Fault detection observer design using time and frequency domain specifications

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Abstract: This paper deals with a multi-objective fault detection observer design problem in time and frequency domain for time invariant systems. Firstly, to improve the abilities of designed observer, different design criteria are proposed, which are evaluated by some suitable performance indices in time and frequency domain. Details about the relationships among different criteria are analyzed for two cases when the fault appears and disappears. With selected residual evaluation function and threshold, a formula with an envelope in time domain to realize fast fault detection is proposed for some typical faults. The designed observer considers the trade-off between fast transients of the residual for specified faults and the traditional criterion $H_\infty$ for general faults and disturbances. Compared with $H_-/H_\infty$ frequency design method, the effectiveness of the proposed method is demonstrated by the numerical simulation with a vehicle lateral dynamics system.

Keywords: Fault detection, Frequency domains, Time domains, Multiple-criterion optimisation.

1. INTRODUCTION

As fault detection and isolation (FDI) becoming critically important in the more and more complex and integrated system such as aircrafts and petrochemical plants, a great deal of works about FDI has been done in the recently decades. And great progress has been made in searching for model based diagnosis techniques. Since the exact model of the plant is difficult to get, and various disturbances and noises will affect the system, the robustness of the system becomes an important issue to consider. Different from the robustness in control theory, the FDI also has to be sensitive to the faults. Difficult to decouple the faults and disturbances Massoumnia et al. (1989), it is reasonable to consider the trade-off between the robustness to the model uncertainty, disturbances and the sensitivity to the faults to get a satisfactory performance of a FDI system Chen and Patton (1999); Ding (2008). It is also critical to be able to detect the possible faults as early as possible so that some solutions could be taken to prevent significant performance degradation or significant damages. Therefore, the objective of the fast fault detection should be considered in the FDI design.

Currently, mixed-norm FDI problems has received much attention, and a great deal of methods are proposed. Among them, the worst case, minimum influence of faults on residual and maximum effects from the disturbances, is investigated in many literature Ding (2008); Casavola et al. (2008); Hou and Patton (1996); Wang et al. (2007b); Bouattour et al. (2011); Liu et al. (2005); Rambeaux et al. (1999); Yang et al. (2013); Ding et al. (2000); Chen et al. (1996); Chen and Patton (1999); Wang et al. (2007a). Typically, for the worst-case of the influence of faults on residual, the smallest singular value ($H_\infty$) is considered as a suitable sensitivity measurement. And the biggest singular value ($H_\infty$) is always considered as a suitable performance index to evaluate the maximum robustness of the FDI system. The mixed criterion $H_-/H_\infty$ is typically designed for unknown faults and unknown disturbances case. Ding (2008); Casavola et al. (2008); Hou and Patton (1996); Wang et al. (2007b); Bouattour et al. (2011); Liu et al. (2005); Rambeaux et al. (1999); Yang et al. (2013); Ding et al. (2000); Chen et al. (1996); Chen and Patton (1999) propose to use the technique LMI (linear matrix inequality) and ILMI (iterative LMI) to solve this mixed-norm $H_-/H_\infty$ optimization problem. In Wang et al. (2007a), pole assignment approach is utilized to transform the fault detection problem into an unconstrained optimization problem, which could be solved by a gradient based optimization method. The eigenvalues could be chosen to improve the rapidity of the residual responses. With the aid of nonsmooth optimization method, Yang et al. (2013) design a switched observer for multi model system. Without using Lyapunov variables, whose number grows quadratically when the system state size increases, the nonsmooth optimization method can calculate faster than the ILMI for the worst case design. To realize a faster fault detection, Yang et al. (2013) propose to increase the fast transients of the residual responses from fault by optimizing the eigenvalues of the transfer function from fault to residual.

The traditional method for improving the transients of the responses utilizes suitable frequency domain specifications to realize the corresponding constraints in time domain. This paper considers the transients of the residual in time domain directly, and the designed fault detection observer will not only consider the mixed criterion $H_-/H_\infty$ for
general faults and disturbances, but also have fast tran-
sients of the residual for some specified faults, such as
steps, ramps or other typical fault signal. With selected
evaluation function and threshold, the rapidity of the fault
detection will only depends on transients of the residual
responses. After the analyses of the relationship among
the different factors of fault detection in the case when
the fault appears and disappears, a procedure to generate
an appropriate setting for the envelopes in time domain
is proposed. With the aid of the Sdotool in Matlab, the
criteria in time and frequency domain could be solved to
design a multi-objectives fault detection observer. Numeri-
cal simulations are used to illustrate the effectiveness of
the results.

