Design of Triple-Level Multiple Models Fuzzy Logic Controller for Adaptive Speed Control with Unknown External Disturbances

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Abstract:

This paper presents an Triple-Level Adaptive Multiple Models Fuzzy Logic Controller (TAM-FLC) with model identification used for speed control in an unmanned vertical parking process. Adaptive Multiple Models fuzzy control ensures good performance with fast convergence and minimal chattering despite of variation in external disturbances. In contrast to conventional fuzzy logic controller where intensive controller tuning is required to achieve optimal control performance, the addition of Triple-Level adaptation in the proposed TAM-FLC increases the controller robustness to uncertainties in system parameters, and also the use of multiple models increase the accuracy of controller significantly. The proposed TAM-FLC is proven to stabilize and experimental results is given to prove the improvement compared with previous designs.

1. INTRODUCTION

Speed control is one of the most widely explored research topics in the area of autonomous driving systems. The conventional approach of PID control has been implemented in various applications (Jan, 2008; Ren, 2008). Although more advanced control theories have been developed and proved, PID controller still remains as the best option considering the balance among control performance, implementation cost and operational complexities. However, it is quite challenging to design an optimal PID controller. First of all, intensive efforts are required to tune the PID controller if the exact plant model cannot be identified. Unfortunately, in many practical systems, the plant parameters cannot be directly measured, especially in the case of non-linear systems. Secondly, a plant in operation is affected by unpredictable disturbances. Such indeterminacies cannot be taken care of in the controller design and may negatively affect the control performance. Last but not least, stability of PID controllers cannot be promised with unknown system parameter and external disturbances.

Recently, Fuzzy Control has been used to replace traditional PID control in many applications. Fuzzy Control is a rule-based decision making which simulates the way of human reasoning. Unlike traditional controllers where numerical parameters are used, fuzzy controllers define linguistic variables using fuzzy sets. This underlying fuzzy logic improves robustness in dealing with system uncertainties and external disturbances. The work by (Malhotra, 2011; Chitra, 2006) demonstrates successful application of fuzzy logic in motor speed control. In this paper, a different approach is proposed to control the motor speed in the intelligent autonomous parking system presented in (Wang & Zhu, 2012). The proposed Triple-Level Adaptive Multiple Models Fuzzy Logic Controller (TAM-FLC) can be viewed as a combination of two sub-systems which function simultaneously to accomplish the control objective. On one side, an identification model is built to identify the plant characteristics of the electric motor. On the other side, a multiple model based fuzzy logic controller with adaptive parameters is used to adjust the motor speed. The detailed design of the TAM-FLC is discussed in Section 3. In the end, experiments are conducted to verify the improvement achieved by using the new TAM-FLC.

2. BACKGROUND

In this section, a brief review on previous works is given. Firstly, a Base Fuzzy Logic Controller (BFLC) is designed, which makes intelligent decisions on the vehicle speed during an unmanned vertical parking process. The BFLC discussed here is a further improvement of the previous Fuzzy On-board Controller (FOBC) in (Wang & Zhu, 2012). The second part introduces a Hybrid Fuzzy Logic Controller (HFLC), where a Supervisory Fuzzy Logic Controller (SFLC) is imposed on the original BFLC to counteract the indeterminacies to achieve better performance (Wang & Zhu, 2013). These gives some preliminaries for the new designed approach to be discussed in section 3.

2.1 Base Fuzzy Logic Controller

The ultimate goal of vertical parking is to turn the vehicle around a right angle while moving backward. It may be intuitive to maintain the vehicle at a constant low speed during the process. However this conservative

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approach delays parking and eventually impacts the overall system efficiency as parking is the most critical and time-consuming step in the Multi-Functional Autonomous Parking System in (Wang & Zhu, 2012). Therefore it is designed such that the speed varies in different stages, while the general principle is to start at a relatively large speed and reduce the speed gradually. A simple feedback control with fuzzy rule base can be implemented to achieve the goal. Assume the final position is defined as θ_f , and the current position defined as θ . Thus error is the angular difference between current position and final position, thus $e(t) = \theta - \theta_f$, which is the input linguistic variable of fuzzy rule base. The output is an electrical signal i_b sent to the electrical motor, which governs the rotational speed of motor, eventually the linear speed of the vehicle. Both input and output linguistic variables can be mapped to the following fuzzy sets: zero, small, medium and large. The respective membership function is given in Fig.1. The simple fuzzy rule base can be summarized as:

IF e(t) is LARGE, then i_b is LARGE

- IF e(t) is MEDIUM, then i_b is MEDIUM
- IF e(t) is SMALL, then i_b is SMALL

IF e(t) is ZERO, then i_b is ZERO.



