Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

On Experiment Design for Identification of Ill-Conditioned Systems

Kurt E. Häggblom

Process Control Laboratory, Department of Chemical Engineering,
Åbo Akademi University, FI-20500 Turku, Finland.
E-mail: khaggblo@abo.fi

Abstract: In this paper experiment design for identification of ill-conditioned systems is studied. A short overview of recently proposed techniques is presented. These are mainly based on a singular value decomposition (SVD) of an estimated gain matrix. A summary of this approach with some extensions is given. Another approach is to find a D-optimal solution; the result is essentially the same as found by SVD methods. A result is that it is very important properly to excite the so-called low-gain direction of the system. The methods are motivated by the desire to guarantee integral controllability in model based control designs such as model predictive control (MPC). The dynamics of the process have not been a consideration in these works. However, it is well known from practical studies and simple models that high gains tend to be associated with slow dynamics and low gains with fast dynamics. For experiment design, it is useful to know how general this behaviour is. In this paper it is shown analytically that this indeed is a general property. Simple examples from the literature are used to support the result. Some possible modifications to existing design methods are given for situations, where the dynamics are not aligned with the gain directions.

Keywords: System identification, Multivariable systems, Ill-conditioned systems, Gain directionality, Experiment design, Input signals.

1. INTRODUCTION

A successful system identification requires data that are truly representative of the system to be identified. To obtain such data, the experiment design for the identification is of utmost importance. In this respect, multiple-input multiple-output (MIMO) systems are much more challenging than single-input single-output (SISO) systems. In the major textbooks on system identification, surprisingly little is said on the subject of experiment design for MIMO system identification.

1.1 Multi-SIMO, Multi-MISO, and MIMO Identification

Isermann and Münchhoff (2011, p. 429) mention that one approach is to perturb one input after another and to identify each input-output dynamics separately as a set of SISO models. However, Andersen et al. (1989) have shown that such a multi-SIMO (single-input multiple-output) approach yields a poor description of the so-called low-gain direction of a 2×2 model for a binary distillation column. Isermann and Münchhoff also note that it is beneficial to perturb all inputs simultaneously because it saves time and yields coherent models. The input signals should then be uncorrelated, e.g., uncorrelated PRBS (pseudo-random binary sequence) signals. According to Isermann and Münchhoff (2011, p. 443) one can in such a case use SISO parameter estimation methods by treating each output separately.

If the model is to be used for simulation, the above approach is probably adequate, but for prediction and control applications, all outputs should be treated simultaneously (Ljung, 1999, p. 525). The reason is that correlations between the outputs (i.e., "directionality") are not accounted for by a multi-MISO (multiple-input single-output) approach (Koung and MacGregor, 1993; Dayal and MacGregor, 1997). Thus, a MIMO system should be identified as a full MIMO system with all inputs perturbed simultaneously. The general view is that the inputs should then be uncorrelated to ensure identifiability. In practice, it suffices that they are not completely interdependent.

1.2 Ill-Conditioned Systems

An ill-conditioned system is a MIMO system whose gain matrix has a large condition number (Skogestad et al., 1988). This means that the gain matrix is almost singular, which is caused by almost linearly interdependent matrix rows and columns. A consequence of this is that a certain combination of the inputs will be strongly amplified, whereas another, orthogonal, combination will be only weakly amplified. The strongest amplification occurs in the so-called high-gain direction, the weakest in the low-gain direction. For this reason, an ill-conditioned system is said to possess a strong directionality. These properties make identification and control difficult tasks.

Consider a system with the steady-state gain matrix K. For internal model control (IMC) based on a model with the gain matrix $\hat{K}$, Garcia and Morari (1985) have shown that the closed-loop system is robustly stably detunable if and only if

$\mathrm{Re}[\lambda_i(K\hat{K}^{-1})] > 0$, $\forall i$, where $\lambda_i(\cdot)$ is the *i*th eigenvalue of $(\cdot)$. This can be extended to apply for any multivariable controller with integral action based on $\hat{K}$ (Koung and MacGregor, 1993). For an ill-conditioned system, even small errors in $\hat{K}$ can have a strong effect on $\hat{K}^{-1}$, thus causing the integral controllability condition to be violated. However, if the errors in $\hat{K}$ are mostly aligned with the gain directions so that the directions are essentially unaffected, or if the gain directionality is reduced, larger errors can be tolerated.