The paper is organized as follows. First, Section 2 for-
mulates the problem of multi-objective fault detection
observer design in time and frequency domain. Different
performance indexes are proposed to evaluate different
design objectives, either in time or frequency domain.
Then, in Section 3, the proposed multi-objective problem
is solved with a vehicle lateral dynamics system by the
tool of Sdotool in Matlab. Finally, the conclusion is given
in Section 4.

2. PROBLEM FORMULATION

2.1 Residual generation

The linear time invariant (LTI) system with faults and
disturbances is described by

$$
\Sigma_0 \begin{cases}
\dot{x}(t) = Ax(t) + Bu(t) + B_1 f(t) + B_d d(t), \\
y(t) = Cx(t) + Du(t) + D_1 f(t) + D_d d(t),
\end{cases}
$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $y(t) \in \mathbb{R}^m$
represents the output measurement vector, $f(t) \in \mathbb{R}^n$
represents the fault vector, which can be the process faults,
sensor faults, or actuator faults. $d(t) \in \mathbb{R}^n$ is the
unknown input vector, including disturbance, modeling er-
er, process and measurement noise or uninterested fault.
$u(t) \in \mathbb{R}^n$ is the control input vector. The matrices
$A$, $B$, $C$, $D$, $B_1$, $D_1$, $B_d$, $D_d$ are constant with appro-
riate dimensions. Without loss of generality, the following
assumptions are used:

- $(A, C)$ is detectable.
- $f(t)$ and $d(t)$ are $L_2$ norm bounded.

For the generation of the residual, we propose a full-order
observer for LTI model in the following from Chen and
Patton (1999):

$$
\Sigma_1 \begin{cases}
\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)), \\
\dot{\hat{y}}(t) = C\hat{x}(t) + Du(t), \\
r(t) = Q(y(t) - \hat{y}(t)).
\end{cases}
$$

The corresponding residual responses from faults and
disturbances are:

$$
r(s) = Q\{D_I + C^S(sI - A + LC)^{-1}(B_I - LD_I)\} f(s)$$

$$
+ Q\{D_d + C^S(sI - A + LC)^{-1}(B_d - LD_d)\} d(s)$$

$$
= G_{rf}(s, L, Q) f(s) + G_{rd}(s, L, Q) d(s)
$$

(4)

The residual error dynamics equations (3) with the
observer gain $L$ and the residual weighting matrix $Q$
contains the following objectives:

- The residual error dynamics equations (3) with the
observer gain $L$ should be stable (design $L$).
- Maximize the effects of faults on the residual (design
$L$ and $Q$).
- Minimize the effects of disturbances on the residual
(design $L$ and $Q$).
- Faster to detect the fault without false alarm (design
$L$, $Q$ and threshold).

One point we should notice is that the last objective
not only depends on the dynamics of the residuals, but
also depends on the selection of the evaluation function
and threshold. Design the observer ($L$ and $Q$) and
the threshold will give a faster fault detection. This paper
just focuses on the problem that if the evaluation and threshold
are selected, how to produce a suitable residual to realize
the objective of fast fault detection.

2.2 Criteria for evaluation

Considering the robustness to the disturbances or the
unknown signals of the residual, the criterion $H_\infty$ is used
in this paper,

$$
\|H\|_\infty = \sup_{\omega \in \Phi} \sigma(G(\omega))
$$

(5)

where $\sigma(G(\omega))$ denotes the maximum singular value of
matrix $G(\omega)$, and $\Phi$ is the evaluated frequency range,
which could be infinite or finite.

For the problem of fault detection observer design for
unknown faults and disturbances, we are more interested
in the “worst-case” of the fault detection, so we use $H_-$
index to evaluate the minimum sensitivity of faults to the
residual.

**Definition 1.** The index $H_-$ of a transfer function $G(s)$ is defined by

$$
\|G(s)\|_- = \inf_{\omega \in \Phi} \sigma_i(G(\omega))
$$

(6)

where $\sigma_i(G(\omega))$ denoting the minimum singular value of
matrix $G(\omega)$, and $\Phi$ is the evaluated frequency range,
which can be either infinite or finite.