Fig. 1. Membership Functions for Inputs/Output in Speed Control



Fig. 2. Position Illustration for Vertical Parking

Define clockwise direction as the positive direction. As illustrated in Fig.2, starting position A is defined as $\theta_a =$ 0° and the final position is defined as $\theta_f = 90^{\circ}$. The vehicle is turning in clockwise direction, hence the angular velocity is positive. Note that the steering angle of the front wheel is towards the counter-clockwise direction, hence the steering angle ρ is negative. Similarly, if the starting position is B, where $\theta_b = 180^{\circ}$. Both error e(t) and angular velocity ω are negative, while the steering angle ρ of front wheels is positive. Thus it can be concluded that the steering angle and the angular position error should always be of opposite signs. Note that this conclusion is only applicable when the vehicle is moving backward. If the vehicle is moving in the forward direction, the error and steering angle should be of the same sign in order for the vehicle to approach the set position.

Intuitively, moving forward is considered as moving in the positive direction. Fuzzy sets of BFLC output is defined in alignment with the notation and have the following values: negative large (NL), negative small (NS), zero (ZO),



Fig. 3. Membership Functions for Inputs/Output in BFLC

positive small (PS), and positive medium (PM). A positive output directs the vehicle to move forward, while negative output moves the vehicle backward. The membership function of the control signal is presented in Fig.3. The output is designed to be asymmetric. Moving forward is only a correction effort and shall take a small value. The two fuzzy inputs can be mapped to the same fuzzy sets, PL, PM, PS, ZO, NS, NM, NL, with their membership functions illustrated in Fig.3. Take the following IF-THEN rule as an example.

IF e(t) is NS, ρ is NS, i_b is PS.

A negative error indicates that there is an overshoot where the vehicle has already passed the target position. Moving backward with a negative steering angle will further increase the overshoot. Based on the above IF-THEN rule, the controller shall force the vehicle to move forward to compensate the overshoot. The complete fuzzy rule base of revised BFLC is summarized in Table I.

ρ	NL	NM	NS	zo	PS	PM	PL
NL	PS	PS	PS	zo	NS	NS	NM
NM	РМ	PS	PS	zo	NS	NM	NL
NS	РМ	PM	PS	zo	NM	NM	NM
zo	zo	zo	zo	zo	zo	zo	zo
PS	NM	NM	NM	zo	PS	РМ	РМ
PM	NL	NM	NS	zo	PS	PS	РМ
PL	NM	NS	NS	zo	PS	PS	PS
	1.4						

Table I: IF-THEN rules for BFLC

2.2 Hybrid Fuzzy Logic Controller

The fuzzy rule base of BFLC is robust such that the vehicle is able to reach the target position under various conditions. However, the actual control performance, in terms of time, overshoot, final position, varies due to indeterminacies arising from external environment, one of which is the road surface friction. To further improve the control performance, a Supervisory Fuzzy Logic Controller (SFLC) is proposed in (Wang & Zhu, 2013). The same principle can be applied to the revamped BFLC.

The output of BFLC is an electrical signal i_b governing the rotational speed of servo motor. Ideally there exists a linear one-to-one mapping between i_b and the actual vehicle speed. In reality, the speed may deviate from the desired value due to variation in friction. The Hybrid Fuzzy Logic Controller (HFLC) is designed to manage the complexities. Fig.4 illustrates the overall structure of the proposed HFLC, which has a Supervisory Fuzzy Logic Controller (SFLC) imposed on the original BFLC.



Fig. 4. Control Diagram of HFLC

The elegant design of HFLC (Wang & Zhu, 2013) demonstrates significant improvement in control performance compared with BFLC. However the improvement starts to diminish when the external disturbances exceed certain limits. The reason is that, be it the main control signal or the corrective control signal, the fuzzy sets are pre-defined from experiments. Therefore, the maximum adjustment can be achieved is also limited to a certain level. In order to overcome the constraints, a Triple-Level Adaptive Multiple Models Fuzzy Logic Controller (TAM-FLC) is proposed.

