAMIC SCALING BASED CONTROL REDESIGN

The dynamics of the x subsystem from (1) can be written as

$$\dot{x} = f(x, h_\xi(y,\xi) + u_d) \tag{14}$$

where $u_d \stackrel{\triangle}{=} [\mu(x,\eta,u) - h_\xi(y,\xi)]$ and h_ξ denotes the nominal control law from (3) for the nominal system. The principal effect of the dynamic input perturbation (represented by the uncertain function $\mu(x,\eta,u)$ with η being the state of the nonlinear appended dynamics subsystem, i.e., input unmodeled dynamics) in the context of the dynamics (14) is the introduction of the term involving $u_d = [\mu(x,\eta,u) - h_\xi(y,\xi)]$. If it were possible to directly set $\mu(x,\eta,u) = h_\xi(y,\xi)$ by solving the equation $\mu(x,\eta,u) = h_\xi(y,\xi)$ for u (in terms of y and ξ), then the resulting closed-loop system would be effectively the nominal closed-loop system and hence globally stable with asymptotic convergence of x to zero. However, solving the equation $\mu(x,\eta,u) = h_\xi(y,\xi)$ for u is, in general, not directly possible since μ is uncertain and non-affine and furthermore involves η which is the unmeasured state of the uncertain input unmodeled dynamics $\dot{\eta} = q_\eta(x,\eta,u)$. The singular perturbation approach (Saxena et al. [1984], Hovakimyan et al. [2007], Yurkevich [2008], Chakrabortty and Arcak [2009]) in such a situation is to attempt to overcome the non-affine nature of the input appearance by introducing a *fast* dynamic extension of the form $a_\epsilon \dot{u} = -\mathrm{sign}\left(\frac{\partial\mu(x,\eta,u)}{\partial u}\right)[\mu(x,\eta,u) - \tilde{u}]$ with $\tilde{u} = h_\xi(y,\xi)$ and with a_ϵ being a small enough constant. The introduction of the singularly perturbed dynamic extension has a stabilizing effect on the signal $u_d = [\mu(x,\eta,u) - \tilde{u}]$ since this dynamic extension will generate a term of the form $-(1/a_\epsilon)|\frac{\partial\mu(x,\eta,u)}{\partial u}|u_d$ in $\dot{u}_d$, which with a_ϵ being a small constant would represent a high gain stabilizing term in the dynamics of u_d. If μ is an uncertain function, then a multiple state singularly perturbed dynamic extension can be utilized instead to estimate the uncertain function $\mu(x,\eta,u)$ and thereafter to force this estimate to converge to the nominal input $\tilde{u} = h_\xi(y,\xi)$. Given any compact set in which the initial conditions of the closed-loop system lie, then, by picking a_ϵ small enough, stability and asymptotic convergence can be proved using Tikhonov's theorem. This classical singular perturbation approach thus addresses semiglobal results. However, in keeping with the spirit of this paper and the dynamic scaling approach, we seek global results here by utilizing a dynamic scaling parameter instead of a small constant a_ϵ. Also, $\mu(x,\eta,u)$ being an uncertain function cannot be directly utilized in the control design; furthermore, η is an unmeasured state component. Here, denoting $\tilde{y} = \tilde{h}(y)$, a dynamic extension is designed as:

$$\dot{\zeta} = \dot{r}_u\tilde{y} + r_u f_{h2}(y)h_\xi(y,\xi) + r_u f_{h1}(y) \quad ; \quad u = \zeta - r_u\tilde{y} \tag{15}$$

where r_u is a dynamic scaling parameter introduced specifically to handle the uncertain function $\mu(x,\eta,u)$. The dynamics of r_u will be designed later in Section 4 such that $r_u(t)$ is a monotonically non-decreasing function of time; furthermore, r_u will be initialized with $r_u(0) \geq 1$. In the implementation of (15), the notation $\dot{r}_u$ refers to the function specifying the derivative of r_u, as will be designed later in Section 4 (equation (33)). From (15), the control law designed above for u yields:

$$\dot{u} = \dot{\zeta} - \dot{r}_u\tilde{y} - r_u\dot{\tilde{y}} \quad = -r_u f_{h2}(y)u_d. \tag{16}$$

