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Feedback Control for Linear Switched Systems Consisting of Controllable and Uncontrollable Subsystems with Stochastic Switch Signal and Uncertain Time Delay ⋆

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Abstract: For linear switched systems consisting of controllable and uncontrollable subsystems with stochastic switch signal and uncertain time delay in detection, the authors investigate its feedback controller design problem and develop a nonsynchronized feedback controller which guarantees that the expectation of state norm converges to zero. Furthermore, the authors show its effectiveness by analyzing the expectation of system state norm. In addition, an illustrative numerical example is also presented to demonstrate the utility of the proposed controller.

Keywords: Linear switched system, uncontrollable subsystem, stochastic process, controller design, nonsynchronized feedback control, uncertain time delay in detection.

1. INTRODUCTION

Switched systems consist of a finite number of subsystems. And there are logical rules that orchestrate switching between these subsystems. Such systems are common across a diverse range of application areas. For example, switched systems modeling plays a major role in the field of power systems where interactions between continuous dynamics and discrete events are an intrinsic part of power system dynamic behavior. An important problem among researches on switched systems is stability and consensus. And it has been widely studied in the past decades (see Serres et al.(2011), Amato et al.(2001), Mitra et al.(2001), Zhao et al.(2012), Xie et al.(2009), and references therein). For example, Serres et al.(2011) investigates sufficient conditions for the convergence to zero of the trajectories of linear switched systems. sufficient conditions for the convergence to zero of the trajectories of linear switched systems are investigated in Serres et al.(2011). And a collection of results that use weak dwell-time, dwell-time, strong dwell-time, permanent and persistent activation hypothesis are provided. Amato et al.(2001) addresses the issue of structural stability results of switched linear systems and provide sufficient and non-conservative results for stability of such systems. Mitra et al.(2001) addresses the issue of structural stability results of switched linear systems and provide sufficient and non-conservative results for stability of such systems. The stability and stabilization problems for a class of switched linear systems with mode-dependent average dwell time are investigated by Zhao et al.(2012).

However, a common assumption in most existing literatures except Xie et al.(2009), Ji et al.(2007), and Xie et al.(2008) is that the detection of the switching signal is instantaneous. And as pointed out in Xie et al.(2009) and Xie et al.(2008), in many real switched systems, the switch signal is created by some unknown or non-deterministic function (called *a switch stimulus*), for example, unknown abrupt phenomena such as component and interconnection failures. The changing of switching signal may not be detected instantaneously, but only after a time period. All the above results become ineffective in such a case. But one fundamental assumption of Ji et al.(2007) and Xie et al.(2008) is that the time delay in the switching detection is available. The results in these papers may become not feasible sometimes because of uncertain time delay. And in this paper, we consider the problem of design a feedback controller for the linear switched systems consisting of controllable and uncontrollable subsystems with stochastic switch signal and time delay in detection.

It is very important to note that the influence on system stability produced by stochastic switch signals and uncontrollable subsystems is also included in our consideration here. Both when the switch stimulus will produce another switch signal and which subsystem the system will switch to may be unknown. And it is noted that in our model the system may not switch to another subsystem after receiving a switch signal. This is very different from the assumptions in the existing literature. The reason for considering such situation is that in many real switched systems it is indeed universal. For example, the switch signals may be not strong enough to influence the system. And in many cases such *pseudo switch signals* are also included in statistical data such as statistic of time intervals between consecutive signals from the switch stimulus.

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Since our controller design is based on statistical data, it is necessary to consider such situation. In addition, uncontrollable subsystem is also included in our model. The reason for considering unstable subsystems is theoretical as well as the fact that uncontrollable subsystems cannot be avoided in many applications. The fact that for uncontrollable subsystems, there does not exist a feedback gain such that the closed-loop system is Hurwitz stable leads to the difficulty in controller design. And a common assumption in most of the existing literature expect Zhai et al.(2000) is that all the subsystems are controllable. But unlike the problem in Zhai et al.(2000), it is difficult to analyze the problem here via the well known and widely used average dwell time approach since the switch signal is stochastic and the activation time period ration of any subsystems is uncertain.

