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Regeneratively-constrained LQG control of passive networks ⋆

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Abstract: In this paper we consider the synthesis of optimal feedback controllers for a stochastically-excited passive electromechanical network, subject to the constraint that in stationarity, the feedback law must be realizable with a regenerative actuation system. Regenerative systems are similar to passive systems but their dynamic constraints are more relaxed, in the sense that they only need to conserve energy in the stationary average sense, rather than at every time instant. In this paper, we examine the design of optimal LQG controllers for passive networks controlled with regenerative actuation. We show that this problem may be posed as a multi-objective LMI problem. We also characterize how close a regeneratively-constrained optimal LQG feedback law is to a passive transfer function. The concepts are demonstrated on a simple example related to vibration suppression.

Keywords: Regeneration, passivity, vibration, optimal control

1. INTRODUCTION

In many control applications, the power and energy required for a given control design plays an important role in the assessment of its favorability. In applications where the energy available for control is stored locally (e.g., in a battery, supercapacitor, pressure accumulator, etc.) such issues become central to the viability of a control law. Such applications include many vibration suppression technologies, in which local energy storage is used to achieve energy-autonomy. This may be desirable from the point of view of reliability, such as for earthquake response control systems in civil structures, for which reliance on the external power grid introduces a significant vulnerability during seismic events. Energy-autonomy may also be desirable merely as a means of efficient system design, such as in automotive suspension control applications.

The price paid for energy-autonomy is that the domain of feasible control laws is constrained to include only those that do not exhaust their storage. To put this concept in more precise terms, consider the generic system diagram in Figure 1, illustrating a passive electromechanical network $\mathcal{N}$ excited exogenously through some vector $a(t) \in \mathbb{R}^{n_a}$ of dynamic inputs, and resulting in a vector $z(t) \in \mathbb{R}^{n_z}$ of performance quantities. We assume the system to be controlled through n_p ports, characterized by a vector $v(t) \in \mathbb{R}^{n_p}$ of potential variables, and a colocated vector $u(t) \in \mathbb{R}^{n_p}$ of flow variables. (For example, if $v(t)$ is a voltage vector then $u(t)$ is the vector of currents flowing into these voltages.) Vector $y(t) \in \mathbb{R}^{n_y}$ is comprised of the measurements available for feedback. We are then concerned with the design of a causal feedback law $K : y \to u$ that minimizes z under some measure.

The most straight-forward technique for designing an energy-autonomous feedback control law is to restrict the design domain to passive feedback laws, i.e.,

$$\mathcal{K}_p = \left\{ K : y \to u \,\middle|\, \int_0^T u^T(t)v(t)dt \leqslant 0, \quad \forall T \geqslant 0, y, v \in \mathcal{L}_2 \right\} \quad (1)$$

This is the domain of feedback laws which never inject cumulative energy into the network. The linear subdomain $\mathcal{K}_p^\ell \subset \mathcal{K}_p$ is comprised of all negative-real [1] colocated feedback laws; i.e.,

$$\mathcal{K}_p^\ell = \left\{ K : v \to u \,\middle|\, \hat{K}(s) + \hat{K}^H(s) \leqslant 0, \ \forall \operatorname{Re}(s) \geqslant 0 \right\} \quad (2)$$

It is a classical result (see, e.g., Darlington, 1999), that any such feedback law can be realized via a network of passive components. For example, in an electrical implementation, the feedback law could be implemented with ideal resistors, capacitors, inductors, transformers, and gyrators. At least on a theoretical level, a feedback law $K \in \mathcal{K}_p^\ell$ can therefore be implemented through classical network design, thus forgoing altogether the need for active control and an energy storage subsystem.

In the area of vibration control, such passive linear feedback implementations have been investigated in many contexts. Many mechanical techniques, such as tuned mass dampers, are ubiquitous in mechanical engineering

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[1] Note that, due to our sign convention for v and u, $K : v \to u$ constitutes positive feedback, and positive $u^T v$ denotes injection of power into the network. Thus K_p^ℓ is the domain of negative-real transfer functions. This is in contrast to the more usual convention of negative feedback between u and v which leads to the characterization of K_p^ℓ as the domain of all positive-real transfer functions.

