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Optimal DOB Design for Balancing Input/Output Disturbances Response

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Abstract: In order to further improve the overall disturbance rejection response when both input and output disturbances are taken into account, we present a design method based on the disturbance observer (DOB) control structure. In this paper, optimal analytical DOB design is provided under the combined 2-norm objective. Results show that the proposed observer can optimally eliminate the input/output disturbance with the best combined performance criterion.

1. INTRODUCTION

The issue of disturbance rejection has been considered as one of the most significant aspects in industry (Morris, M., et al). Therefore, it has been paid a great deal of attention both in research and applications. After the practice of the past decades, internal model control (IMC) theory has been widely recognized as one of the most effective strategies for the disturbance rejection and has been successfully applied to different types of situations in practice. A number of articles providing control schemes or tuning strategies in terms of IMC principle have been proposed to improve output load disturbance rejection performance for different types of plants (Morris, M., et al) , (Zhang, WD., et al., 2012). In order to separately acquire optimal load disturbance rejection performance without decreasing the nominal servo ability, the two-degree-of-freedom (2DOF) structure is usually adopted (Liu, T., et al., 2005). After years, since the inherent shortcoming of the conventional IMC, attention of the search for new filters and alterative procedures to cope with the input disturbance rejection issue has been raised. Campi et al. firstly provided a modified filter which gives easy adjustment for unstable linear time-invariant continuous systems to balance the closed-loop bandwidth and robustness (Campi, M., et al). Then, Horn et al. suggested another type of the IMC-filter to enhance the input disturbance attenuation (Horn, I. G., et al). From a broader perspective, improved IMC-based filters against different cases of disturbances available for both stable and unstable plants were proposed by Lee et al (Lee, Y., et al., 1998, 2000). For the inevitable negative effect of slow poles, Liu proposed new insight into IMC-based filter design (Liu, T., et al., 2010, 2011). Furthermore, the input/output disturbance trade-off problem is discussed recently. By appropriately selecting the weighting function, Alcantara et al demonstrated the IMC-based control scheme for balancing input/output disturbance response (Alcantara, S., et al., 2011, 2013).

The DOB-based control scheme is a widely adopted structure for load disturbance rejection problem after originally proposed by Ohnishi (Ohnishi, K., et al). There have been plenty of issues providing different structures of the observers that are utilized for specific conditions to effectively observe the unexpected external disturbance and generate appropriate compensation signals to optimally restrain the impact of the disturbance to the system (Chen, XK., et al) (Yoon, YD., et al). Therefore, the proposed article aims to present a simple and effective observer formula in terms of the combined 2-norm. The input/output disturbance rejection problem is discussed based on the DOB-based control structure. A brief introduction of this paper is presented as follows: in Section 2, we first discuss the problem of system parameterization and the set of filters that can guarantee internal stability of the system are provided; then, a novel analytical result of the input/output disturbance rejection criterion is given and the optimal solution for the DOB is proposed. Based on the given results, Section 3 specifically analyzes a typical first order process. The simulation example is given to show the disturbance rejection ability of the proposed control scheme. In section4, we summarize the main ideas and make some concluding remarks. At last, for the sake of notation simplicity, we tend to drop arguments (e.g., G_p instead of $G_p(s)$, $G_p(-)$ instead of $G_p(-s)$) when there is no danger of confusion.

CONVENTIONAL DOB BASED CONTROL SCHEME

The block diagram of the standard DOB-based control scheme is shown [12] in Fig. 1, where $G_p e^{-\tau s}$ is the real