The integral controllability condition can also illustrated geometrically. Assume that the signs of the inputs and the outputs are chosen to make all gains of K positive. This is usually possible when the rows (or columns) of K are nearly linearly dependent. Figure 1 illustrates the column vectors of such a system. The closer the vectors are to each other, the stronger is the directionality of the system. If the errors in $\hat{K}$ are uncorrelated, the corresponding vectors of the model, $\hat{k}_1$ and $\hat{k}_2$, end within the dashed circles. If it is possible to draw a straight "singularity line" between the vectors such that both uncertainty regions are intersected, integral controllability cannot be guaranteed (Koung and MacGregor, 1993). This also means that there is a possibility that $\det K$ and $\det \hat{K}$ have different signs. Figure 1 illustrates this situation. However, *if the uncertainties* tend to be correlated such that they *are more aligned with the gain vectors*, as illustrated by Fig. 2, *larger ellipsoidal uncertainties can be tolerated* without violation of the integral controllability condition.

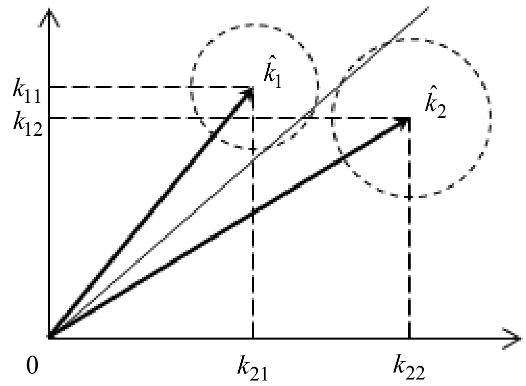

Fig. 1. Illustration of uncorrelated uncertainties.

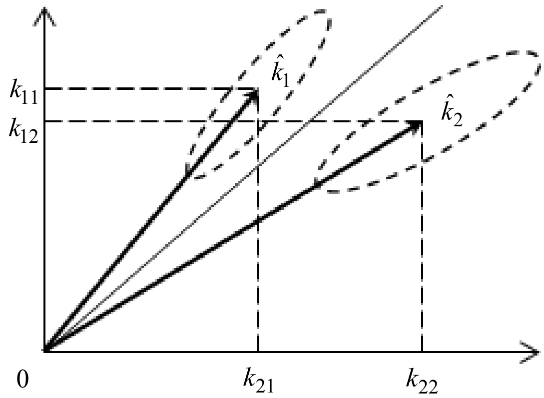

Fig. 2. Illustration of correlated uncertainties.

1.3 Experiment Design for Ill-Conditioned Systems

Koung and MacGregor (1993) have shown that the model uncertainty for an ill-conditioned 2×2 system can be shaped in the desired way if the low- and high-gain directions are explicitly excited. To arrive at this result, they employed a singular value decomposition (SVD) of the steady-state gain matrix. The SVD also showed that the strength of the directional excitations (e.g., the amplitudes of the "rotated" PRBS signals) should be inversely proportional to the respective singular values to make the outputs equally informative. This design yields linearly dependent input signals in each orthogonal gain direction. These signals, with different amplitudes, can be combined to excite both directions simultaneously in such a way that the overall sample correlations between the inputs are negligible, which is necessary for identifiability.

A potential problem with the combination of input signals is that they might yield output variations larger than intended. Of course, this has to be taken into account in the choice of input amplitudes by scaling them properly (Conner and Seborg, 1994). This consideration has been taken further by explicit inclusion of linear constraints on input and output variables (Bruwer and MacGregor, 1996; Zhan et al., 2006). The solution is obtained by minimization of a D-optimality criterion subject to the constraints.