Define

$$V_u = \frac{1}{2\Pi(r_u)}\ln(1 + u_d^2) \tag{17}$$

where $\ln$ denotes the natural logarithm. $\Pi : \mathcal{R}^+ \longrightarrow \mathcal{R}^+$ is any function (e.g., $\Pi(r_u) = 1 + \tanh(kr_u)$ with k being any positive constant) such that $\Pi(r_u) \geq 1$ for all $r_u \geq 1$, $\Pi(r_u)$ is a continuously differentiable monotonically increasing function of r_u over $[1,\infty)$, $\Pi'(r_u) = \frac{\partial\Pi(r_u)}{\partial r_u} > 0$ for all $r_u \in [1,\infty)$, and $\Pi(r_u)$ is a bounded function, i.e., some positive constant $\overline{\Pi}$ exists such that $\Pi(r_u) \leq \overline{\Pi}$ for all $r_u \in [1,\infty)$. The dynamics of $u_d = [\mu(x,\eta,u) - h_\xi(y,\xi)]$ can be written as:

$$\begin{aligned}
\dot{u}_d = &-r_u f_{h2}(y)\frac{\partial\mu(x,\eta,u)}{\partial u}u_d + \frac{\partial\mu(x,\eta,u)}{\partial x}f(x,\overline{u}) \\
&+ \frac{\partial\mu(x,\eta,u)}{\partial\eta}q_\eta(x,\eta,u) - \frac{\partial h_\xi(y,\xi)}{\partial y}\frac{\partial h(x)}{\partial x}f(x,\overline{u}) \\
&- \frac{\partial h_\xi(y,\xi)}{\partial\xi}f_\xi(y,\xi).
\end{aligned} \tag{18}$$

4. STABILITY ANALYSIS AND SCALING PARAMETER DYNAMICS

Using Assumption A1 and noting that $\overline{u} = \mu(x,\eta,u) = h_\xi(y,\xi) + u_d$, we have

$$\dot{V}_{x\xi} \leq -\alpha_{x\xi}(x,\xi) + \gamma_1(y,\xi)\tilde{\gamma}(u_d^2). \tag{19}$$

Note that $|u_d|$ can be bounded as $|u_d| \leq \overline{u}_d(y,u,\xi)$ where $\overline{u}_d(y,u,\xi)$ is a known function given as

$$\overline{u}_d(y,u,\xi) \stackrel{\triangle}{=} \Big[\overline{\mu}(y,u) + |h_\xi(y,\xi)|\Big]. \tag{20}$$

Using the polynomial upper bound on $\tilde{\gamma}$ in Assumption A1, we derive the inequality

$$\tilde{\gamma}(u_d^2) = \frac{\tilde{\gamma}(u_d^2)}{\ln(1+u_d^2)} \ln(1+u_d^2) \le \overline{\tilde{\gamma}}(y,u,\xi)\ln(1+u_d^2) \quad (21)$$

where

$$\overline{\tilde{\gamma}}(y,u,\xi) = [1+\overline{u}_d^2(y,u,\xi)] \sum_{i=1}^{n_\epsilon} p_{\epsilon,i}\overline{u}_d^{2i-2}(y,u,\xi). \quad (22)$$

Note that in writing the equations (21) and (22), we have utilized the inequality

$$\epsilon^i \le \epsilon^{i-1}(1+\epsilon)\ln(1+\epsilon) \quad (23)$$

which holds for all $\epsilon \ge 0$ and all integers $i \ge 1$. Hence, using (19) and (21), we have

$$\dot{V}_{x\xi} \le -\alpha_{x\xi}(x,\xi) + \gamma_1(y,\xi)\overline{\tilde{\gamma}}(y,u,\xi)\ln(1+u_d^2). \quad (24)$$