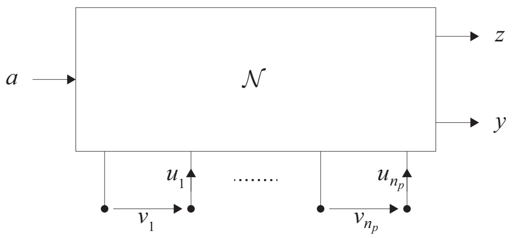

Fig. 1. n_p-ports passive network $\mathcal{N}$ with exogenous input a, peformance output z, and feedback output y

[Den Hartog, 1985] as well as civil structures [Housner et al., 1997]. Recently, formal passive synthesis techniques from classical electrical network theory have been revisited and linked with these mechanical designs, through the use of inerters [Smith, 2002]. Passive electrical feedback is common in vibration suppression applications to aerospace systems, in which piezoelectric or electromagnetic transducers are shunted with tuned resonant RLC networks [Moheimani, 2003, Behrens et al., 2005].

Although they require no energy, linear passive feedback implementations are disadvantageous for a few reasons. Most immediately, the design domain $\mathcal{K}_p^\ell$ may be too restrictive to achieve acceptable closed-loop performance. Additionally, passive designs do not afford the features of adaptivity. Moreover, all the existing synthesis techniques whereby the electrical network is derived from a desired $K \in \mathcal{K}_p^\ell$ result in rather elaborate networks, even for moderately-sized problems [Anderson and Vongpanitlerd, 1973]. For all these reasons, energy-autonomous active feedback control may be preferable over passive systems.

Beginning with the work of Jolly and Margolis [1997], a number of investigations have been conducted in the area of vibration suppression, using a type of energy-autonomous device called a *regenerative actuator*. Similar concepts have been developed for automotive suspensions by Okada et al. [1997] and Li et al. [2013], for aerospace applications by Onoda et al. [2003], for civil structure applications by Nerves and Krishnan [1996], Scruggs and Iwan [2005] and Gonzalez-Buelga et al. [2014], and in application-independent contexts (Nakano et al., 2003). Figure 2 shows an diagram of an electromagnetic regenerative actuation system, in which (u, v) constitute the current and voltage vectors, and are proportional to the linear forces and velocities of the devices through coupling factors; i.e., $f_i = \kappa_i i_i$, $v_i = \kappa_i \dot{x}_i$. The regenerative actuation system interfaces the mechanical system in which the transducers are embedded, with a supercapacitor or flywheel. The system operates entirely off the stored energy $E_s(t)$ in the supercapacitor, replenishing its supply with energy it extracts from the mechanical system. It must be controlled to ensure that $E_s(t) \geqslant 0$.

The key concept that distinguishes the regenerative system in Figure 2 from a passive system, is the fact that in its undisturbed state, $E_s > 0$. Furthermore, in its typical operation, we assume that the rate at which $E_s(t)$ changes is slow, compared to the time constants of the problem at hand. To put this more precisely, we assume that if τ is a characteristic time constant for a given application, that $|\dot{E}_s(t)| \ll E_s(t)/\tau$. Under this assumption, for deterministic problems the set of feedback laws realizable by a

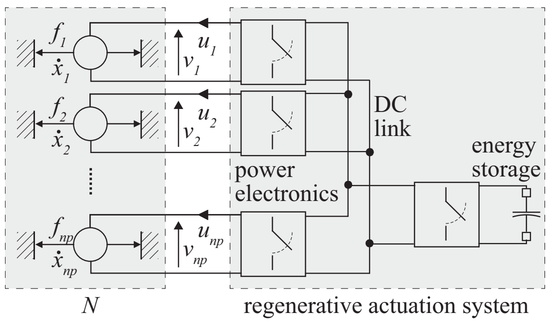

Fig. 2. Electromechanical example of a regenerative actuation system

regenerative system is

$$\mathcal{K}_r = \left\{ K : y \to u \,\middle|\, \lim_{T\to\infty} \frac{1}{T} \int_0^T u^T(t)v(t)dt \leqslant 0, \ \forall a \in \mathcal{A} \right\} \tag{3}$$

The subset $\mathcal{K}_r^\ell$ of linear feedback laws is defined analogously. We note, however, that unlike in the passive case, a $K \in \mathcal{K}_r^\ell$ need not be colocated; i.e., it is not necessary that $y = v$ for a linear feedback law to be regenerative.