Another problem is that the designs by necessity are based on an approximate gain matrix. If the directionality of the gain matrix is different from that of the true system, the intended strong excitation in the low-gain direction may, in fact, excite the high-gain direction to a significant degree. Therefore, it is advisable to detune the excitation(s) in the low-gain direction(s). For a 2×2 system with the condition number 141.4, Bruwer and MacGregor (1996) suggested a detuning factor as low as $c \approx 0.1$ for the low-gain excitation. Using quite advanced theoretical considerations, Darby and Nikolaou (2009) derived the detuning factor $c_i = (\hat{\sigma}_i / \hat{\sigma}_1)^{2/3}$ for the *i*th gain direction. Here, $\hat{\sigma}_i$ is the *i*th singular value of $\hat{K}$.

1.4 Design Modifications

A slight modification to the basic experiment design of Koung and MacGregor (1993) has been suggested by Zhu and Stec (2006) and later used by Vaillant et al. (2013). The low-gain direction is still excited by correlated high-amplitude input signals, but the high-gain direction is not explicitly excited. Instead, low-amplitude uncorrelated signals are added to the low-gain signals. In practice, this will excite the high-gain direction.

In the above methods, PRBS-type signals are used as perturbation signals. Rivera et al. (2007) have suggested the use of more plant-friendly multi-sine signals as perturbations in the various gain directions.

1.5 Applications

In the research mentioned above, simple linear models, mainly 2nd order 2×2 systems, have been used for illustration. Similar methods have been applied to the identification of a pilot-scale two-product distillation column by Häggblom and Böling (1998, 2013). The general conclusion is that explicit excitation of the gain directions is superior to other types of excitations (Häggblom and Böling, 2013). Even gain-directional step signals outperform uncorrelated PRBS signals. For distillation columns it is easy to find the gain directions because they are very accurately given by the product flow gains, which are easy to determine, in the so-called *LV*-structure (Häggblom, 1995).

1.6 Contribution of This Paper

The main motivation behind the mentioned design methods is to shape the uncertainty of the steady-state gain matrix so as to maximize the range for integral controllability when using, e.g., model predictive control (MPC). *The dynamics of the system has not been a consideration in the experiment design.* However, there is also a dynamic directionality as exemplified by distillation columns, for which it has been noted that the dynamics of the high-gain direction are significantly slower than the dynamics of the low-gain direction. (Skogestad and Morari, 1988a; Andersen et al., 1989). For many simplified linear models given in the literature, these directions coincide exactly because of the way they have been derived. But how general is this property?

In this paper it is shown that *there is a general connection between high gains and slow dynamics, and vice versa*. This is shown by examples from the literature, but a theoretical explanation is also provided. However, the gain and dynamic directions coincide exactly only in special cases. Based on this, a modified experiment design, aimed at providing better information about dynamics than the steady-state gain-based designs, is considered. As a prelude to this, a summary of the author's interpretation/development of the experiment design principles for identification of ill-conditioned systems based on the steady-state gain matrix is given.

2. EXPERIMENT DESIGN FOR OPEN-LOOP MIMO IDENTIFICATION

2.1 Singular Value Decomposition

Consider a system with an input vector u, an output vector y, denoted $\bar{y}$ at steady state, and a non-singular steady-state gain matrix K of size $m \times m$. A singular value decomposition of K yields

$$\bar{y} = Ku = W\Sigma V^{\mathrm{T}} u \,, \tag{1}$$

where V and W are orthogonal matrices and Σ is a diagonal matrix of singular values, σ_i, $i = 1, 2, \ldots, m$, $\sigma_1 \geq \sigma_2 \ldots \geq \sigma_m > 0$. The orthogonality means that $V^{\mathrm{T}} V = I$ and $W^{\mathrm{T}} W = I$ (Golub and Van Loan, 2013).