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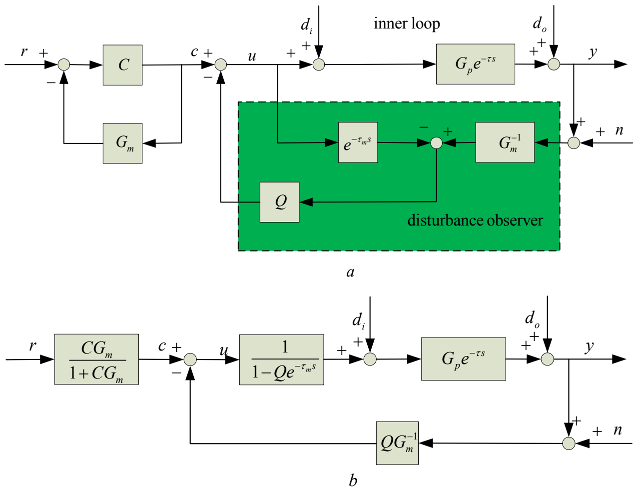

Fig. 1: Block diagram of the standard DOB-based Control Scheme. (a) Original structure for design; (b) Equivalent structure for implementation.

plant. G_p is the plant free of time delay and $e^{-\tau s}$ is the constant time delay of the system. G_m is the nominal model of G_p; C is the controller. r, d, y and n denote the reference input, input disturbance, system output and measurement noise, respectively. Q is the DOB to be designed. The original structure in Fig. 1a is not causal and sometimes is not internally stable (the control plant contains right half plane zeros). An equivalent structure shown in Fig. 1b is causal and internally stable. Under nominal conditions, $G_p = G_m$ and $\tau = \tau_m$. The nominal model G_m herein can be expressed as follows:

$$G_m = \frac{KN_+N_-}{M_+M_-} \tag{1}$$

Specifically

$$M_- = \prod_{i=1}^{n_s}(\tau_i s+1)^{k_i};M_+ = \prod_{j=n_s+1}^{n_s+n_u}(-\tau_j s+1)^{k_j} \quad \forall \tau_i,\tau_j > 0 \tag{2}$$

where K is a real constant denoting the static gain. The subscript minus sign (-) denotes the roots in the left half part of complex plane (LHP); correspondingly, the subscript plus sign (+) denotes the roots located in the right half plane (RHP). The nominal plant satisfies the following two assumptions:

Assumption 1 $N_-(0) = N_+(0) = M_-(0) = M_+(0) = 1$ and the degree condition satisfies
$\deg\{N_-\} + \deg\{N_+\} \le \deg\{M_-\} + \deg\{M_+\}$

Assumption 2 the nominal plant G_m described as (1) has no imaginary axis poles.

For control system, the first and foremost target is to guarantee internal stability of the system. Therefore, we first discuss parameterization problem. Then, the following subsection provides the observer parameterization formula in this article.

2.1 Observer Parameterization

A closed-loop system is internally stable if bounded signals injecting the system at any point generate bounded responses at any other point. For the system configuration in Fig. 1, the following Theorem gives the set of all the observers that guarantees the system internal stability.

Theorem 1 The control scheme is depicted as Fig. 1. The observer which guarantees the system internal stability can be parameterized as

$$Q = \frac{Q_1 N_+}{K} \tag{3}$$

Q_1 is any stable transfer function that makes Q proper. Besides, Q_1 satisfies

$$\lim_{s\to 1/\tau_j}\frac{\mathrm{d}\,k}{\mathrm{d}\,s^k}\left[1-\frac{Q_1N_+}{K}e^{-\tau_m s}\right] = 0\,;0 \le k < k_j \tag{4}$$

and there is no right half plane zero-pole cancellation in C. Specifically, the observer should also satisfy the following equation to obtain zero-steady error for a unit load disturbance:

$$\lim_{s\to 0}\frac{KN_+N_-}{M_+M_-}e^{-\tau_m s}(1-\frac{Q_1N_+}{K}e^{-\tau_m s}) = 0 \tag{5}$$

which leads to

$$Q_1 = \frac{K+sQ_2}{F} \tag{6}$$

where Q_2 is an any stable transfer function and F is also a stable transfer function to make it strictly proper. Generally, F is given as $F = 1/(\lambda s+1)^n$, where λ is the low-pass filter time constant and n is the degree constant to be settled properly.