3. TRIPLE-LEVEL ADAPTIVE MULTIPLE MODELS FUZZY LOGIC CONTROLLER

The process of adaptation ensures optimal performance in spite of variation in plant parameter and external disturbances. Auto-adaptation also saves many efforts in controller tuning and can be easily used on different plants. The use of multiple models and even multiple levels adaptive fuzzy control enhance the accuracy of the controller and result in a faster and smoother convergence. Fig.5 shows the control diagram and the working principle of Triple-Level Adaptive Multiple Models Fuzzy Logic Controller (TAM-FLC) is discussed in extensive details in this section.



Fig. 5. Control Diagram for TAM-FLC

The principal ideas involved in Triple-Level Adaptive Multiple Models Fuzzy Logic Controller (TAM-FLC) are discussed extensively as following, which include the mathematical preliminaries of adaptive fuzzy control and its extention to multiple model based adaptive fuzzy controller.

3.1 System Model Description

Unlike the Base Fuzzy Logic Controller (BFLC) and Supervisory Fuzzy Logic Controller (SFLC) where no mathematical model of the plant is required to design the controllers, the Triple-Level Adaptive Multiple Models Fuzzy Logic Controller (TAM-FLC) need to find a mathematical model of the system, on which the adaptive control law can be applied. In terms of vehicle speed control, the plant model should define the relationship between motor speed, driving force, motor parameters, and friction. One good choice is to use the mechanical model of electrical motor as the plant system to be controlled, which is given in the form below:

$$J\dot{\omega_p} + B\omega_p + T_i = T_m \tag{1}$$

where ω_p is the rotational speed of the motor, and T_m and T_i are torques acting on the motor. T_m is the torque generated by control signal such that $T_m = ki_a$ while T_i represents the effects of disturbances. The motor itself is characterized by the inertia of the rotor J and the viscous damping coefficient B. However, the accurate plant model cannot be determined, since system coefficients J and B cannot be directly measured and T_i varies in different environments.

3.2 Model Identification (1st level Adaptation)

With some of parameters of the plant system are unknown, there is a necessity to identify the system in order to control it. The identification is deisgned to be on-line such that prior modelling process is not needed. This method is also known as indirect adaptive control approach, which takes the following two steps:

1) Identify the mechanical model of motor using an identification model; 2) Design an adaptive fuzzy logic controller based on the identified plant model.

By reformatting equation (1), it gives:

$$\dot{\omega_p} = a_p \omega_p + b_p u_p + D_t \tag{2}$$

where
$$a_p = -\frac{B}{J}$$
, $b_p = \frac{k}{J}$, $u_p = i_a$, $Dt = -\frac{T_i}{J}$.

The objective is to control the vehicle speed, despite any internal and/or external uncertainties. Mathematically the control objective can be expressed as the model equation below:

$$\dot{\omega_m} = a_m \omega_m + b_m r \tag{3}$$

where r is the reference input, a_m and b_m are model parameters, and $\dot{\omega_m}$ is the desired motor speed if controller is tuned to compensate all indeterminacies.

In order to identify plant parameters a_p and b_p , an identification model is built as:

$$\dot{\widehat{\omega}}_p = a_m \widehat{\omega}_p + [\widehat{a}_p - a_m] \omega_p + \widehat{b}_p u(t) + D_t \tag{4}$$

where \hat{a}_p is an estimate of the unknown plant parameter a_p The unknown motor parameter can be identified through an adaptation process. The adaptive law must be carefully examined to ensure the estimation error converges. Lyapunov method is used to design the adaptive law and system stability is gauranteed. Define the Lyapunov candidate function as:

$$V(e_p, \widetilde{a}_p, \widetilde{b}_p) = \frac{1}{2} [e_p{}^2 + \widetilde{a}_p^2 + \widetilde{b}_p^2]$$
(5)

where

$$e_p(t) = \widehat{\omega}_p(t) - \omega_p(t) \tag{6}$$

$$\widetilde{a}_p = \widehat{a}_p(t) - a_p \tag{7}$$

$$b_p = b_p(t) - b_p \tag{8}$$

Taking the derivative of V w.r.t t gives:

$$\dot{V} = e_p \dot{e}_p + \widetilde{a}_p \dot{\widetilde{a}}_p + \widetilde{b}_p \dot{\widetilde{b}}_p \tag{9}$$

Substitute equation (3), (4) and (6) into (9) and the derivative can be re-written as:

$$\dot{V} = a_m e_p{}^2 + \tilde{a}_p (e_p u_p + \dot{\tilde{a}}_p) + \tilde{b}_p (e_p u + \dot{\tilde{b}}_p)$$
(10)

The system is stable if the Lyapunov function is locally positive definite and its derivative is locally semidefinite negative. To fulfil the requirement the adaptive law is defined such that:

$$\dot{\tilde{a}}_p = -e_p(t)\omega_p(t) \tag{11}$$

$$\widetilde{b}_p = -e_p(t)u(t) \tag{12}$$

Since a_m is always negative for a stable reference model, $\dot{V} = a_m e_p^2$ is also negative.