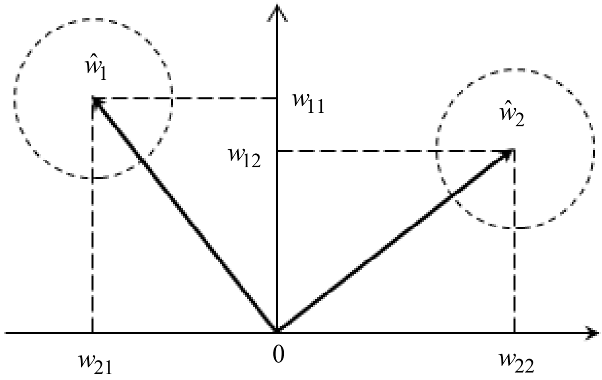

Fig. 3. Illustration of vectors in orthogonal system.

2.2 A Variable Transformation

A new input ξ is defined by

$$\xi = \Sigma V^{\mathrm{T}} u \,. \tag{2}$$

The steady-state output is then given by

$$\bar{y} = W\xi \,. \tag{3}$$

By using m linearly independent vectors ξ as inputs, W can conceptually be identified instead of K. Of course, ξ cannot be applied directly, but it can be realized in the true system by the input

$$u = \hat{V}\hat{\Sigma}^{-1}\xi \,, \tag{4}$$

where $\hat{V}$ and $\hat{\Sigma}$ are estimates of V and Σ, respectively. This use of variable transformations is analogous to the use of variable transformation for synthesizing control structures with desired properties (Häggblom and Waller, 1988, 1990).

2.3 Integral Controllability

Because W is orthogonal with all eigenvalues and singular values equal to 1, the hypothetical identification of W is a very easy task. The fact that the columns of W are orthogonal means that even comparatively large identification errors will not compromise the integral controllability condition based on W, i.e., $\mathrm{Re}[\lambda_i(W\hat{W}^{-1})] > 0$. This is also illustrated in Fig. 3. Hence, it is not required to know $\hat{\Sigma}$ and $\hat{V}$ accurately in (4). In fact, the accuracy of $\hat{\Sigma}$ is quite unimportant, since it only affects the size of the perturbations in each direction and not the actual directions.

2.4 Design Options

One way of forming m linearly independent vectors of ξ is to use $\xi_i \neq 0$ and $\xi_j = 0$, $j \neq i$, for $i = 1, \ldots, m$. If the ith column of W is denoted w_i, this yields the output $\bar{y} = w_i \xi_i$ with the 2-norm $\|\bar{y}\|_2 = |\xi_i|$. Hence, $|\xi_1| = |\xi_2| = \ldots = |\xi_m|$ makes all excitations equally "informative." By choosing $|\xi_i|$ differently, certain directions can be detuned (or amplified).

Note that ξ_1 excites the highest gain direction and ξ_m excites the lowest gain direction because of the order of the singular values in (1).

Assume that an amplitude $a_i > 0$ is selected for the perturbation ξ_i. The 2-norm of the outputs produced by ξ_i is then proportional to a_i. This amplitude should be selected with possible detuning in mind. The perturbation ξ_i can be a (series of) step change(s), a PRBS, a multisine signal, or any other type of perturbation considered adequate. The directions *i* can be excited separately (in sequence), or all together provided that all ξ_i, $i = 1,\ldots,m$, are mutually independent (i.e., uncorrelated). It is even possible to mix different types of signals. In all situations, the input to the true system is calculated by (4). Examples of excitations calculated in this way are given in Häggblom and Böling (2013).

3. CONNECTIONS BETWEEN STEADY-STATE GAINS AND DYNAMICS

3.1 From State-Space Model to Transfer Function

To show the connection between steady-state gains and dynamics, a state-space description,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B^{\mathrm{T}}u(t) \\ y(t) &= Cx(t) , \end{aligned} \tag{5}$$

is used. Here the input matrix is denoted by the transpose B^{T} to streamline the notation in the following development. The transfer function for this system is given by

$$G(s) = C(s - A)^{-1}B^{\mathrm{T}} . \tag{6}$$

Assume that the system matrix is diagonalizable. This is always true for a system with distinct real poles, but sometimes also with repeated real poles. Thus, it assumed that

$$A = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix} , \tag{7}$$

where λ_ℓ, $\ell = 1,2,\ldots,n$, are the eigenvalues of A. Denote the ℓ th column of B and C by b_ℓ and c_ℓ, respectively. The transfer function can then be expressed as

$$G(s) = \sum_{\ell=1}^{n} c_\ell (s - \lambda_\ell)^{-1} b_\ell^{\mathrm{T}} . \tag{8}$$