When analyzing the stability of switched systems, the most general way is to analyze the state norms or the Lyapunov functions (see Serres et al.(2011), Mitra et al.(2001), Zhai et al.(2000), Zhao et al.(2012), Xie et al.(2009), and references therein). However, it is difficult to analyze the state norms, nor the Lyapunov functions since the stochastic switch signal and uncertain time delay lead to the uncertain system state, meanwhile the existence of uncontrollable subsystems leads to the difficulty in utilizing the Lyapunov functions. Here, we analyze the expectations of system state norms instead since we note that when dealing with practical problems, empirical data such as statistical data are always available. And in this paper, we consider to develop our feedback controller based on some statistical data. Here, we need to know the expectation of the time delay, expectations of switching durations and the probabilities that the system switches to each subsystems and it does not switch. All these data are available when dealing with many practical problems. By utilizing these data, we consider to develop a feedback controller that guarantees the grow rates of the state norm expectation is less than 1 after a switching duration. Then the expectation of state norm will convergent. And because there exist time delays in detection, our controller is also a nonsynchronized feedback controller. It is obvious that we need to develop feedback gains via different methods for controllable uncontrollable subsystems. For controllable subsystems, by choosing appropriate gains, the closed-loop systems can be Hurwitz stable and the state norms decrease. For uncontrollable subsystems, we consider to choose appropriate gains that guarantees the system states inside required bounds over given time intervals. It is noted that in Amato et al.(2006) a sufficient condition for the design of a dynamic output feedback controller with which the linear closed-loop system states do not exceed a certain threshold of a given bound during a given time interval is presented. Motivated primarily by worked in Amato et al.(2006) and Amato et al.(2001), we consider to ensure the grow rate of state norm expectations bounded over time durations from the time that a switch signal is detected to the time that another new switch signal is produced. And it is also noted that time delay in detection may lead to the mismatches between the controller and the switched system it controls (called *plant*) and such mismatches may lead to unstable subsystems. Since we do not know which subsystem the system dwell during these time intervals, here we analyze the possible norm grow rate produced by time delay directly. Specially, by synthesizing the results developed in Amato et al.(2006), the tradition way to design feedback controllers and analysis of influence from uncertain time delay, we develop a nonsynchronized feedback controller which guarantees that the expectation of state norm converges to 0. And we show its effectiveness by analyzing the expectation of system state norm. In addition, we also present an illustrative numerical example to demonstrate the utility of the proposed controller.

The contents of the paper are as follows. In Section 2 we state the problem formulation and preliminaries. In Section 3 we present our main results, including design of a nonsynchronized feedback controller for the given linear switched systems consisting of controllable and uncontrollable subsystems with stochastic switch signal and uncertain time delay in detection and showing its effectiveness by analyzing the expectation of system state norm. In Section 4, an illustrative numerical example is presented to demonstrate the utility of the proposed controller. Finally, in Section 5 we draw some conclusions.

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{I}$ denotes the set of integers, $\bar{\mathbb{Z}}_+$ denotes the set of positive integers, $\mathbb{N}_0$ denotes the set of nonnegative integers, $(\cdot)^{\mathrm{T}}$ denotes transpose. Furthermore, we write $\mathrm{d}x$ for the differential of x, $V^{'}(x)$ for the *Fréchet* derivative of V at x, $\|\cdot\|$ for a vector norm, $\|\cdot\|_F$ for the Frobenius matrix norm, $(\cdot)^{\dagger}$ for Moore-Penrose inverse, $\mathbb{P}(E)$ for the probability of the event E, $\mathbb{E}(x)$ for the expectation of random variable x, $\lfloor a \rfloor$ for the largest integer no larger than a, $\lambda_{max}(\cdot)$ (resp., $\lambda_{min}(\cdot)$) for the maximum (resp., minimum) eigenvalue of a Hermitian matrix.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider the linear switched systems consisting of controllable and uncontrollable subsystems with stochastic switch signal and uncertain time delay in detection given by