Comparing (3) to (1), we see two essential differences: (*i*) We relax the requirement on the inner product of (u, v), such that negativity is only enforced on the entire trajectory, rather than on every subdomain $[0, T]$ therein, and (*ii*) The domain over which the negativity condition must hold is a specified domain $\mathcal{A}$ of possible disturbances, a, rather than the domain of all y and $v \in \mathcal{L}_2$. The above imply that the determination of whether a given $K \in \mathcal{K}_r$ depends on the specification of a disturbance set $\mathcal{A}$, and also, implicitly, on the dynamics of the specific network in which the actuation system is embedded. For example, a given K may be regenerative for all disturbances with frequency content confined to some range $\omega \in [\omega_1, \omega_2]$ but not over other ranges [Jolly and Margolis, 1997]. Likewise, a given K may be regenerative for the impulse response of one system, but not for another.

The present paper examines the design of regeneratively-constrained feedback laws in a stochastic setting. To do this, we must define $\mathcal{K}_r$ in a probabilistic setting. For our analysis, $\mathcal{A}$ becomes the probability space associated with a stationary stochastic process with known spectrum $S_a(\omega)$, and

$$\mathcal{K}_r = \left\{ K : y \to u \ \middle| \ \mathcal{E}(u^T v) \leqslant 0, \ a \sim \mathcal{A} \right\} \tag{4}$$

where $\mathcal{E}(\cdot)$ denotes the expectation in stationarity.

Of particular interest in this paper will be the optimization of regeneratively-constrained feedback laws in the context of multi-criterion LQG control. Specifically, we are interested in the following optimization problem:

$$K^\star = \underset{K \in \mathcal{K}_r^\ell}{\operatorname{argmin}} \left\{ \max_{i=1\ldots n_z} \mathcal{E}(z_i^2) \ , \ a \sim \mathcal{A} \right\} \tag{5}$$

The remainder of the paper is organized as follows: Section 2 illustrates the straight-forward solution to this problem using LMI techniques. Section 3 develops a means for assessing how close $K^\star$ is to $\mathcal{K}_p^\ell$; i.e., how "close" the optimal regenerative controller is to being passive. Section 4 considers some simple examples in the context of vibration suppression. Finally, Section 5 ends with some concluding remarks.

2. REGENERATIVELY-CONSTRAINED OPTIMAL STOCHASTIC CONTROL

We assume a finite-dimensional linear time-invariant dynamical model for the passive network; i.e.,

$$\mathcal{N}:\begin{cases}\dot{x}_N &= A^N x_N + B_a^N a + B_u^N u\\ v &= C_v^N x_N + D_{vu}^N u\\ y &= C_y^N x_N + D_{ya}^N a\\ z &= C_z^N x_N + D_{za}^N a + D_{zu}^N u\end{cases} \tag{6}$$

Due to the Kalman-Yakubovic-Popov lemma [Anderson and Vongpanitlerd, 1973], we note that because $\mathcal{N}$ is positive-real (i.e., passive), it follows that $\exists W = W^T \geqslant 0$ such that

$$\begin{bmatrix}(A^N)^T W + W A^N & W B_u^N - (C_v^N)^T\\ (B_u^N)^T W - C_v^N & -D_{vu}^N - (D_{vu}^N)^T\end{bmatrix} \leqslant 0 \tag{7}$$

We will further strengthen this condition to weakly-strict positive-real (WSPR) [Brogliato et al., 2007] by requiring that $(A^N, W^{1/2})$ be observable, which by Lasalle's theorem, guarantees A^N to be Hurwitz.

As discussed in the introduction, we will assume the disturbance $a(t)$ to be a stationary stochastic process. We will further assume its power spectrum can be approximated to sufficient accuracy as strictly-proper and rational. These assumptions imply the existence of a state space model for $a(t)$ as

$$\mathcal{A}:\begin{cases}\dot{x}_A &= A^A x_A + B^A w\\ a &= C^A x_A\end{cases}, \quad S_w(\omega) = I \tag{8}$$

where $S_w(\omega)$ is the power spectrum of the generating white noise signal $w(t)$.

Augmenting systems $\mathcal{N}$ and $\mathcal{A}$, we arrive at the composite plant model

$$\mathcal{P}:\begin{cases}\dot{x} &= Ax + B_u u + B_w w\\ v &= C_v x + D_{vu} u\\ y &= C_y x\\ z &= Ex + Fu\end{cases} \tag{9}$$

with appropriate definitions for the augmented variables.