Defining a scaled Lyapunov function $\tilde{V}_\eta = \frac{V_\eta}{\varphi(\xi)}$, and using Assumption A3, (10) and (11) in Assumption A5 and the condition on $\varphi(y(t))$ that it is nondecreasing as a function of time, we get:

$$\begin{aligned}\dot{\tilde{V}}_\eta &\le -\frac{\alpha_\eta(|\eta|)}{\varphi(\xi)} + \frac{\beta_\eta(x,u)}{\varphi(\xi)} \\ &\le -\frac{\alpha_\eta(|\eta|)}{\varphi(\xi)} + [k_{\eta 1}+2k_{\eta 2}]\alpha_{x\xi}(x,\xi) + 2\beta_{\eta 1}(y,\xi)u_d^2. \quad (25)\end{aligned}$$

Using (17) and (18), we have

$$\begin{aligned}\dot{V}_u = &-r_u f_{h2}(y)\frac{\partial\mu(x,\eta,u)}{\partial u}\frac{u_d^2}{(1+u_d^2)\Pi(r_u)} \\ &- \frac{\Pi'(r_u)\dot{r}_u}{2\Pi^2(r_u)}\ln(1+u_d^2) \\ &+ \frac{u_d}{(1+u_d^2)\Pi(r_u)}\Big\{\frac{\partial\mu(x,\eta,u)}{\partial x}f(x,\overline{u}) \\ &+ \frac{\partial\mu(x,\eta,u)}{\partial\eta}q_\eta(x,\eta,u) - \frac{\partial h_\xi(y,\xi)}{\partial y}\frac{\partial h(x)}{\partial x}f(x,\overline{u}) \\ &- \frac{\partial h_\xi(y,\xi)}{\partial\xi}f_\xi(y,\xi)\Big\}. \quad (26)\end{aligned}$$

Using the inequality (from Assumption A4) that $f_{h2}(y)$ is lower bounded by a positive constant $\underline{f}_{h2}$, the inequality (from Assumption A2) that $\frac{\partial\mu(x,\eta,u)}{\partial u}$ is lower bounded by a positive constant $\underline{\mu}$, and the inequalities in Assumption A5, the inequality (26) can be reduced to the inequality

$$\begin{aligned}\dot{V}_u \le &-r_u\underline{f}_{h2}\underline{\mu}\frac{u_d^2}{(1+u_d^2)\Pi(r_u)} - \frac{\Pi'(r_u)\dot{r}_u}{2\Pi^2(r_u)}\ln(1+u_d^2) \\ &+ 2c_u\frac{u_d^2}{(1+u_d^2)\Pi(r_u)}\Big\{k_{11}\beta_{11}(y,\xi,u) + \frac{k_{13}}{8}\beta_{13}(y,\xi,u) \\ &+ k_{21}\beta_{21}(y,\xi,u) + \frac{k_{22}}{4c_\eta\underline{V}_\eta}\beta_{22}(y,\xi,u)\varphi(\xi) \\ &+ k_{31}\beta_{31}(y,\xi,u)\Big\} \\ &+ \frac{3}{8c_u}\alpha_{x,\xi}(x,\xi) + \frac{1}{c_u}\tilde{\gamma}_1(u_d^2) + \frac{c_\eta\underline{V}_\eta V_\eta}{2c_u\varphi(\xi)} \quad (27)\end{aligned}$$

which holds for any positive constants c_u and c_η. Analogous to (21), an upper bound for $\tilde{\gamma}_1(u_d^2)$ can be written as $\tilde{\gamma}_1(u_d^2) \le \overline{\tilde{\gamma}}_1(y,u,\xi)\ln(1+u_d^2)$ where

$$\overline{\tilde{\gamma}}_1(y,u,\xi) = [1+\overline{u}_d^2(y,u,\xi)]\sum_{i=1}^{n_{\epsilon,1}} p_{\epsilon,i,1}\overline{u}_d^{2i-2}(y,u,\xi). \quad (28)$$