Proof: Omitted. For details explanations, the readers can refer to (Zhang, et al, 2006).

□

At the end of this subsection, we provide a lemma which will be utilized for the design of the optimal observer in the next part. The symbol L_2 denotes the family of all the rational transfer functions with no poles on the imaginary axis. Then,

Lemma 1 Let H_2 denote the subset of L_2, $\mathrm{H}_2^{\perp}$ the set of rational transfer functions analytic in $\mathrm{Re}\,s \le 0$. Every function F in L_2 can be uniquely expressed as

$$F = F_1 + F_2, F_1 \in H_2, F_2 \in H_2^{\perp}$$

then

$$\|F_1+F_2\|_2^2 = \|F_1\|_2^2 + \|F_2\|_2^2 \tag{7}$$

2.2 Optimal Observer Design

The subject of this subsection is to design optimal observer. With the context in mind, the aim of this article is to take care of the input/output disturbance trade-off issue. The general idea is to obtain the optimal balanced input/output disturbance rejection observer in terms of the combined control objective, which is considered as follows:

$$J=(1-\varepsilon)\left\|G_m e^{-\tau_m s}\left(1-Qe^{-\tau_m s}\right)\frac{1}{s}\right\|_2^2+\varepsilon\left\|\left(1-Qe^{-\tau_m s}\right)\frac{1}{s}\right\|_2^2 \quad (8)$$

The combined control objective given above takes into account of both input disturbance rejection performance and output disturbance rejection performance. They are balanced by the adjustable weighting parameter ε, which determines the importance degree of each type of the disturbance to the system. Especially, when $\varepsilon=0$, the objective is simplified to the optimal input disturbance rejection. With the increase of ε, the effect of the output disturbance becomes significant. To obtain the optimal formula of (8), we rewrite the expression as

$$J=\left\|\begin{Bmatrix}\sqrt{1-\varepsilon}\left[G_m e^{-\tau_m s}\left(1-Qe^{-\tau_m s}\right)\right]\\ \sqrt{\varepsilon}\left(1-Qe^{-\tau_m s}\right)\end{Bmatrix}\frac{1}{s}\right\|_2^2 \quad (9)$$

The time delay is all-pass, and the following transfer function matrix is also all-pass:

$$\Phi=\begin{bmatrix}\dfrac{N_+^2(-)M_+}{N_+^2 M_+(-)}e^{2\tau_m s} & 0\\ 0 & \dfrac{N_+(-)}{N_+}e^{\tau_m s}\end{bmatrix} \quad (10)$$

for

$$\Phi^{\mathrm{T}}(-)\Phi=I \quad (11)$$

In light of the definition of the 2-norm, an all-pass transfer function does not affect the value of the 2-norm. Therefore,

$$J=\left\|\Phi\begin{Bmatrix}\sqrt{1-\varepsilon}\left[G_m e^{-\tau_m s}\left(1-Qe^{-\tau_m s}\right)\right]\\ \sqrt{\varepsilon}\left(1-Qe^{-\tau_m s}\right)\end{Bmatrix}\frac{1}{s}\right\|_2^2 \quad (12)$$

After a series of equivalent manipulations, we obtain

$$J=\left\|\begin{Bmatrix}\sqrt{1-\varepsilon}\begin{pmatrix}\dfrac{KN_+^2(-)N_-e^{\tau_m s}-KN_+-sX}{N_+M_+(-)M_-}+\dfrac{sX}{N_+M_+(-)M_-}\\ +\dfrac{K(\lambda s+1)^n-KN_+^2(-)N_-}{M_+(-)M_-(\lambda s+1)^n}-\dfrac{N_+^2(-)N_-}{M_+(-)M_-(\lambda s+1)^n}sQ_2\end{pmatrix}\\ \sqrt{\varepsilon}\left(\dfrac{N_+(-)e^{\tau_m s}-N_+}{N_+}+\dfrac{(\lambda s+1)^n-N_+(-)}{(\lambda s+1)^n}-\dfrac{N_+(-)}{K(\lambda s+1)^n}sQ_2\right)\end{Bmatrix}\frac{1}{s}\right\|_2^2 \quad (13)$$