For this augmented system model, we re-state the control design objective (5) as the minimization of

$$J \triangleq \max_{i\in\{1\ldots n_z\}} \mathcal{E}\left\{(E_i x + F_i u)^2\right\} \tag{10}$$

(where E_i and F_i are the i^{th} rows of their respective matrices), subject to the average generated power constraint

$$\bar{p} \triangleq -\mathcal{E}\left\{u^T\left(C_v x + D_{vu} u\right)\right\} > 0 \tag{11}$$

For the optimization domain, we will assume the feedback law is strictly proper, and that it is (at most) the same order as $\mathcal{P}$. As such, we assume a controller of the form

$$\mathcal{K}:\begin{cases}\dot{x}_K &= A_K x_K + B_K y_K\\ u &= C_K x_K\end{cases}, \quad \dim x_K = \dim x \tag{12}$$

Controllers from the above set which also satisfy (11) comprise the intersection $\mathcal{K}\cap\mathcal{K}_r$.

The above problem falls into a broad class of multi-criterion LQG problems that can be handled efficiently through the use of semidefinite programming; i.e., Linear Matrix Inequality (LMI) methods. Prior to illustrating this, however, we will need one more result, which is necessary to turn the above into a convex optimization. This result, stated in the theorem below, was proven shown by Scruggs [2010].

Theorem 1. Assume $\mathcal{N}$ to be WSPR, and that $R \triangleq \frac{1}{2}\left(D_{vu} + D_{vu}^T\right) > 0$. Then

$$\bar{p} = \bar{p}_0 - \mathcal{E}\left\{(u - Gx)^T R (u - Gx)\right\} \tag{13}$$

where $\bar{p}_0 = -B_w^T T B_w > 0$, $G = -R^{-1}\left(\frac{1}{2}C_v + B_u^T T\right)$, and $T = T^T$ satisfies Riccati equation

$$0 = A^T T + TA - \left(\tfrac{1}{2}C_v^T + TB_u\right) R^{-1}\left(\tfrac{1}{2}C_v + B_u^T T\right) \tag{14}$$

Using this theorem, we can restate constraint (11) as

$$\mathcal{E}\left\{(u - Gx)^T R (u - Gx)\right\} < \bar{p}_0 \tag{15}$$

To frame the optimization problem as a convex linear program, first define

$$S = \mathcal{E}\left\{\begin{bmatrix} xx^T & xx_K^T\\ x_K x^T & x_K x_K^T\end{bmatrix}\right\} \tag{16}$$

Then for some quantity γ, we have that $J < \gamma$ if S simultaneously satisfies the following three inequalities:

$$\begin{bmatrix} A & B_u C_K\\ B_K C_y & A_K\end{bmatrix} S + S\begin{bmatrix} A & B_u C_K\\ B_K C_y & A_K\end{bmatrix}^T + \begin{bmatrix} B_w & 0\\ 0 & 0\end{bmatrix} < 0 \tag{17}$$

$$[E_i\ \ F_i C_K]\, S \begin{bmatrix} E_i^T\\ C_K^T F_i^T\end{bmatrix} < \gamma \quad , \quad i = 1\ldots n_z \tag{18}$$

$$\operatorname{tr}\left\{\left([-G\ \ C_K]\right) S \left(\begin{bmatrix} -G^T\\ C_K^T\end{bmatrix}\right) R\right\} < \bar{p}_0 \tag{19}$$

Equivalently, defining $\hat{A} = \left[\begin{smallmatrix} A & B_u C_K\\ B_K C_y & A_K\end{smallmatrix}\right]$, $J < \gamma$ if $\exists P = P^T > 0$ and $Q = Q^T$ such that

$$0 > \begin{bmatrix} \hat{A}^T P + P\hat{A} & P\begin{bmatrix} B_w\\ 0\end{bmatrix}\\ \begin{bmatrix} B_w^T & 0\end{bmatrix} P & -I\end{bmatrix} \tag{20}$$

$$0 < \begin{bmatrix} \gamma & [E_i\ \ F_i C_K]\\ \begin{bmatrix} E_i^T\\ C_K^T F_i^T\end{bmatrix} & P\end{bmatrix}, \quad i = 1\ldots n_z \tag{21}$$

$$0 < \begin{bmatrix} Q & [-G\ \ C_K]\\ \begin{bmatrix} -G^T\\ C_K^T\end{bmatrix} & P\end{bmatrix} \tag{22}$$

$$\bar{p}_0 > \operatorname{tr}\{QR\} \tag{23}$$

Our optimization problem is them to minimize γ over the domain $\{P, Q, A_K, B_K, C_K, \gamma\}$, and subject to constraints (20), (21), (22), and (23).