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t),\ x(0) = x_0,\ t \geq 0, \tag{1}$$

$$\gamma(t) = \sigma(t - \tau), \tag{2}$$

$$\sigma(t) = \begin{cases} \sigma(t^-), & t \neq t_k,\ k = 0, 1, 2, \ldots, \\ \beta, & t = t_k,\ k = 0, 1, 2, \ldots, \end{cases} \tag{3}$$

where $x(t) \in \mathbb{R}^n, t \geq t_0$ is the state vector, $u(t) \in \mathbb{R}^m, t \geq t_0$ is the control input, $\sigma(t) : [0, \infty) \to \mathcal{I}_N = \{1, 2, \ldots, N\},\ N > 1$ is a piecewise constant function of time, called a *switch signal*. It is defined by (3) where $0 = t_0 < t_1 < t_2 < \cdots$ are the switching moments. And $\beta \in \mathcal{I}_N$ is a random variable whose distribution is given in TABLE 1 where $\mathbb{P}(\beta = i) = p_i,\ i \in \mathcal{I}_N, \sum_{i=1}^{N} p_i = 1$ and $p_i > 0$ are known constants. What's more, $\sigma(t) = i$ means that the ith subsystem $(A_i, B_i),\ i = 1, 2, \ldots, N$ is activated. Such a signal is created by some unknown or non-deterministic function called a *switch stimulus* (e.g. unexpected fault, change of working points etc.). The switch stimulus send a signal to the system at $t_k,\ k = 0, 1, 2, \ldots$. Then the value of $\sigma(t)$ may change at the same time and the system may switch to another subsystem. And the value of $\sigma(t)$ will not change when $t \neq t_k,\ k = 0, 1, 2, \ldots$. But it is very important to note that it is possible the value of $\sigma(t)$ may not change at $t_k,\ k = 0, 1, 2, \ldots$. Furthermore, here we assume that the event that the switch stimulus produces a switch signal and the event that one detects a signal are independent events, that is, all the signals and detections are independent with each other. Specially, both how long later the switch stimulus will produce another switch signal(denoted by the switching duration h_m defined as $h_m = t_{m+1} - t_m,\ m = 0, 1, 2, \ldots)$ and what signal the switch stimulus will produce(denoted by β) are independent random variables. What's more, how long later one can detect the signal after the stimulus produces it (denoted by τ) is also an independent random variable. Hence, it is obvious that the switch signal in our model is a stochastic switch signal. And we assume that distributions of h_m are unknown but their expectations are known. Specially, for all $m = 1, 2, \ldots$, $\mathbb{E}(h_m) = \tau_i$ when $\sigma(t_m) = i,\ i \in \mathcal{I}_N$ where $\tau_i > 0$ are known constants. Moreover, $\gamma(t)$ is the detection function of $\sigma(t)$. The time delay $\tau > 0$ implies that one cannot detect whether the system get a signal from the stimulus or not instantaneously, but after a time period τ. And one can know which subsystem is activated after getting the signal. It is very important to note that τ is not a known constant here, but a random variable. Here we assume its distribution is unknown, but its expectation is given by $\mathbb{E}(\tau) = \tau_0$ where $\tau_0 > 0$ is a known constant. It is obvious that one cannot know when the system get the signal. In addition, $A_i \in \mathbb{R}^{n\times n},\ i \in \mathcal{I}_N$and $B_i \in \mathbb{R}^{n\times m},\ i \in \mathcal{I}_N$ are known matrices. Since both controllable and uncontrollable subsystems exist in (1), we assume without loss of generality that $[A_i, B_i],\ i = 1, 2, \ldots, r$ are uncontrollable subsystems where $0 < r < N$ is a known constant, and the remaining subsystems are controllable. Furthermore, for the given switched system we assume that the required properties for the existence and uniqueness of solutions are satisfied. In addition, we assume that the system state $x(t), t \geq t_0$ is available for feedback.

Throughout this paper, we will need the following assumption.

Assumption 1 $\min_{m=0,1,2,\ldots} \bar{h}_m > \hat{\tau}$ where $\bar{h}_m,\ m = 0, 1, 2, \ldots$ denotes the possible minimum duration of the mth subsystem, and $\hat{\tau}$ denotes the possible maximum value of τ. In other words, $\tau < h_m$ holds for all $m \geq 0$.

Remark 1 Assumption 1 This guarantees that once a subsystem is activated, although we cannot detect which subsystem is being activated instantaneously, we can detect it before another subsystem is activated since its duration time is greater than τ.