where

$$X=N_+Y \quad (14)$$

In order to obtain the optimal solution of (8), we need to first optimize the input disturbance rejection performance objective. We provide the optimal result in Appendix A. Here, Y is

$$Y=\sum_{a=1}^{n_s+n_u}\left\{\left.\frac{KN_+^2(-)N_-e^{\tau_m s}-KN_+}{sN_+}\right|_{s=-1/\tau_a}\prod_{b=1,b\neq a}^{n_s+n_u}\frac{\tau_a(\tau_b s+1)}{\tau_a-\tau_b}\right\} \quad (15)$$

Therefore, J can be separated as

$$J=J_1+J_2 \quad (16)$$

where

$$J_1=\left\|\begin{Bmatrix}\sqrt{1-\varepsilon}K\left(\dfrac{KN_+^2(-)N_-e^{\tau_m s}-KN_+-sX}{N_+M_+(-)M_-}\right)\\ \sqrt{\varepsilon}\left(\dfrac{N_+(-)e^{\tau_m s}-N_+}{N_+}\right)\end{Bmatrix}\frac{1}{s}\right\|_2^2$$

and

$$J_2=\left\|\begin{Bmatrix}\sqrt{1-\varepsilon}\begin{pmatrix}\dfrac{K(\lambda s+1)^n-KN_+^2(-)N_-}{M_+(-)M_-(\lambda s+1)^n}+\dfrac{sX}{N_+M_+(-)M_-}\\ -\dfrac{N_+^2(-)N_-}{M_+(-)M_-(\lambda s+1)^n}sQ_2\end{pmatrix}\\ \sqrt{\varepsilon}\left(\dfrac{(\lambda s+1)^n-N_+(-)}{(\lambda s+1)^n}-\dfrac{N_+(-)}{K(\lambda s+1)^n}sQ_2\right)\end{Bmatrix}\frac{1}{s}\right\|_2^2$$

For convenience, we set

$$W_1=\frac{K(\lambda s+1)^n-KN_+^2(-)N_-}{M_+(-)M_-(\lambda s+1)^n}+\frac{sX}{N_+M_+(-)M_-} \quad (17)$$

$$W_2=\frac{(\lambda s+1)^n-N_+(-)}{(\lambda s+1)^n} \quad (18)$$

Then,

$$J_2=\left\|\left\{\begin{bmatrix}\sqrt{1-\varepsilon}W_1\\ \sqrt{\varepsilon}W_2\end{bmatrix}-\begin{bmatrix}\sqrt{1-\varepsilon}\dfrac{sN_+^2(-)N_-}{M_+(-)M_-(\lambda s+1)^n}\\ \sqrt{\varepsilon}\dfrac{sN_+(-)}{K(\lambda s+1)^n}\end{bmatrix}Q_2\right\}\frac{1}{s}\right\|_2^2 \quad (19)$$

Furthermore, we perform an inner-outer factorization such that

$$P=\begin{bmatrix}\sqrt{1-\varepsilon}\dfrac{sN_+^2(-)N_-}{M_+(-)M_-(\lambda s+1)^n}\\ \sqrt{\varepsilon}\dfrac{sN_+(-)}{K(\lambda s+1)^n}\end{bmatrix}=\Theta_i\Theta_o \quad (20)$$

where Θ_i is an inner matrix function, and Θ_o is the outer. According to the definition of the inner-outer factorization, Θ_i satisfies that

$$\Theta_i^{\mathrm{T}}(-)\Theta_i=I \quad (21)$$

Based on Θ_i, the following matrix is given

$$\Psi=\begin{bmatrix}\Theta_i^{\mathrm{T}}(-)\\ I-\Theta_i\Theta_i^{\mathrm{T}}(-)\end{bmatrix} \quad (22)$$