We now make use of a standard coordinate transformation which converts the above matrix inequalities into linear form [Scherer et al., 1997]. Let

$$P = \begin{bmatrix} Y & N\\ N^T & \times\end{bmatrix} \qquad P^{-1} = \begin{bmatrix} X & M\\ M^T & \times\end{bmatrix} \tag{24}$$

where "×" denotes terms that are unnecessary to be solved. Now, define similarity transformation matrix

$$\Pi = \begin{bmatrix} X & I\\ M^T & 0\end{bmatrix} \tag{25}$$

Recasting the above problem with the coordinate change $\xi = \Pi^{-1}\left[x^T\ \ x_K^T\right]^T$ gives

$$0 > \begin{bmatrix} \Delta_1 + \Delta_1^T & A + \tilde{A}^T & B_w \\ A^T + \tilde{A} & \Delta_2 + \Delta_2^T & YB_w \\ B_w^T & B_w^T Y & -I \end{bmatrix} \quad (26)$$

$$0 < \begin{bmatrix} \gamma & E_i X + F_i \tilde{C} & E_i \\ XE_i^T + \tilde{C}^T F_i^T & X & I \\ E_i^T & I & Y \end{bmatrix}, \quad i = 1...n_z \quad (27)$$

$$0 < \begin{bmatrix} Q & \tilde{C} - GX & -G \\ \tilde{C}^T - XG^T & X & I \\ -G^T & I & Y \end{bmatrix} \quad (28)$$

where the transformed optimization variables for the controller are

$$\tilde{A} = NA_K M^T + NB_K C_y X + YB_u C_K M^T + YAX \quad (29)$$

$$\tilde{B} = NB_K, \quad \tilde{C} = C_K M^T \quad (30)$$

where

$$\Delta_1 = AX + B_u \tilde{C} \qquad \Delta_2 = YA + \tilde{B} C_y \quad (31)$$

We thus have that the optimization problem becomes to minimize γ over the domain $\{\gamma, Q, \tilde{A}, \tilde{B}, \tilde{C}, X, Y\}$, subject to LMI constraints (26), (27), (28), and (23). Noting that all matrix inequalities are linear, we thus arrive at a convex optimization that can be solved by standard LMI solvers [Boyd et al., 1994]. We note that, in contrast to many multi-objective LMI optimization problems, it was not necessary to impose any conservatism on the optimization domain (other than bounding the order of the controller) to arrive at a convex problem.

As stated above, the optimal solution may be unbounded, because the feedback measurement signal y has been assumed to be uncorrupted by noise. To force a finite solution, we presume a small level of artificial measurement noise on y, with spectral intensity matrix Ξ. This merely modifies (26) to

$$0 > \begin{bmatrix} \Delta_1 + \Delta_1^T & A + \tilde{A}^T & B_w & 0 \\ A^T + \tilde{A} & \Delta_2 + \Delta_2^T & YB_w & \tilde{B} \\ B_w^T & B_w^T Y & -I & 0 \\ 0 & \tilde{B}^T & 0 & -\Xi^{-1} \end{bmatrix} \quad (32)$$

Following the solution of the above optimization, associated parameters M and N can be found as any matrices satisfying $XY + MN^T = I$. (Different solutions for (M, N) correspond to different state space realizations of the controller.) With M and N found, optimal controller parameters $\{A_K, B_K, C_K\}$ can then found by inverting equations (29)-(30).

2.1 Accounting for Parasitics

Irrespective of what technology (i.e., hydraulic, electronic, etc.) might be used to realize a regenerative actuation system, it will exhibit parasitic losses, which will hamper its ability to recycle the energy it extracts from the network. The set $\mathcal{K}_r$ defined in (3) and (4) (for deterministic and stochastic situations, respectively) presumes idealized hardware with perfect efficiency. However, it is straightforward to extend such definitions to account for basic parasitic loss models [Scruggs et al., 2012]. Here, we do this for the stochastic definition of $\mathcal{K}_r$, with primary reference to electronic implemenations.