In this paper, we consider the problem of design a feedback controller for the linear switched systems consisting of controllable and uncontrollable subsystems with stochastic switch signal and uncertain time delay in detection given by (1)-(3). Since the controller and the switched system it controls (called *plant*) cannot switch synchronously because of the existence of time delay in detection, such a controller is a nonsynchronized feedback controller. Since both the switching durations and time delay are random variables, it is not feasible to analyze the exact value of system state norm, nor its estimation. In this paper, we analyze the expectation of the system state norm instead. This is very different from the most general way. But it is also feasible and useful in many applications, especially when dealing the systems with a lot of uncertainties and disturbances since it is difficult or unfeasible to analyze the state norm. Furthermore, if the expectation of the system state norm converges to 0 or it is bounded in a required bound, the system can be stable most of the time and the unstability seldom happens. For many applications, such property is enough for satisfying the performance requirements. In particular, here we give a definition on such stability property of the given switched system.

Table 1. The distribution of the random variable β

$\mathbb{P}(\beta = 1)$	$\mathbb{P}(\beta = 2)$	$\cdots$	$\mathbb{P}(\beta = i)$	$\cdots$	$\mathbb{P}(\beta = N)$
p_1	p_2	$\cdots$	p_i	$\cdots$	p_N

Definition 1 For the switched system given by (1)-(3), if the expectation of the system state norm converges to 0, that is, $\mathbb{E}(\| x(t) \|) \to 0$ as $t \to \infty$, the system is said to be expected stable.

Hence, our design goal is to design a nonsynchronized feedback controller that guarantees expected stability of the given system. Specially, for the system given by (1)-(3), we consider to design a control input $u(t) = K_{\gamma(t)}x(t),\ t \geq 0$ where $K_{\gamma(t)} \in \mathbb{R}^{m\times n}$ are gain matrices that will be designed later and the control input guarantees that $\mathbb{E}(\| x(t) \|) \to 0$ as $t \to \infty$.

It is noted that one of the difficulties in our controller design is that the existence of uncontrollable subsystems leads to the difficulty in developing the feedback gain matrices by analyzing Lyapunov functions. This is because that for uncontrollable subsystems, there does not exist feedback gain matrices such that the closed-loop system is Hurwitz stable, which leads to the difficulty in utilizing both common quadratic Lyapunov functions and piecewise Lyapunov functions. And it is well known that it is difficult to develop feedback gain matrices such that the system state converges for uncontrollable subsystems. Moreover, we note that in Amato et al.(2006), a sufficient condition for the design of a dynamic output feedback controller with which the linear closed-loop system states do not exceed a certain threshold of a given bound during a given time interval is presented. Motivated primarily by the works in Amato et al.(2006) and Amato et al.(2001), we consider to develop feedback gain matrices such that the grow rate of system state norm is bounded for uncontrollable subsystems.

And the next theorem is needed for the statement of our main results presented in the next section.

Theorem 1. Consider the linear system given by

$$\dot{x}_f(t) = A_f x_f(t) + B_f u_f(t),\ x_f(t_{f0}) = x_{f0},\ t \geq t_{f0}, \quad (4)$$

where $x_f(t) \in \mathbb{R}^{n_f}, t \geq t_{f0}$ is the state vector, $u_f(t) \in \mathbb{R}^{m_f}, t \geq t_{f0}$ is the control input, $A_f \in \mathbb{R}^{n_f\times n_f}$and $B_f \in \mathbb{R}^{n_f\times m_f}$ are known matrices. For three given positive scalars c_{f1}, c_{f2}, T_f, with $c_{f1} < c_{f2}$, and a given positive define matrix R_f such that

$$x_{f0}^{\mathrm{T}} R_f x_{f0} \leq c_{f1}, \quad (5)$$

if there exist a nonnegative scalar α_f, a positive definite matrix $Q_f \in \mathbb{R}^{n_f \times n_f}$ and a matrix $N_f \in \mathbb{R}^{m_f \times n_f}$ such that

$$A_f \tilde{Q}_f + \tilde{Q}_f A_f^{\mathrm{T}} + B_f N_f + N_f^{\mathrm{T}} B_f^{\mathrm{T}} - \alpha_f \tilde{Q}_f < 0, \quad (6)$$