And Ψ satisfies

$$\Psi^{\mathrm{T}}(-)\Psi=I \quad (23)$$

Ψ is also an all-pass matrix that will not affect the 2-norm. therefore

$$J_2=\left\|\Psi\left\{\begin{bmatrix}\sqrt{1-\varepsilon}W_1\\ \sqrt{\varepsilon}W_2\end{bmatrix}-\begin{bmatrix}\sqrt{1-\varepsilon}\dfrac{sN_+^2(-)N_-}{M_+(-)M_-(\lambda s+1)^n}\\ \sqrt{\varepsilon}\dfrac{sN_+(-)}{K(\lambda s+1)^n}\end{bmatrix}Q_2\right\}\frac{1}{s}\right\|_2^2 \tag{24}$$

then

$$J_2=\left\|\begin{Bmatrix}\Theta_i^{\mathrm{T}}(-)\begin{bmatrix}\sqrt{1-\varepsilon}W_1\\ \sqrt{\varepsilon}W_2\end{bmatrix}-\Theta_oQ_2\\ \left[I-\Theta_i\Theta_i^{\mathrm{T}}(-)\right]\begin{bmatrix}\sqrt{1-\varepsilon}W_1\\ \sqrt{\varepsilon}W_2\end{bmatrix}\end{Bmatrix}\frac{1}{s}\right\|_2^2=J_3+J_4 \tag{25}$$

where

$$J_3=\left\|\left\{\Theta_i^{\mathrm{T}}(-)\begin{bmatrix}\sqrt{1-\varepsilon}W_1\\ \sqrt{\varepsilon}W_2\end{bmatrix}-\Theta_oQ_2\right\}\frac{1}{s}\right\|_2^2 \tag{26}$$

and

$$J_4=\left\|\left\{\left[I-\Theta_i\Theta_i^{\mathrm{T}}(-)\right]\begin{bmatrix}\sqrt{1-\varepsilon}W_1\\ \sqrt{\varepsilon}W_2\end{bmatrix}\right\}\frac{1}{s}\right\|_2^2 \tag{27}$$

for

$$\Theta_i^{\mathrm{H}}=\Theta_o^{-\mathrm{H}}\begin{bmatrix}-\sqrt{1-\varepsilon}\dfrac{sN_+^2(-)N_-}{M_+(-)M_-(\lambda s+1)^n} & -\sqrt{\varepsilon}\dfrac{sN_+(-)}{K(\lambda s+1)^n}\end{bmatrix} \tag{28}$$

We set U_1 and U_2 as follows

$$U_1=-\Theta_o^{-\mathrm{H}}\left((1-\varepsilon)\frac{sN_+^2N_-(-)}{M_+M_-(-)(-\lambda s+1)^n}W_1+\varepsilon\frac{sN_+}{K(-\lambda s+1)^n}W_2\right) \tag{29}$$

$$U_2=\begin{bmatrix}\sqrt{1-\varepsilon}KW_1\\ \sqrt{\varepsilon}W_2\end{bmatrix}-\Theta_iU_1 \tag{30}$$

Then

$$J_2=\underbrace{\left\|\left(U_1(s)-\Theta_o(s)Q_2(s)\right)\frac{1}{s}\right\|_2^2}_{J_3}+\underbrace{\left\|U_2(s)\frac{1}{s}\right\|_2^2}_{J_4} \tag{31}$$

In order to get the optimal solution for (8), we mean

$$J^*=\inf J=J_1+J_4+\inf J_3 \tag{32}$$

Therefore, the optimization issue is converted to optimize J_3, which means that the combined objective is simplified as the search of the optimal solution for a single performance criterion.