A composite Lyapunov function is defined as:

$$V = V_{x\xi} + c_\eta\tilde{V}_\eta + c_u V_u \quad (29)$$

with c_u being any positive constant and c_η being any positive constant such that $c_\eta \le \frac{1}{8(k_{\eta 1}+2k_{\eta 2})}$. Hence, using (24), (25), (27), and (28), the inequality $\underline{V}_\eta V_\eta \le \alpha_\eta(|\eta|)$ from Assumption A3, the inequality $u_d^2 \le (1+\overline{u}_d^2(y,u,\xi))\ln(1+u_d^2)$ which follows from (23), and the inequality that $\Pi(r_u) \ge 1$ from the design of the function Π, we obtain the following inequality:

$$\begin{aligned}\dot{V} \le &-\frac{1}{2}\alpha_{x\xi}(x,\xi) - c_u r_u\underline{\tilde{\mu}}\frac{u_d^2}{(1+u_d^2)\Pi(r_u)} - \frac{c_\eta}{2\varphi(\xi)}\alpha_\eta(|\eta|) \\ &- \frac{c_u\Pi'(r_u)\dot{r}_u}{2\Pi^2(r_u)}\ln(1+u_d^2) + \frac{Q_{u1}(y,\xi,u)}{\Pi(r_u)}\frac{u_d^2}{1+u_d^2} \\ &+ Q_{u2}(y,\xi,u)\ln(1+u_d^2) \quad (30)\end{aligned}$$

where $\underline{\tilde{\mu}} = \underline{f}_{h2}\underline{\mu}$ and

$$\begin{aligned}Q_{u1}(y,\xi,u) = 2c_u^2\Big\{&k_{11}\beta_{11}(y,\xi,u) + \frac{k_{13}}{8}\beta_{13}(y,\xi,u) \\ &+ k_{21}\beta_{21}(y,\xi,u) + \frac{k_{22}}{4c_\eta\underline{V}_\eta}\beta_{22}(y,\xi,u)\varphi(\xi) \\ &+ k_{31}\beta_{31}(y,\xi,u)\Big\} \quad (31)\end{aligned}$$

$$\begin{aligned}Q_{u2}(y,\xi,u) = &\gamma_1(y,\xi)\overline{\tilde{\gamma}}(y,u,\xi) + \overline{\tilde{\gamma}}_1(y,u,\xi) \\ &+ 2c_\eta\beta_{\eta 1}(y,\xi)[1+\overline{u}_d^2(y,u,\xi)]. \quad (32)\end{aligned}$$

The dynamics of the parameter r_u are designed as

$$\dot{r}_u = \lambda(R_u(y,\xi,u) - r_u)\Omega_u(y,\xi,u,r_u) \quad (33)$$

$$R_u(y,\xi,u) = \max\Big(\overline{R}_u, \frac{2Q_{u1}(y,\xi,u)}{c_u\underline{\tilde{\mu}}}, \frac{2[Q_{u2}(y,\xi,u)+\nu_u](1+\overline{u}_d^2(y,u,\xi))\overline{\Pi}}{c_u\underline{\tilde{\mu}}}\Big) \quad (34)$$

$$\Omega_u(y,\xi,u,r_u) = \max\Big(\overline{\Omega}_u, \frac{4Q_{u1}(y,\xi,u)\Pi(r_u)}{c_u\Pi'(r_u)}, \frac{4[Q_{u2}(y,\xi,u)+\nu_u]\Pi^2(r_u)}{c_u\Pi'(r_u)}\Big) \quad (35)$$

where $\overline{R}_u$ and $\overline{\Omega}_u$ are nonnegative constants that can be arbitrarily picked and ν_u is any positive constant. $\lambda : \mathcal{R} \to \mathcal{R}^+$ is any nonnegative continuous function such that $\lambda(s) = 1$ for any $s > 0$ and $\lambda(s) = 0$ for any $s < -\epsilon_r$ with ϵ_r being some positive constant. The form of the dynamics of the scaling parameter r_u in (33) is motivated by the idea of ensuring that the derivative of the parameter (i.e., $\dot{r}_u = \Omega_u$) is large until the parameter itself becomes large.