In addition, the error convergence can be shown by:

$$-\int_{t_0}^{\infty} \dot{V}(e(\tau)\tilde{a}_p(\tau)\tilde{b}_p(\tau)d\tau = V(t_0) - V(\infty) = 0 \quad (13)$$

hence:

$$0 \leqslant \int_0^\infty (e_p)^2(\tau) d\tau < \infty \tag{14}$$

where $e \in \mathcal{L}^2$

As \dot{e} is bounded, it follows by Barbalat's lemma that $\lim_{t\to\infty} e_p(t)=0$

3.3 Multiple Models Fuzzy Logic Controller

The stability analysis above proves that the designed identification model can be used to identify the motor parameter a_p and b_p . With the system parameters known, a desired feedback control signal can be derived by combining (3) and (4), which is:

$$u_p^* = \theta^*(t)\omega_p(t) + \phi^*r + D_p \tag{15}$$

$$\theta^* = b_p^{-1}(a_m - a_p) \tag{16}$$

$$\phi^* = b_n^{-1} b_m \tag{17}$$

$$D_p = -b_p^{-1} D_t \tag{18}$$

Howerver, the adaptation process discussed in section 3.2 only identifies the internal parameters, the external disturbance still remains unknown. The theoretical optimal control signal u^* cannot be found. The conventional adaptive law does not give us enough information to decide the output of the fuzzy controller. Besides, to overcome the weakness of low accuracy by using fuzzy logic, multiple models are used to enhance the accuracy and speed of the controller. The 2nd and 3rd levels of the Triple-Level Adaptive Multiple Models Fuzzy Logic Controller (TAM-FLC) are designed to address the issues. As the plant model of electrical motor can be modelled as a linear system, a small fuzzy rule base is enough for the decision making. In addition, the output fuzzy set is designed to be adaptive, meaning that parameters $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n$ are adjustable. Adaptive fuzzy output ensures stable performance even when the road surface friction varies. Further, multiple models are built to identify the controller by restricting the unknown parameters in a convex hull.

The defuzzified output of tradtional fuzzy controller can be expressed as:

$$u_{fz}(\alpha) = \alpha^T \xi \tag{19}$$

where $\alpha = [\alpha_1, \alpha_2, ..., \alpha_m]^T$ is a parameter vector and $\xi = [\xi_1, \xi_2, ..., \xi_m]$ is a regressive vector with ξ_i defined as

$$\xi_i = \frac{w_i}{\sum_{i=1}^m w_i} \tag{20}$$

According to Universal Approximation Theorem, there exists an optimal fuzzy control signal such that

$$u^*(t) = u^*_{fz}(\alpha^*) + \epsilon = \alpha^{*T}\xi + \epsilon \tag{21}$$

where ϵ is the approximation error bounded by $|\epsilon| < E$. The actual fuzzy logic controller with adjustable parameters can be viewed as an identification model of the optimal fuzzy controller, and equivalently an estimator of u^* , which can be expressed as

$$\widehat{u}_{fz}(\widehat{\alpha}) = \widehat{\alpha}^T \xi \tag{22}$$

Multiple models fuzzy logic controllers: Instead of using single identification model of fuzzy controller, N models are used to identify the desired fuzzy logic controller to improve the accuracy and speed simultaneously.

The N identification controllers $\Sigma_1, \Sigma_2, ..., \Sigma_N$ can be expressed as:

$$\Sigma_i : \widehat{u}_{fz}^{(i)}(\widehat{\alpha}^{(i)}) = (\widehat{\alpha}^{(i)})^T \xi, i \in 1, 2, ..N$$
(23)

where $(\widehat{\alpha}^{(i)})^T = [\alpha_1^{(i)}, \alpha_2^{(i)}, ..., \alpha_m^{(i)}]^T$ is the parameter for i^{th} identification controller, their initial values can be chosen to set up a convex hull such that the single identification controller described by (22) can be expressed as a linear combination of N multiple identification controllers, i.e.

$$\widehat{u}_{fz}(\widehat{\alpha}) = \widehat{\alpha}^T \xi = \sum_{i=1}^m (\gamma_i(\widehat{\alpha}^{(i)})^T) \xi$$
(24)

for $\sum_{i=1}^{m} \gamma_i = 1$ and $\gamma_i \ge 0$. This indicates that the desired fuzzy controller will fall in the convex hull of the N identification controllers.