2.3 Inner-Outer Factorization

This subsection discusses the inner-outer factorization issue. Commonly, we can follow the steps provides by (Francis) to obtain the inner outer factorization of (20). Especially, for the first order plant plus with time delay which is expressed as

$$G(s)=\frac{-\alpha s+1}{\tau s+1}e^{-\theta s} \tag{33}$$

(20) is

$$P=\begin{bmatrix}\sqrt{1-\varepsilon}\dfrac{(\alpha s+1)^2s}{(\tau s+1)(\lambda s+1)^n}\\ \sqrt{\varepsilon}\dfrac{(\alpha s+1)s}{(\lambda s+1)^n}\end{bmatrix}=\Theta_i\Theta_o \tag{34}$$

Specifically, for (34), we have

$$\Theta_i(s)=\begin{bmatrix}\dfrac{\sqrt{1-\varepsilon}(\alpha s+1)}{\sqrt{\alpha^2-(\alpha^2-\tau^2)\varepsilon}s+1}\\ \dfrac{\sqrt{\varepsilon}(\tau s+1)}{\sqrt{\alpha^2-(\alpha^2-\tau^2)\varepsilon}s+1}\end{bmatrix} \tag{35}$$

And

$$\Theta_o(s)=\frac{(\alpha s+1)\left(\sqrt{\alpha^2-(\alpha^2-\tau^2)\varepsilon}s+1\right)s}{(\tau s+1)(\lambda s+1)^n} \tag{36}$$

EXAMPLES AND SIMULATIONS STUEIES

In this section, an example is given to clarify the optimal observer deriving procedures and illustrate the effectiveness of the proposed result. Consider the following plant

$$G(s)=\frac{-2s+1}{s+1} \tag{37}$$

According to (35) and (36), we have

$$\Theta_o(s)=\frac{(2s+1)\left(\sqrt{4-3\varepsilon}s+1\right)s}{(s+1)(\lambda s+1)} \tag{38}$$

$$\Theta_i(s)=\begin{bmatrix}\dfrac{\sqrt{1-\varepsilon}(2s+1)}{\sqrt{4-3\varepsilon}s+1}\\ \dfrac{\sqrt{\varepsilon}(s+1)}{\sqrt{4-3\varepsilon}s+1}\end{bmatrix} \tag{39}$$

According to (15), Y is

$$Y(s)=\frac{2}{3} \tag{40}$$

and

$$W_1=-\frac{\left[(2/3)\lambda-4\right]s^2+\left[\lambda-(10/3)\right]s}{(s+1)(\lambda s+1)} \tag{41}$$

$$W_2=\frac{(\lambda-2)s}{\lambda s+1} \tag{42}$$

Therefore, when $\lambda=0.1$, the optimal DOB is given as

$$Q^*=\frac{\left(5.3497-4\sqrt{4-3\varepsilon}\right)s^3-0.5116s^2+\left(\sqrt{4-3\varepsilon}-3.0817\right)s+1}{0.2\sqrt{4-3\varepsilon}s^3+\left(2.1\sqrt{4-3\varepsilon}+0.2\right)s^2+\left(\sqrt{4-3\varepsilon}+2.1\right)s+1}$$

assume $\varepsilon=0.5$, which means that the input disturbance and output disturbance perform the equivalent impact on the system. Table 1 compares the proposed method with the optimal solutions for separate performance criterion, where $\lambda=\sqrt{4-3\varepsilon}$. Fig 2 shows that results. From the results, it can be seen that the proposed performs better disturbances rejection ability for the smallest values over the whole range.

Table 1. Comparison of the proposed with other methods

	Q_o	Q_i	$Q_{i/o}$
Input	8	$\frac{64}{9}$	$\frac{8(\lambda+2)^2}{9\lambda}$
Output	4	$\frac{40}{9}$	$\frac{4(1+2\lambda)(2+\lambda)}{9\lambda}$
I/O	$8-4\varepsilon$	$\frac{64}{9}-\frac{24}{9}\varepsilon$	$\frac{8(\lambda+2)^2}{9\lambda}-\frac{4(\lambda+2)}{3\lambda}\varepsilon$