We assume the power electronics in a regenerative force actuation system implement high-bandwidth current track-

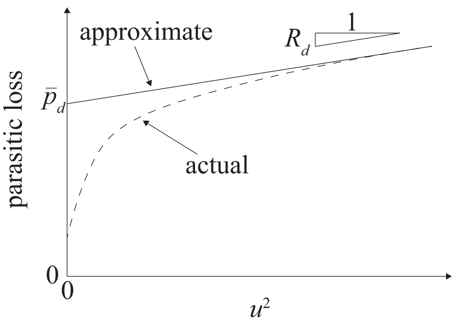

Fig. 3. Actual and approximate loss models for scalar u

ing at each of the transducers, through the use of pulse width modulation or hysteretic switching control. The drives that interface each transducer with the storage bus facilitate these switching actions through high-frequency gating of MOSFETs. If ideal circuit components were used, this mode of switching control would exhibit zero dissipation; i.e., there are no dissipative components in the idealized electronics. However, in reality, parasitic losses manifest themselves in many ways. There are conductive losses in the conversion system, due to resistances in the circuit components, including the MOSFETs and diodes. There are also impulsive transition losses, incurred each time a MOSFET switches on or off. Additionally, there is a gating energy necessary to charge the gate capacitance of a MOSFET each time it is switched on, and this energy is generally not recovered when the MOSFET is switched off again. This gives rise to potentially-significant parasitic gating losses.

A highly accurate depiction of all these parasitic losses would result in a considerably more complex characterization of $\mathcal{K}_r$. However, it can be shown that for many systems, these gating losses are predominantly a function of u, and exhibit a semi-concave functionality with respect to u^2. This is shown for scalar u in Figure 3. Because of this, there exist parameters $(\bar{p}_d, R_d)$, representing effective static power loss and transmission resistance respectively, which over-bound the true loss model. This is also shown in Figure 3. As such, we can conservatively account for parasitics through a modification of the average power flow $\bar{p}$ from (11) to

$$\bar{p} \triangleq -\mathcal{E}\left\{u^T\left(C_v x + D_{vu} u\right)\right\} - \mathcal{E}\left\{\bar{p}_d + u^T R_d u\right\} \quad (33)$$

with corresponding changes to the definition of $\mathcal{K}_r$.

In the analysis above, substitution of (33) for (11) has only two consquences. First, it modifies R to be defined as

$$R = \tfrac{1}{2}\left(D_{vu} + D_{vu}^T\right) + R_d \quad (34)$$

Second, it modifies the definition of $\bar{p}_0$ to

$$\bar{p}_0 = -B_w^T T B_w - \bar{p}_d \quad (35)$$

With these changes in place, Theorem 1 still holds, and the development which follows is still valid.

3. REGENERATIVE VS PASSIVE CONTROL

One of the reasons to use regenerative technology, rather than passive techniques, is that it provides a larger design domain. However, such technology will obviously be more expensive, and its use in an application must be justified by some evidence that its capabilities are indeed in excess of those achievable by passive systems. In this paper, we reduce this down to the most basic question: How close is the optimal $K^\star \in \mathcal{K}_r^\ell$ to the set $\mathcal{K}_p^\ell$? Clearly, in order for this to make sense, we must have $y = v$, because linear passive feedback laws must be colocated. Beyond this, however, the answer will depend on all the problem data. In this paper we address this issue by checking to see whether the optimal $K^\star$, obtained via the techniques described in the previous section, is also in $\mathcal{K}_p^\ell$. If it is not, then we wish to find some measure of how close it is to $\mathcal{K}_p^\ell$.

Technically, if $K^\star$ is passive then the KYP lemma must be true; i.e., $\exists W_K = W_K \geqslant 0$ such that

$$A_K^T W_K + W_K A_K \leqslant 0 \quad , \quad W_K B_K = -C_K^T \tag{36}$$

However, this precise condition may not exactly hold for the optimal $K^\star$, even if it is, for all practical purposes, passive. For example, consider a problem with one actuator, thus where the $K^\star(s)$ is scalar. In order to be passive, we must have that $K^\star(s)$ be stable, and at each finite ω, $K^\star(j\omega) + K^\star(-j\omega) \leqslant 0$. However, it may be the case that this condition is satisfied at all ω where $|K^\star(j\omega)|$ is significant, but perhaps not at extremely high frequencies, where $|K^\star(j\omega)|$ is extremely small. In such circumstances, there exists a passive system K^p which is extremely close to $K^\star$ (i.e., for which $\|K^p - K^\star\|_\infty < \epsilon_0$ for ϵ_0 equal to some tolerance), which gives performance which is nearly indistiguishable.