For the problem of fault detection, the rapidity to detect
fault is an important criterion to design the observer. One
factor to affect the time to detect fault is the threshold...
selection. Nevertheless, the threshold is normally dependent on the disturbances but not the faults. This paper will consider to improve the rapidity of the fault detection by residual generation with selected threshold to detect faults faster. The adaptive residual threshold evaluation is introduced in Frank and Ding (1997), which depends upon the nature of the system uncertainties and varies with the system disturbances. The time windowed root mean square (RMS):

$$J_{RMS} = \|r\|_{RMS} := \left( \frac{1}{T} \int_{t}^{t+T} r^T (\tau) r (\tau) \, d\tau \right)^{\frac{1}{2}}$$

(7)

where $T$ is the finite time window.

In the absence of any faults, the threshold should be bigger than $\|r\|_{RMS}$:

$$J_{th} = \sup_{f=0} J_r (t)$$

Under fault-free condition, we have the following relationships:

$$J_{th} = \|r (t)\|_{RMS}, f=0 = \|G_{rd} d (t)\|_{RMS} \leq \|G_{rd} \|_{\infty} \|d (t)\|_{RMS} \leq \|G_{rd} \|_{\infty} \max (\|d (t)\|_{RMS})$$

To detect the fault, the logic decision unit we consider could be:

$$\begin{cases} J_{RMS} > J_{th} & \text{alarm} \\ J_{RMS} \leq J_{th} & \text{no alarm} \end{cases}$$

In order to improve the fast transients of the residual from faults, Yang et al. (2013) proposed to optimize the eigenvalues of the transfer function from fault to residual. In a low order systems (e.g. second order system), the constraints of the eigenvalues are appropriate to increase the rapidity of the responses. However, in some cases, especially for a high order systems, optimizing the eigenvalues cannot give a good transients of the residual. Normally, a high order transfer function could be separated as some different low order transfer functions:

$$G_{rf} (s) = \sum_{i=1}^{n} g_i (s) = \sum_{i=1}^{n} \frac{a_i (s)}{b_i (s)}$$

where $b_i (s)$ is a first order or second order transfer function, the real part of whose eigenvalues is represented as $\lambda_i$ ($\lambda_i < 0$ when $G_{rf} (s)$ is stable). We assume that

$$\max_{i=1, \ldots, n} (\lambda_i) = \lambda_j$$

In general, the eigenvalues, which has the maximum real part, will determine the major transients of the residual. However, if the weights in $a_j$ is much smaller than the weights in $a_i (i \neq j)$, the dynamics of the responses will not mainly depend on the part of $g_j (s)$. In other words, the $\lambda_j$ can not be used to evaluate the transients of the residual response of the fault exactly. In this case, to develop the transients of the responses, we not only should consider

$$r_{i, min} (t) \leq r_i (l, q, t) \leq r_{i, max} (t), \quad \forall t \geq 0, \ i \in I := \{1, \ldots, n_r\}$$

As shown in Fig. 1, normally, the low envelope $r_{i, min} (t)$ in (8) is a constraint to produce a suitable residual for fast fault detection. Therefore, the residual should react as fast as possible when fault appears. It means that the constraint of the rise time $C_{rise}$ should be as small as possible, which may be result in a large overshoot. The constraint of the upper envelop $r_{i, max} (t)$ will restrict the large overshoot, thus it will affect the effects of minimizing the $C_{rise}$.

To evaluate the effects of fast fault detection, the time $\{t_{detect} \| r (t_{detect}) \|_{RMS} \geq J_{th}, \| r (t_{detect} - \xi) \|_{RMS} < J_{th} \}$

where $\xi$ is a tiny positive value.

However, as shown in Fig. 1, a phenomenon appears when the fault disappears. After the fault disappears, the residual is still nonzero if the transients of the residual is not good. In this case, the fault detection observer will give alarms even when there is no fault. To eliminate this kind of false alarms, we propose to add the constraint of the upper envelop $r_{i, max} (t)$ into the time domain constraints
The details how to design this constraint will be introduced in the next part. When the fault disappears, it is also important for the observer to detect that there is no fault. In this way, we define the time $t_{\text{disappear}}$ of the observer detect that the fault disappears:

$$
\{t_{\text{disappear}} \mid \| r(t_{\text{disappear}}) \|_{\text{rms}} \leq J_{th}, \quad \| r(t_{\text{disappear}} - \xi) \|_{\text{rms}} > J_{th} \}
$$

In other words, to improve the rapidity of the fault detection without false alarm when fault disappears, we should focus on the time $t_{\text{detect}}$ and $t_{\text{disappear}}$ together. Therefore, in the following discussion, minimizing $t_{\text{detect}}$ and $t_{\text{disappear}}$ becomes an important criterion to design the observer. In fact, considering to a step fault signal, the dynamics of the residual from the fault when fault appears and disappears (without disturbances) are upside down, which means that the trends of $t_{\text{detect}}$ and $t_{\text{disappear}}$ are same when the dynamics of the residual changes. Therefore, these two criteria could be considered as one criterion.