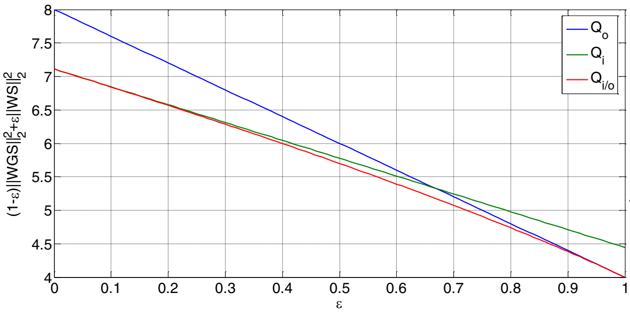

Fig. 2: Comparison of the proposed with other methods for the whole range of ε

To further demonstrate the result, we present a simulation to test the effectivness. All the optimal methods are augmented by the same filter

$$F=\frac{1}{\lambda_f s+1} \tag{43}$$

To compare the optimal performance, $\lambda_f=0.01$ for all the cases. For the limitation of the layout, the simulations that filters are tuned for all the systems with fixed robust level will be not presented. Here, we only compare the optimal performance. For further studies, more details will be given in the following publication. The input load disturbance is entered at $t_i=55s$ and the output disturbance at $t_o=105s$. The simulations are shown in Fig 3. The results are shown in Table 2. Form Table 2, it can be seen that, just as what we have analyzed, for input disturbance, the observers Q_i performs the best ISE specification; for output disturbance, the observer Q_o performs the best; the proposed observer $Q_{i/o}$ performs the best overall ISE specification. But for the IAE, Q_o performs the best for all the cases, since the disturbance rejection response by Q_o has the largest overshoot and undershoot values, which dominant the ISE values. Also, it performs the smallest transient time, resulting in the best IAE performances.

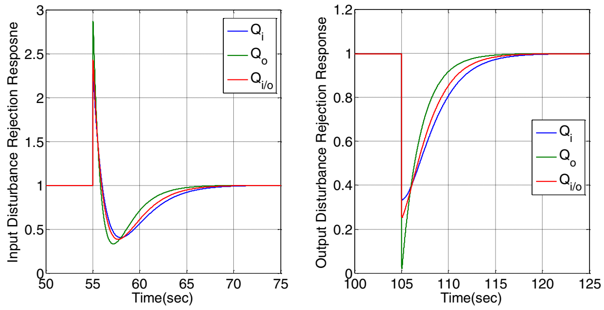

Fig. 3: Simulations for the optimal performance of the proposed with other methods

Table 2. Simulation results for different methods

methods	Input. Dist		Output. Dist		Total Values	
	ISE	IAE	ISE	IAE	ISE	IAE
Q_i	**1.783**	3.814	1.114	2.674	2.898	6.488
Q_o	2.001	**3.346**	**1.002**	**2.007**	3.004	**5.354**
$Q_{i/o}$	1.807	3.595	1.051	2.394	**2.858**	5.990

CONCLUSIONS

This note has provided a DOB control scheme for the open loop plant with constant time delay when both input. Optimal analytical solutions of the DOB have been proposed, in terms of the combined 2-norm. The dominant contribution of this article is that a novel optimal analytical result of the input/output disturbance rejection criterion has been derived. Then, the proposed control scheme is analyzed for the first order plant. The design procedures and a specific example are provided. Simulation results verify that the proposed scheme can eliminate the influence of input/output load disturbance and obtain the best overall ISE performance specification.