Considering the two cases that (a) $r_u \ge R_u$ and (b) $r_u < R_u$, it can be shown that in either of the two cases (in the first case, $r_u \ge R_u$, and in the second case, $\dot{r}_u = \Omega_u$):

$$\dot{V} \le -\frac{1}{2}\alpha_{x\xi}(x,\xi) - \frac{c_\eta}{2}\underline{V}_\eta\tilde{V}_\eta - 2\nu_u V_u. \quad (36)$$

Local existence of solutions is guaranteed by the assumptions on the functions appearing in the system dynamics and the continuity (by construction) of functions appearing in the overall dynamic controller. Let the maximal interval of existence of solutions be $[0,t_f)$ where $t_f \in (0,\infty]$. By (36), V is bounded on $[0,t_f)$. From the definition of V, and noting that $\Pi(r_u)$ is a bounded function of r_u, this implies directly that x, ξ, $\tilde{V}_\eta$, and $|u_d|$ are bounded. The boundedness of x implies boundedness of y and $\tilde{y}$. Also, the boundedness of ξ implies boundedness of $\varphi(\xi)$; also, by Assumption A5, $\varphi(\xi)$ is lower bounded by a positive constant φ_0; boundedness of $\tilde{V}_\eta(\eta)$ and $\varphi(\xi)$ implies boundedness of V_η and hence of η. Boundedness of u_d and $\tilde{u} = h_\xi(y,\xi)$ implies boundedness of $\mu(x,\eta,u) = u_d + \tilde{u}$ and therefore of u by Assumption A2. Hence, from (34), $R_u(y,\xi,u)$ is bounded over $[0,t_f)$ implying boundedness of r_u since, by the dynamics of the scaling parameter r_u in (33), it is known that $r_u(t) \ge 1$ for all $t \in [0,t_f)$, $r_u(t)$ is a monotonically non-decreasing function of time, and $\dot{r}_u$ is zero if $r_u \geq (R_u + \epsilon_r)$. Finally, boundedness of ζ also follows from boundedness of u, r_u, and $\tilde{y}$ from the definition of u in (15). Hence, all the closed-loop signals $(x, \xi, \eta, \zeta, r_u, u)$ are bounded over $[0, t_f)$; therefore $t_f = \infty$ and solutions exist for all time. The stability analysis above holds globally, i.e., for any initial conditions $(x(0), \xi(0), \eta(0), \zeta(0), r_u(0)) \in \mathcal{R}^n \times S_{\xi,0} \times S_\eta \times \mathcal{R} \times [1, \infty)$. Furthermore, from (36), it is seen that while u, ζ, and ξ do not necessarily converge to zero in general, the signals $x(t)$, $\eta(t)$, and $u_d(t)$ all go to zero asymptotically as $t \to \infty$. The overall dynamic output-feedback controller is given by the combination of the dynamics of ξ in (3), the dynamic extension (ζ) and the definition of the control input u in (15), and the scaling parameter dynamics in (33).

5. CONCLUSION

A general class of uncertain nonlinear systems with dynamic input unmodeled dynamics was considered in this paper and a dynamic scaling based control redesign procedure was proposed to provide global output-feedback asymptotic stabilization. The design of the scaling parameter dynamics is analogous to the scaling parameter dynamics in our dual dynamic high-gain scaling based control designs for feedforward and strict-feedback systems and shares the same motivating philosophy that the time derivative of the scaling parameter is designed to be large (relative to an appropriately designed function of the available state variables of the closed-loop system) until the scaling parameter itself becomes sufficiently large (also relative to an appropriately designed function of the available state variables of the closed-loop system). The control redesign procedure proposed in this paper to address dynamic input uncertainties is applicable to a wide class of uncertain nonlinear systems including triangular (both upper triangular and lower triangular) and non-triangular system structures as long as a nominal control law for the unperturbed system is available with appropriate stability margins (in a Lyapunov inequality sense) and structural inequality bounds as given in Section 2 and provides a general tool for handling uncertain dynamic input perturbations. The analysis of specific classes of triangular and non-triangular system structures and the corresponding structural conditions on the functions appearing in the overall system dynamics remains a topic for further research.

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