Also the output error for each of the i^{th} identification controller is expressed as:

$$\widetilde{u}_{fz}^{(i)} = ((\widehat{\alpha}^{(i)})^T - \alpha^{*T})\xi = (\widetilde{\alpha}^{(i)})^T \xi$$
(25)

2nd level controller:

The system model and its estimator can be re-written as:

$$\dot{\omega_p} = a_p \omega_p + b_p u^* \tag{26}$$

$$\dot{\widehat{\omega}}_p^{(i)} = a_m \widehat{\omega}_p^{(i)} + (a_p - a_m)\omega_p + b_p \widehat{u}^{(i)} \tag{27}$$

where $\hat{u}^{(i)} = \hat{u}_{fz}^{(i)} + \eta^{(i)}(t)$. $\eta^{(i)}(t)$ is the transient error between $\hat{u}^{(i)}$ and $\hat{u}_{fz}^{(i)}$ and it can be proven that $\eta^{(i)}(t)$ is bounded. $\hat{\omega}_{p}^{(i)}$ is the resulted motor speed given the controller is $\hat{u}^{(i)}$

Subtract (27) from (3), the following is obtained:

$$\dot{e_p}^{(i)} = a_m e_p^{(i)} + b_p (\widehat{u}_{fz}^{(i)} + \eta^{(i)}(t) - \epsilon^{(i)})$$
(28)

where $e_p^{(i)} = \hat{\omega}_p^{(i)} - \omega_m$.

Choose a Lyapunov candidate function as:

$$V = \frac{1}{2} (e_p^{(i)})^2 + (\widetilde{\alpha}^{(i)})^T \widetilde{\alpha}^{(i)} \ge 0$$
(29)

therefore, the derivative of Lyapunov candidate is:

$$\dot{V} = e_p \dot{e}_p^{(i)} + (\tilde{\alpha}^{(i)})^T \dot{\tilde{\alpha}}^{(i)}$$
(30)

$$\dot{V} = a_m (e_p^{(i)})^2 + (\widetilde{\alpha}^{(i)})^T (e_p^{(i)} b_p \xi + \dot{\widetilde{\alpha}}^{(i)}) + e_p^{(i)} b_p (\eta^{(i)}(t) - \epsilon^{(i)})$$
(31)

To ensure stability, the adaptive law is chosen such that the second term on the right side of (31) is always zero. i.e.

$$\dot{\widetilde{\alpha}}^{(i)} = -e_p^{(i)} b_p \xi \tag{32}$$

In addition, define

$$\eta^{(i)}(t) = -Esgn(e_p) \tag{33}$$

which ensures that $e_p^{(i)}b_p\eta^{(i)}(t) - \epsilon^{(i)}$ is always negative. Therefore, \dot{V} is always negative and the system is stable.

3rd level controller:

Though the adaptive law at 2nd level has ensured the stability of the controller, the parameter information of each identification controller is still used seperately during the control process. One of the idea to further enhance the controller's performance is to combine the information of multiple controllers instead of just using one. Intriguing by this, the 3rd level controller will be implemented based on the convex combination property as shown in (24). By subtracting (22) from (24), it gives us:

$$\sum_{i=1}^{m} \gamma_i((\widehat{\alpha}^{(i)})^T - \widehat{\alpha}^T) = 0$$
(34)

where $\sum_{i=1}^{m} \gamma_i = 1$. For further simplicity, define $\hat{\alpha}^{(i)} - \hat{\alpha} = \phi_i, \Phi = [\phi_1 - \phi_m, \phi_2 - \phi_m, ..., \phi_{m-1} - \phi_m]$ and $\gamma = [\gamma_1, \gamma_2, ..., \gamma_{m-1}]$. Thus, (34) becomes

$$\Phi^T \gamma = -\phi_m \tag{35}$$

multiplying both sides by Φ , we have

$$\Phi \Phi^T \gamma = -\Phi \phi_m \tag{36}$$

Therefore, 3rd level controller can be constructed by using the differential equation to estimate γ , which is also the weight of each identification controller in the linear convex combination. The 3rd level control law is:

$$\dot{\widehat{\gamma}}(t) = -\Phi \Phi^T \widehat{\gamma}(t) - \Phi(t)\phi_m(t)$$
(37)

Hence,

$$\dot{\widetilde{\gamma}}(t) = -\Phi \Phi^T \widetilde{\gamma}(t) \tag{38}$$

where $\widetilde{\gamma} = \widehat{\gamma} - \gamma$ is the estimation error.