2.3 Transformation for calculation

Applying the criteria $H_\infty$ and $H_\infty$ into the residual model (4), the problem of fault detection observer design can be formulated as follows:

i) $A - LC$ is asymptotically stable;

ii) $\max_{L,Q} \| G_{rf} \|_\infty = \max_{L,Q \in \{L \in \omega \mid |w|, \omega \}} \sup_{t \geq 0} g(G_{rf})$,

$$
= \max_{L,Q \in \{L \in \omega \mid |w|, \omega \}} \inf_{s_I \geq A + LC} \| QD_f + QC(sI - A + LC)^{-1} \times (B_I - LD_f) \|_\infty
$$

iii) $\min_{L,Q} \| G_{rd} \|_\infty = \min_{L,Q \in \{L \in \omega \mid |w|, \omega \}} \sup_{t \geq 0} \theta(G_{rd})$

$$
= \min_{L,Q \in \{L \in \omega \mid |w|, \omega \}} \sup_{t \geq 0} \| (QD_d + QC(sI - A + LC)^{-1} \times (B_d - LD_d) \|_\infty
$$

iv) $\gamma$

$$
\begin{align*}
\gamma & \geq 0 \\
\gamma & \geq 0 \\
\end{align*}
$$

Remark 2. To transform the above multi-objective optimization problem to a more easily solved formulation, the traditional frequency design method always combines the $\max_{L,Q} \| G_{rf} \|_\infty$ and $\min_{L,Q} \| G_{rd} \|_\infty$ together to be $\min_{L,Q} \| G_{rd} \|_\infty$ and $\min_{L,Q} \| G_{rf} \|_\infty$:

$$
\min_{L,Q} \| G_{rd} \|_\infty \quad \text{and} \quad \min_{L,Q} \| G_{rf} \|_\infty
$$

2.4 Quantitative analysis for the criteria

One critical problem is how to set the upper and lower envelopes ($r_{\max}(t)$ and $r_{\min}(t)$) to produce a satisfied residual to detect fault. From a practical point of view, the residual should be designed to detect the fault faster and without any false alarms when fault disappears. Therefore, under these objectives, it is straightforward to propose the specifications like rise time, peak time, settling time, overshoot, damping, threshold. In this paper, we consider to design the residual for any deterministic fault of practical interest such as ramps, step, sinusoid, etc with selected threshold. The relationship among the different criteria and different objectives are shown in Fig. 2.

Remark 3. The relationship between the fast fault detection and short rise time. It is reasonable to design the observer to make the residual react fast when the fault appears or disappears. In the field of fault detection, a smaller fault detection time $t_{\text{detect}}$ is more interesting for the design. Only with the constraints of small rise time, there could be an oscillation in the residual, which will cause a series of peaks and troughs in the residual. The fault will be detected when the evaluated residual $\| r(t) \|$ is bigger than the threshold $J_{th}$. However, the first trough may be smaller than the threshold $J_{th}$, and the time interval between the first jump and first trough is two small for the observer to react for the alarm. Thus, besides the minimization of the rise time, there should be a constraint to bring down the oscillation of the residual.

Remark 4. The effects of the high overshoot on the rate of false alarm when fault disappears. A small rise time may cause a high overshoot. A disadvantage of the high overshoot will reveal when the fault disappears, which will result in false alarm. As shown in Fig. 1, a higher overshoot means that there will be a more violent oscillation after the fault disappears. If peak of the oscillating evaluated residual (after fault disappears) is bigger than the threshold, there will be a fault alarm even the fault disappears. Therefore, in the ideal case, to eliminate this kind of false alarms, there should be no overshoot for the residual response in time domain. One point we should notice is that it will be allowed to design the residual with some overshoots with a higher threshold. The selection of threshold gives other freedoms to optimize the transients of the residual, which will be discussed in another paper.

Remark 5. The effects of the finite time window $T$ in (7). A big finite time window will not increase the fault detection time $t_{\text{detect}}$, but will delay the time $t_{\text{disappear}}$. 

![Fig. 2. Relationships among different criteria](image-url)
It will also decrease the effects of the fast transients of the residual. To illustrate the effects of the design with constraints in time domain, the finite time window \( T \) will be small in the design.