To address the above issue, we assume that the "passive equivalent" of $K^\star$, if it exists, has a realization of the same order as $K^\star$, i.e.,

$$K^p \sim \left[\begin{array}{c|c} A_p & B_p \\ \hline C_p & D_p \end{array}\right] \quad , \quad \dim(A_p) = \dim(A_K) \tag{37}$$

and we seek to find a realization of $K^p \in \mathcal{K}_p^\ell$ which minimizes $\|K^p - K^\star\|_\infty$. Stated in generality, this problem is nonconvex, due to the constraint on the domain. However, if we assume $A_p = A_K$ and $B_p = B_K$, we recover convexity.

Under this assumption we have that $\|K^p - K^\star\|_\infty < \epsilon$ if $\exists T_p = T_p^T > 0$ satisfying

$$\begin{bmatrix} A_K^T T_p + T_p A_K & T_p B_K & C_p^T - C_K^T \\ B_K^T T_p & -\epsilon I & D_p^T \\ C_p - C_K & D_p & -\epsilon I \end{bmatrix} < 0 \tag{38}$$

In order for K^p to also satisfy passivity, we must have that $\exists W_p = W_p^T > 0$ satisfying

$$\begin{bmatrix} A_K^T W_p + W_p A_K & W_p B_K + C_p^T \\ B_K^T W_p + C_p & D_p^T + D_p \end{bmatrix} < 0 \tag{39}$$

To find the best approximation of $K^\star$ within the parametrization for K^p, we then minimize ϵ over the domain $\{\epsilon, C_p, D_p, T_p, W_p\}$, subject to constraints $T_p > 0$, $W_p > 0$, (38), and (39). Upon execution of this optimization, we conclude that $K^\star$ is "effectively passive" if $\epsilon < \epsilon_0$, for ϵ_0 defined to be some tolerance. If $\epsilon > \epsilon_0$, then its value implies a distance of $K^\star$ to $\mathcal{K}_p^\ell$.

We note that the above technique is equivalent to recovering passivity for $K^\star$ by shifting the zeros of its transfer function.

The above algorithm can be streamlined with the one described in the previous section, by placing it directly in terms of the optimal parameters $\{\tilde{A}^\star, \tilde{B}^\star, \tilde{C}^\star, X^\star, Y^\star\}$. Specifically, it is straight-forward to show that the above are equivalent to finding feasible matrices $\tilde{T}_p > 0$ and $\tilde{W}_p$, and transformed passive system parameters $\tilde{C}_p = C_p M^T$ and $\tilde{D}_p = D_p$, satisfying the following LMIs, which are equivalent to (38) and (39) respectively:

$$\begin{bmatrix} \bar{A}^T \tilde{T}_p Z + Z^T \tilde{T}_p \bar{A} & Z^T \tilde{T}_p \tilde{B}^\star & \tilde{C}_p^T - \tilde{C}^{\star T} \\ \tilde{B}^{\star T} \tilde{T}_p Z & -\epsilon I & D_p^T \\ \tilde{C}_p - \tilde{C}^\star & D_p & -\epsilon I \end{bmatrix} < 0 \tag{40}$$

$$\begin{bmatrix} \bar{A}^T \tilde{W}_p Z + Z^T \tilde{W}_p \bar{A} & Z^T \tilde{W}_p \tilde{B}^\star + \tilde{C}_p^T \\ \tilde{B}^{\star T} \tilde{W}_p Z + \tilde{C}_p & \tilde{D}_p^T + \tilde{D}_p \end{bmatrix} < 0 \tag{41}$$

where

$$\bar{A} = \tilde{A}^\star - \tilde{B}^\star C_y X^\star - Y^\star B_u \tilde{C}^\star - Y^\star A X^\star \tag{42}$$

$$Z = I - Y^\star X^\star \tag{43}$$

Minimizing ϵ subject to the above inequalities, in lieu of (38) and (39), obviates the need to solve for M, N, A_K, B_K, and C_K to find ϵ.

We note also, as evident from (40) and (41), that these inequalities are guaranteed to be infeasible (for $\tilde{T}_p > 0$ and $\tilde{W}_p > 0$) if $\bar{A}Z^{-1}$ is non-Hurwitz. In this circumstance, the optimal $K \in \mathcal{K}_r^\ell$ will be open-loop unstable (while still stabilizing the closed loop), thus precluding the possibility of a passive feedback law with the same poles.