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Appendix A

Theorem 2 The control scheme is depicted as Fig. 1. The DOB to optimize the input load disturbance rejection criterion in terms of H_2 norm is given as

$$Q_{opt} = \frac{(sYF+K)N_+}{KN_+^2(-)N_-} \tag{44}$$

where

$$Y = \sum_{a=1}^{n_s+n_u} K\left\{ \frac{N_+^2(-)N_-e^{\tau_m s} - N_+}{sN_+} \bigg|_{s=-1/\tau_a} \prod_{b=1,b\neq a}^{n_s+n_u} \frac{\tau_a(\tau_b s+1)}{\tau_a-\tau_b} \right\} \tag{45}$$

Proof:

$$\left\| WG_m e^{-\tau_m s}\left(1-Qe^{-\tau_m s}\right)\right\|_2^2$$

$$= \left\| \frac{1}{s}\frac{KN_+N_-}{M_+M_-} - \frac{KN_+^2N_-}{sM_+M_-F}e^{-\tau_m s} - \frac{N_+^2N_-Q_2}{M_+M_-F}e^{-\tau_m s}\right\|_2^2$$

$$= \left\| \frac{KN_+^2(-)N_-}{sN_+(s)M_+(-)M_-}e^{\tau_m s} - \frac{KN_+^2(-)N_-}{sM_+(-)M\_F} - \frac{N_+^2(-)N_-Q_2}{M_+(-)M\_F}\right\|_2^2$$

$$= \left\| \frac{KN_+^2(-)N_-e^{\tau_m s} - KN_+}{sN_+M_+(-)M_-} + \frac{K}{sM_+(-)M_-} - \frac{KN_+^2(-)N_-}{sM_+(-)M\_F} - \frac{N_+^2(-)N_-Q_2}{M_+(-)M\_F}\right\|_2^2$$

$$= \left\| \frac{KN_+^2(-)N_-e^{\tau_m s} - KN_+}{sN_+M_+(-)M_-} + \frac{K-KN_+^2(-)N_-}{sM_+(-)M\_F} - \frac{N_+^2(-)N_-Q_2}{M_+(-)M\_F}\right\|_2^2$$

$$= \left\| \frac{KN_+^2(-)N_-e^{\tau_m s} - KN_+ - sX}{sN_+M_+(-)M_-} + \frac{X}{N_+M_+(-)M_-} + \frac{K-KN_+^2(-)N_-}{sM_+(-)M\_F} - \frac{N_+^2(-)N_-Q_2}{M_+(-)M\_F}\right\|_2^2$$

$X(s)$ should be selected to possess the following properties

- $sM_+(-s)M_-(s)$ is a factor of $KN_+^2(-s)N_-(s)e^{\tau_m s} - KN_+(s) - sX(s)$;
- $N_+(s)$ is a factor of $X(s)$.

To satisfy these two conditions, firstly, $X(s)$ states as

$$X = N_+Y \tag{46}$$

Without a loss of generality, the unique Y which could satisfy the aforementioned conditions is expressed as

$$Y = \sum_{a=1}^{n_s+n_u} \left\{ \frac{KN_+^2(-)N_-e^{\tau_m s} - KN_+}{sN_+} \bigg|_{s=-1/\tau_a} \prod_{b=1,b\neq a}^{n_s+n_u} \frac{\tau_a(\tau_b s+1)}{\tau_a-\tau_b} \right\}$$

Therefore, the expression is rearranged as

$$= \left\| \frac{KN_+^2(-)N_-e^{\tau_m s} - KN_+ - sX}{sN_+M_+(-)M_-}\right\|_2^2 + \left\| \frac{X}{N_+M_+(-)M_-} + \frac{K-KN_+^2(-)N_-}{sM_+(-)M\_F} - \frac{N_+^2(-)N_-Q_2}{M_+(-)M\_F}\right\|_2^2$$

By minimizing the left hand side part, which is equals to set it as zero, we can acquire

$$Q_2 = \frac{sYF+K-KN_+^2(-)N_-}{sN_+^2(-)N_-} \tag{47}$$

which leads to

$$Q_{opt} = \frac{(sYF+K)N_+}{KN_+^2(-)N_-} \tag{48}$$

and the optimal disturbance rejection criterion is

$$J_{opt} = \left\| \frac{KN_+^2(-)N_-e^{\tau_m s} - KN_+ - sX}{sN_+M_+(-)M_-}\right\|_2^2 \tag{49}$$

□