$$\mathrm{cond}(Q_f) < \frac{c_{f2}}{c_{f1}} e^{-\alpha_f T_f}, \quad (7)$$

$$\tilde{Q}_f = R_f^{-\frac{1}{2}} Q_f R_f^{-\frac{1}{2}}, \quad (8)$$

where $\mathrm{cond}(Q_f) = \lambda_{max}(Q_f)/\lambda_{min}(Q_f)$ denotes the condition number of Q_f, the linear system is FTS with respect to $(c_{f1}, c_{f2}, T_f, R_f)$, that is, there exist

$$x_f^{\mathrm{T}}(t) R_f x_f(t) < c_{f2}, \ \forall t \in [t_{f0}, t_{f0} + T_f], \quad (9)$$

with a state feedback controller given by

$$u_f(t) = K_f x_f(t), \ t \geq t_{f0}, \quad (10)$$

where $K_f = N_f \tilde{Q}_f^{-1}$.

Proof. It is a direct consequence of *Theorem 5* in Amato et al.(2006), hence, is omitted.

Another difficulty in our controller design is that during some time intervals the feedback gains and the subsystems may be *mismatched* because of the existence of time delay. For example, the feedback gain for ith subsystem may be applied to the jth subsystem where $i \neq j$. And such mismatches may lead to the growth of system norm. Further it is difficult to utilize the most general used Lyapunov functions or the results in *Theorem 1* because one cannot know which subsystem is activated. Hence, we consider to analyze the system state norm directly. And by synthesizing the traditional feedback controller and the analysis above, we develop our nonsynchronized feedback controller.

3. MAIN RESULTS

In this section, we consider to design a nonsynchronized feedback controller for the given switched system with stochastic switch signal and uncertain time delay in detection.

We first define a special constant $\rho(D)$ for a given matrix $D \in \mathbb{R}^{n \times n}$ to denote the smallest constant γ such that the inequality given by $\| e^{Dt} \| \leq e^{\gamma t}$ holds for all $t > 0$.

Definition 2 For a given matrix $D \in \mathbb{R}^{n \times n}$, there always exists a special constant $\rho(D)$ given by

$$\rho(D) = \min_{\xi \in \mathcal{S}_D} \xi, \ \mathcal{S}_D = \{\alpha | \ \| e^{Dt} \| \leq e^{\alpha t}, \ t > 0\}.$$

Since $\rho(D)$ is easy to compute using algebraic matrix theory, it is convenient for realistic applications.

Then we state the main results in this section.

Theorem 2. Consider the linear switch system given by (1)-(3), and assume that *Assumption 1* holds. In addition, assume there exist positive defined matrices $P_i \in \mathbb{R}^{n \times n}$, $Q_i \in \mathbb{R}^{n \times n}$, $i = 1, 2, \ldots, r$, matrices $M_i \in \mathbb{R}^{m \times n}$, $i = 1, 2, \ldots, r$, $F_i \in \mathbb{R}^{m \times n}$, $i = r+1, r+2, \ldots, N$ and positive constants $\alpha_i > 0$, $0 < \phi < 1$ such that $A_i + B_i F_i$, $i = r+1, r+2, \ldots, N$ are Hurwitz stable and the following inequalities hold:

$$A_i \tilde{Q}_i + \tilde{Q}_i A_i^{\mathrm{T}} + B_i M_i + M_i^{\mathrm{T}} B_i^{\mathrm{T}} - \alpha_i \tilde{Q}_i < 0, \ i = 1, 2, \ldots, r, \quad (11)$$

$$\sum_{i=1}^{r} p_i \eta_i e^{\tau_0 \varphi_i} + \sum_{i=r+1}^{N} p_i e^{(\tau_i - \tau_0)\lambda_{max}(A_i + B_i F_i) + \tau_0 \varphi_i} \leq \phi, \quad (12)$$

$$\eta_i > \sqrt{e^{\alpha_i(\tau_i - \tau_0)} \frac{\lambda_{max}(P_i)\lambda_{max}(Q_i)}{\lambda_{min}(P_i)\lambda_{min}(Q_i)}}, \ i = 1, 2, \ldots, r, \quad (13)$$

where

$$\tilde{Q}_i = P_i^{-\frac{1}{2}} Q_i P_i^{-\frac{1}{2}}, \ i = 1, 2, \ldots, r, \quad (14)$$