4. EXAMPLE

Consider the three-degree-of-freedom structure in Figure 4a, in which a single regenerative actuator is used to suppress the vibrations. The structure is nondimensionalized, with all springs and masses equal to 1, and dashpots equal to 0.01. The acceleration $a = \ddot{x}_0$ is taken as an exogenous stationary stochastic disturbance with power spectral density

$$S_a(\omega) = \frac{\omega^2}{(\omega^2 - 1)^2 + (0.25\omega)^2} \tag{44}$$

We further assume $z = [x_2 - x_1 \ \ddot{x}_3 \ u]^T$ and $y = v = \dot{x}_2 - \dot{x}_1$. For this example, the unconstrained multi-criterion LQG optimal performance is 1.701. We are interested in knowing how the presence of the regenerative constraint, and the severity of the parasitic losses, hamper the ability of the system to achieve acceptable performance.

Figure 4b shows a surface plot of J values for unconstrained, over regenerative performance. We see that for an extremely efficient regenerative system, the optimal unconstrained performance is achieved, because $\bar{p} > 0$. As the parasitics increase, there is a degradation in performance, as the constraint $\bar{p} \geqslant 0$ becomes activated. Ultimately this constraint imposes a boundary on feasibility, beyond which there is no controller (no matter how poorly it might perform) that can recover more energy than it expends.

Figure 4c shows a plot of the values of ϵ for this example, normalized by $\|K^\star\|_\infty$, for the same domain of $(\bar{p}_d, R)$ values. We see that over the majority of the feasible domain, $K^\star$ is open-loop unstable. Where it is not unstable, $\epsilon = 1$. This is due to the fact that in these regions $K^\star$ has high gain at low frequencies, and exhibits positive feedback in the low-frequency range.

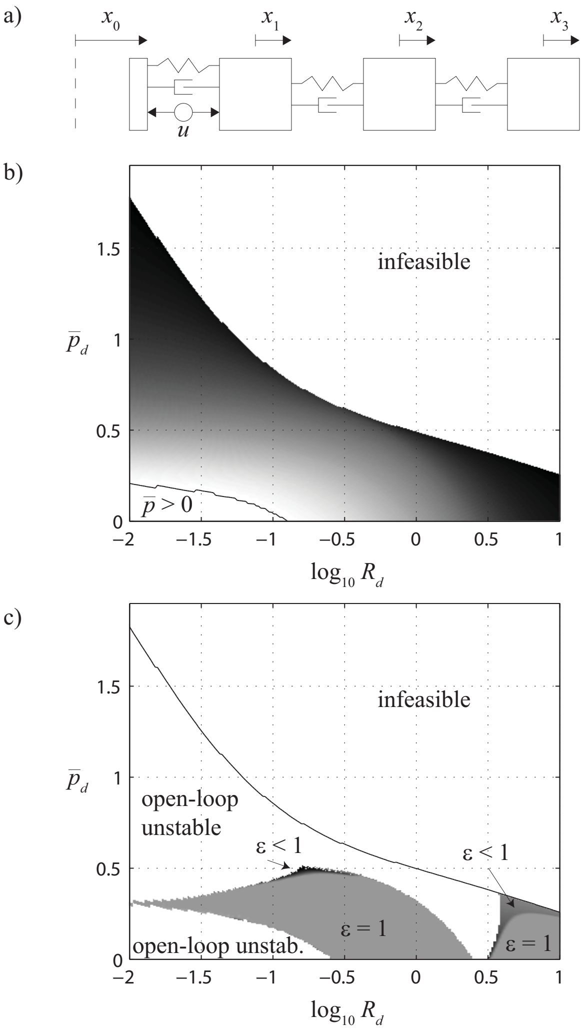

Fig. 4. First example: (a) system diagram, (b) surface plot of performance ratio, relative to unconstrained LQG, with values ranging from 1 (white) down to 0 (black) (c) surface plot of ϵ solutions, normalized by $\|K^\star\|_\infty$

5. CONCLUSIONS

The focus of this paper has been on the use of regenerative actuation to control stochastically-excited passive systems, and has highlighted some interesting potential advantages of regenerative actuators:

- For a sufficiently efficient regenerative system, it may be possible to realize a fully-active LQG control law.
- Regenerative actuation systems can implement open-loop-unstable controllers, which affords them a capability which is fundamentally beyond passive systems.
- Even when regenerative controllers are open-loop-stable, they generally are not passive.
- Regenerative systems with high parasitic loss parameters can perform extremely poorly. Although we have not investigated it here, there will almost always be a subset of the feasible $(\bar{p}_d, R_d)$ region, outside of which a regenerative implementation is not useful, even if it is physically possible. However, finding this region requires that the optimal passively-constrained J be found, which is a nonconvex optimization problem, and is beyond the scope of this paper.

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