3.2 Steady-State Gains and Dynamics

Assume that the system is stable with the time constants T_ℓ defined by $\lambda_\ell = -1/T_\ell$, $\ell = 1,\ldots,n$. The steady-state gain matrix can then be expressed as

$$K = \sum_{\ell=1}^{n} c_\ell T_\ell b_\ell^{\mathrm{T}} . \tag{9}$$

From this expression it is clear that *large time constants tend to yield gains of large magnitude whereas small time constants hardly affect the gains at all* unless b_ℓ and c_ℓ happen to be very counteractive. Excitation of a high-gain direction will then tend to have slow dynamics whereas excitation of a low-gain direction tends to have faster dynamics.

3.3 Gain Directions and Dynamics

Substitution of the time constants into (8) yields

$$G(s) = \sum_{\ell=1}^{n} \frac{c_\ell T_\ell b_\ell^{\mathrm{T}}}{T_\ell s + 1} . \tag{10}$$

If b_ℓ happens to be exactly aligned with some v_i (i.e., the *i*th column of V), excitation of the *i*th direction according to the steady-state design will also excite the dynamics governed by T_ℓ. Moreover, an excitation orthogonal to this direction, will completely suppress T_ℓ. This happens to the slow dynamics when the low-gain direction is excited if the dynamics are aligned with the gain directions.

Of course, there are also situations when the dynamics are not aligned with the gain directions. It may even be impossible due to the structure of the model. Consider a 2×2 system with second-order transfer functions. The transfer functions for output 1 can be written

$$g_{11}(s) = \frac{k_{11}^{\mathrm{H}}}{T_{11}s+1} + \frac{k_{11}^{\mathrm{L}}}{T_{12}s+1} , \; g_{12}(s) = \frac{k_{12}^{\mathrm{H}}}{T_{11}s+1} + \frac{k_{12}^{\mathrm{L}}}{T_{12}s+1} , \tag{11}$$

where $T_{11} > T_{12} > 0$. Assume that the input $u^{\mathrm{H}} = [u_1^{\mathrm{H}} \; u_2^{\mathrm{H}}]$ excites the slower dynamics only. Then

$$k_{11}^{\mathrm{L}} u_1^{\mathrm{H}} + k_{12}^{\mathrm{L}} u_2^{\mathrm{H}} = 0 . \tag{12}$$

If the orthogonal input $u^{\mathrm{L}} = [-1/u_1^{\mathrm{H}} \; 1/u_2^{\mathrm{H}}]$ excites the faster dynamics only, $-k_{11}^{\mathrm{H}}/u_1^{\mathrm{H}} + k_{12}^{\mathrm{H}}/u_2^{\mathrm{H}} = 0$, or

$$k_{11}^{\mathrm{H}} u_2^{\mathrm{H}} - k_{12}^{\mathrm{H}} u_1^{\mathrm{H}} = 0 . \tag{13}$$

Combination of (12) and (13) yields

$$\frac{k_{11}^{\mathrm{H}}}{k_{12}^{\mathrm{H}}} = -\frac{k_{12}^{\mathrm{L}}}{k_{11}^{\mathrm{L}}} = -\frac{u_1^{\mathrm{H}}}{u_2^{\mathrm{H}}} \tag{14}$$

as a requirement for the possibility of exciting the dynamics separately. A similar expression can be derived for output 2.