To examine the stability of our 3rd level controller, using a Lyapunov Candidate function as

$$V(\tilde{\gamma}) = \frac{1}{2} \tilde{\gamma}^T \tilde{\gamma} \tag{39}$$

Which follows that

$$\dot{V}(\tilde{\gamma}) = -\tilde{\gamma}(t)^T \Phi \Phi^T \tilde{\gamma}(t) = -||\Phi^T \tilde{\gamma}(t)||^2 \le 0$$
(40)

Hence, since the derivative is non-negative, (40) is stable and $\tilde{\gamma}(t)$ is bounded.

4. EXPERIMENT RESULTS

To test the performance of the proposed Triple-Level Adaptive Multiple Models Fuzzy Logic Controller (TAM-FLC), comparative experiments are using the same vehicle prototype in (Wang & Zhu, 2012) and (Wang & Zhu, 2013)

These data are further processed and forwarded to the software-based controller simulated in MATLAB. The performance of three different controllers, i.e. BFLC, HFLC and TAM-FLC discussed in Section 2 and Section 3, are compared based on tracking error convergence and oscillation. The results are discussed in more details below.

In order to implement the BFLC and HFLC, the parameters in output fuzzy linguistic variables should be determined. The values are tuned by trial and error to achieve a satisfactory control performance. One set of the parameters collected from experiments is given in Table III and IV.





Two different sets of experiments are conducted to compare the performance of different controllers. The reference input of the first experiment is a square wave signal with a period of 50 seconds and that of the second experiment is a sinusoidal signal with a frequency of 0.2 Hz. Performance of the three controllers are evaluated based on the tracking error between desired motor speed ω_m as in model equation and actual motor speed measured $\hat{\omega}_p$. The comparative experimental results are shown in Fig.6 to Fig.8.

Based on the experiment results shown in Fig.6, the tracking error of BFLC is large and follows the same trend as the reference input within each period. It shall not be taken wrongly that the BFLC cannot achieve the control objective. The convergence time required by a BFLC is large compared to the period of reference signal, hence the output cannot stabilize with high frequency disturbances.

Compared with BFLC, the control performance of HFLC is better since the tracking error converges to zero in both cases. However, the convergence time is relatively long and the overshoot is large at the initial stage. This is a constraint arising out of the design where parameters in the fuzzy sets are pre-defined.

It is obvious that among the three, TAM-FLC delivers best control performance in terms of convergence time and overshoot. As illustrated in Fig.8, the convergence time of TAM-FLC is only one third of that of HFLC, while the initial overshoot is also much smaller.



Fig. 6. Tracking Error for BFLC



Fig. 7. Tracking Error for HFLC



Fig. 8. Tracking Error for TAM-FLC

5. CONCLUSIONS AND FUTURE WORK

In this paper, a Triple-Level Adaptive Multiple Models Fuzzy Logic Controller with model identification is proposed for speed control during vertical parking in an unmanned intelligent parking system. As an great improvement of previous work in (Wang & Zhu, 2012) and (Wang & Zhu, 2013), it is focused on design of triplelevel multiple model based fuzzy control to achieve better performance. The rule-base reasoning of a fuzzy logic controller ensures the robustness of system performance under external disturbances. The process of triple-level auto-adaptation results in faster and smoother convergence while at the same time reduce efforts in controller tuning. Furthermore, the use of multiple models enhance the accuracy of the controller and by introducing model identification, variations resulting from uncertainties in plant model can be minimized. Therefore the controller can be used in different systems with minimal changes. A detailed stability analysis of the proposed system is also discussed in the paper.

It shall be noticed that there is an error between the fuzzy control signal \hat{u}_{fz} and the optimal control signal u^* . Although the error is bounded, it can be significant under certain conditions. Hence one important area of future improvement should be focused on minimizing the bounded error by adjusting the adaptive laws. Another potential research area is combined with decision tree (Wang & Zhu, 2014). The decision making process with trained decision tree is based on fuzzy inference which further improves the robustness of the controller.

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