### 2.5 Two types of cases

Let \( D_f = 0 \) in the case of \( D_f = 0 \) in (1), the fault should be an actuator fault. And the corresponding transfer function from the fault to the residual \( G_{rf} \) will be strictly proper. Considering that if the fault is kind of step signal, the corresponding responses of the fault \( f(t) \) on the residual \( r(t) \) at \( t = 0 \) will be zero. In this case, when we design the envelope for the residual in time domain (8), just following the analysis in the previous part is enough to get a suitable response of the residual in time domain.

Let \( D_f \neq 0 \) Different from the case of \( D_f = 0 \), when \( D_f \neq 0 \) in (1), the transfer function \( G_{rf} \) is biproper, which will result in a nonzero initial responses in \( r(t) \) from the fault \( f(t) \). And the corresponding value is dependent on the parameters \( D_f \) of the system. Without suitable design, the nonzero initial value in the response will cause a trend downward in a short time interval. This kind of phenomenon will delay the time to detect the fault (increasing \( t_{detect} \)) and cause false alarm when fault disappears. In order to utilize the nonzero initial value of the response, a constraint of the minimum value for the residual after the residual begins to decrease. In some cases, with this kind of design, we even can realize a zero time fault detection without false alarms when the fault disappears.

### 3. NUMERICAL EXAMPLE

To compare the proposed method with the traditional frequency method, there are two types of criteria to optimize:

- The \( H_-/H_\infty \) frequency design method:
  
  \[
  \min_{L, Q} \frac{\|G_{rd}\|_\infty}{\|G_{rf}\|_\infty} \quad (11)
  \]
  
  subject to \( \text{system is stable} \)

This frequency design method focuses on the unknown faults and disturbances.

- The time and frequency domain design:

The complex criteria just consider the time domain constraints for the specified faults, \( H_- \) for general faults and \( H_\infty \) for the disturbances. As introduced in the previous part, the criteria in time domain and frequency domain should be consider are:

\[
\min_{L, Q} \frac{\|G_{rd}\|_\infty}{\|G_{rf}\|_\infty} \quad (12)
\]

subject to \( r_{min} (t) \leq r_i (L, Q, t) \leq r_{max} (t) \) for all \( \ell \leq t \leq \bar{t} \)

system is stable

A solver named Sdotool in Matlab could be used to tune the observer (observer gain \( L \) and residual weighting matrix \( Q \)) to satisfy time and frequency domain design requirements. Here, we use the custom objective module to minimize \( \|G_{rd}\|_\infty / \|G_{rf}\|_\infty \) and use the module of step response envelope to satisfy the criterion in time domain.

#### 3.1 Fault detection observer design for comparison

To illustrate the effectiveness of the introduced method to design an observer in time and frequency domain, here is an example from Ding (2008). The simulation is implemented using Matlab Sdotool Toolbox and Simulink. The benchmark is about a simplified vehicle lateral dynamic system.
where the residual weighting matrix $Q_2$ is designed to make the steady value of the residual response from fault be 1. With the same value of criterion $\|G_{rf}\|_\infty/\|G_{rf}\|_\infty$ for the general case, we can find that the constraint of fast transients for typical fault does not decrease the ability of fault detection in the worst case.

Fig. 3. Simulation with $H_-/H_\infty$ design

\[ L_2 = [0.1407, 0.8534]^T, \]
\[ Q_2 = 0.014, \]
\[ \left\|G_{rd}\right\|_\infty/\left\|G_{rf}\right\|_\infty = 0.1392 \]

The effects of the design are showed in the Fig. 4. The dynamics of the residual with $L_1$ and $Q_1$ are much better than the residual with $L_1$ and $Q_1$. Comparing the evaluated residual with the selected threshold, the residual with $L_2$ and $Q_2$ can detect the fault as soon as the fault appears, and the corresponding $t_{\text{disappear}}$ is nearly zero.

4. CONCLUSIONS

In this paper, we deal with a multi-objective observer design problem for fault detection in time and frequency domain. The relationships among different criteria are analyzed for two cases when the fault appears and disappears. With selected residual evaluation function and threshold, the context proposes how to format an envelope in time domain to improve the fast transients of the residual for some specified faults. The observer is designed with multi objectives of fast transients of the residual for specified faults and the traditional criterion $H_/H_\infty$ for general faults and disturbances. The effectiveness of the proposed method is compared with $H_/H_\infty$ frequency design method by the numerical simulation with a vehicle lateral dynamics system.

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