Assume that the transfer functions (11) contain no zeroes. This means that

$$k_{11}^{\mathrm{L}} T_{11} + k_{11}^{\mathrm{H}} T_{12} = 0 , \;\; k_{12}^{\mathrm{L}} T_{11} + k_{12}^{\mathrm{H}} T_{12} = 0 , \tag{15}$$

from which it follows that

$$\frac{k_{11}^{\mathrm{H}}}{k_{11}^{\mathrm{L}}} = \frac{k_{12}^{\mathrm{H}}}{k_{12}^{\mathrm{L}}} = -\frac{T_{11}}{T_{12}} . \tag{16}$$

A similar expression can be derived for output 2. Because (16) is incompatible with (14), it means that the *dynamics cannot be aligned with the gain directions if neither transfer function for an output contains a zero*. It is possible, if at least one transfer function contains a zero.

3.4 A Modified Design

Assume that $T_1 \geq \ldots \geq T_n > 0$. If b_1 is not orthogonal to any gain direction v_i, the effect of the slow dynamics cannot be suppressed by the steady-state directional design. However, if (an estimate of) b_1 is available, *an input orthogonal to* b_1 *can be used to suppress the slow dynamics*.

A normal situation is that the system order n is larger than the number of inputs, m. Then some directions will always have higher-order dynamics than first order. Also in this case, there is a possibility of selecting what dynamics to suppress.

4. EXAMPLES

4.1 A Simple Model of a Heat Exchanger

Jacobsen and Skogestad (1994) have derived a linear model of a much simplified heat exchanger. By choosing certain parameters equal, a lot of symmetries were introduced. The model is described by the transfer function

$$G(s) = \frac{89.243}{(100s+1)(2.439s+1)}\begin{bmatrix} -21(4.76s+1) & 20 \\ -20 & 21(4.76s+1) \end{bmatrix}. \quad (17)$$

The steady-state gain-matrix is

$$K = 89.243\begin{bmatrix} -21 & 20 \\ -20 & 21 \end{bmatrix}. \quad (18)$$

A singular value decomposition of K yields

$$\Sigma = 89.243\begin{bmatrix} 41 & 0 \\ 0 & 1 \end{bmatrix}, \quad (19a)$$

$$W = \begin{bmatrix} -0.7071 & -0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}, \quad (19b)$$

$$V = \begin{bmatrix} 0.7071 & 0.7071 \\ -0.7071 & 0.7071 \end{bmatrix}. \quad (19c)$$

The symmetries are very apparent from W and V.

From (19c) it follows that equal input changes with opposite signs will excite the high-gain direction whereas equal identical changes will excite the low-gain direction. From (17) it follows that the high-gain changes give a slow response with the time constant 100 and the low-gain changes a fast response with the time constant 2.439. The slight discrepancies are due to round-off errors in (17).

4.2 A Simplified Model of a High-Purity Distillation Column

Skogestad and Morari (1988a, b) have developed a simple linear distillation model based on the observed response of a simulated binary distillation column to certain changes in the internal flows (reflux and boilup) and the product flows. A significant idealization is that constant molar flows were assumed in the column sections. This model has been used in many MIMO identification studies.

To highlight some properties of the model, its transfer function is expressed in the form

$$G(s) = \begin{bmatrix} \frac{87.8}{1+194s} & -\frac{87.8}{1+194s} + \frac{1.4}{1+15s} \\ \frac{108.2}{1+194s} & -\frac{108.2}{1+194s} - \frac{1.4}{1+15s} \end{bmatrix}. \quad (20)$$

The steady-state gain matrix is

$$K = \begin{bmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{bmatrix}. \quad (21)$$

A singular value decomposition of K yields

$$\Sigma = \begin{bmatrix} 197.21 & 0 \\ 0 & 1.3914 \end{bmatrix}, \quad (22a)$$

$$W = \begin{bmatrix} -0.6246 & -0.7809 \\ -0.7809 & 0.6246 \end{bmatrix}, \quad (22b)$$

$$V = \begin{bmatrix} -0.7066 & -0.7077 \\ 0.7077 & -0.7066 \end{bmatrix}. \quad (22c)$$

The model has the condition number $\kappa = 197.21/1.3914 \approx 142$. The scaled high- and low-gain input directions u^{H} and u^{L}, respectively, are

$$u^{\mathrm{H}} = \begin{bmatrix} -0.7066 & 0.7077 \end{bmatrix}^{\mathrm{T}} / 197.21, \quad (23a)$$

$$u^{\mathrm{L}} = \begin{bmatrix} -0.7077 & -0.7066 \end{bmatrix}^{\mathrm{T}} / 1.3914. \quad (23b)$$

Step changes of size u^{H} and u^{L} yield steady-state outputs

$$\bar{y}^{\mathrm{H}} = \begin{bmatrix} -0.6246 & 0.7809 \end{bmatrix}^{\mathrm{T}}, \quad (24a)$$

$$\bar{y}^{\mathrm{L}} = \begin{bmatrix} -0.7809 & 0.6246 \end{bmatrix}^{\mathrm{T}}. \quad (24b)$$

Consider now the dynamics. It is easy to see that a step change u^{L} in the low-gain direction does almost, but not completely, cancel out the slow dynamics in (20). Likewise, a step change u^{H} in the high-gain direction mostly excites the slow dynamics, but some fast dynamics also remain. The fact that the dynamics are not exactly cancelled out is not due to calculation inaccuracy. The form of (20) reveals that the slow dynamics could be cancelled out by equal changes in the two inputs, but the fast dynamics can never be cancelled out by a change orthogonal to this. In practice, however, the dynamics are excellently separated.

A state-space representation of (20) in the form of (5) is defined by

$$A = \begin{bmatrix} -\frac{1}{194} & 0 \\ 0 & -\frac{1}{15} \end{bmatrix}, B^{\mathrm{T}} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} \frac{87.8}{194} & \frac{1.4}{15} \\ \frac{108.2}{194} & -\frac{1.4}{15} \end{bmatrix}. \quad (25)$$

It is now obvious that the slow dynamics are cancelled out by two equal changes in the inputs whereas the fast dynamics are cancelled by a change in the first input only with the second input unchanged. These were, in fact, the assumptions when the model was derived (Skogestad and Morari, 1988a).

4.3 A Detailed Model of a High-Purity Distillation Column

A more detailed model of the distillation column studied by Skogestad and Morari (1988a, b) is the state-space model (Sadabadi and Poshtan, 2009) of the form (5) with

$$A = \begin{bmatrix} -0.0051 & 0 & 0 & 0 & 0 \\ 0 & -0.0737 & 0 & 0 & 0 \\ 0 & 0 & -0.1829 & 0 & 0 \\ 0 & 0 & 0 & -0.4620 & 0.9895 \\ 0 & 0 & 0 & -0.9895 & -0.4620 \end{bmatrix} \quad (26a)$$

$$B = \begin{bmatrix} 0.624 & -0.172 & -0.108 & -0.139 & -0.056 \\ -0.629 & 0.055 & 0.030 & -0.186 & -1.230 \end{bmatrix} \quad (26b)$$

$$C = \begin{bmatrix} -0.7223 & -0.5170 & 0.3386 & -0.0163 & 0.1121 \\ -0.8913 & 0.4728 & 0.9876 & 0.8425 & 0.2186 \end{bmatrix} \quad (26c)$$

The time constants of this model are $T_1 = 194$, $T_2 = 13.6$, $T_3 = 5.47$, $T_{4,5} = 0.387 \pm 0.830\sqrt{-1}$ (min).

According to (22c), the low-gain direction is excited by (almost exactly) equal input changes. The column vector b_1 in (26b) shows that the same input suppresses the slow dynamics $T_1 = 194$ min. To some degree, T_2 and T_3 are also suppressed because of different element signs in b_2 and b_3. The very fast (and oscillatory) dynamics are excited by the input in the high-gain direction. In this case, *the dynamics are very accurately aligned with the gain directions although this hardly was an issue when the model was developed*.

5. CONCLUSIONS

Existing experiment design principles for identification of ill-conditioned systems have been reviewed and summarized with some extensions. It was shown analytically as well as by examples that *the process dynamics tend to be aligned with the so-called gain directions*. This is of theoretical interest, but also practically useful in the design of identification experiments. A possible modification to existing methods was proposed for situations where the dynamics are misaligned with the gain directions.

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