$$\varphi_i = \max_{j=1,2,\ldots,N} \rho(A_j + B_j K_i), \ i = 1, 2, \ldots, N, \quad (15)$$

$$K_i = \begin{cases} M_i \tilde{Q}_i^{-1}, & i = 1, 2, \ldots, r, \\ F_i, & i = r+1, r+2, \ldots, N, \end{cases} \quad (16)$$

where τ_i, $i = 0, 1, \ldots, N$, p_i, $i = 1, \ldots, N$, are defined in above paragraphes. Then with the state feedback controller given by $u(t) = K_{\gamma(t)} x(t)$, $t \geq 0$, the closed-loop switched system given by (1)-(3) and (11)-(16) is expected stable, that is, $\mathbb{E}(\| x(t) \|) \to 0$ as $t \to \infty$.

Proof. Firstly we analyze the expectation of system state norm.

Specially, note that all the switch signals and detections are independent with each other and **Definition 2**, it follows from

$$x(t_{m+1} + \tau) = x(t_m + h_m + \tau) = e^{\tau[A_{\sigma(t_{m+1})} + B_{\sigma(t_{m+1})} K_{\sigma(t_m)}]} \times e^{(h_m - \tau)[A_{\sigma(t_m)} + B_{\sigma(t_m)} K_{\sigma(t_m)}]} x(t_m + \tau), \ m = 0, 1, 2, \ldots,$$

that

$$\begin{aligned}
&\mathbb{E}(\| x(t_{m+1} + \tau) \|) = \mathbb{E}(\| e^{\tau[A_{\sigma(t_{m+1})} + B_{\sigma(t_{m+1})} K_{\sigma(t_m)}]} \times e^{(h_m - \tau)[A_{\sigma(t_m)} + B_{\sigma(t_m)} K_{\sigma(t_m)}]} \times x(t_m + \tau) \|) \\
&\leq (\sum_{i=r+1}^{N} p_i \| e^{\mathbb{E}(\tau)[A_{\sigma(t_{m+1})} + B_{\sigma(t_{m+1})} K_i]} e^{\mathbb{E}(h_m - \tau)[A_i + B_i K_i]} \|) \\
&\quad \times \mathbb{E}(\| x(t_m + \tau) \|) + \sum_{i=1}^{r} p_i \| e^{\mathbb{E}(\tau)[A_{\sigma(t_{m+1})} + B_{\sigma(t_{m+1})} K_i]} \| \\
&\quad \times \mathbb{E}(\| x(t_m + \mathbb{E}(h_m - \tau) + \tau) \|) \\
&\leq (\sum_{i=r+1}^{N} p_i e^{\tau_0 \rho(A_{\sigma(t_{m+1})} + B_{\sigma(t_{m+1})} K_i)} \| e^{(\tau_i - \tau_0)[A_i + B_i K_i]} \|) \\
&\quad \times \mathbb{E}(\| x(t_m + \tau) \|) + \sum_{i=1}^{r} p_i e^{\tau_0 \rho(A_{\sigma(t_{m+1})} + B_{\sigma(t_{m+1})} K_i)} \\
&\quad \times \mathbb{E}(\| x(t_m + (\tau_i - \tau_0) + \tau) \|) \\
&\leq (\sum_{i=r+1}^{N} p_i e^{\tau_0 \varphi_i} \| e^{(\tau_i - \tau_0)[A_i + B_i K_i]} \|) \times \mathbb{E}(\| x(t_m + \tau) \|) \\
&\quad + \sum_{i=1}^{r} p_i e^{\tau_0 \varphi_i} \times \mathbb{E}(\| x(t_m + (\tau_i - \tau_0) + \tau) \|) \\
&\leq \sum_{i=r+1}^{N} p_i e^{(\tau_i - \tau_0)\lambda_{max}(A_i + B_i F_i) + \tau_0 \varphi_i} \times \mathbb{E}(\| x(t_m + \tau) \|)
\end{aligned}$$

$$+\sum_{i=1}^{r} p_i e^{\tau_0\varphi_i} \times \mathbb{E}(\| x(t_m+(\tau_i-\tau_0)+\tau) \|)$$
$$m=0,1,2,\ldots. \quad (17)$$

Then we analyze the conditions given by (11), (13) and (14). Note that it follows from (14) that

$$\frac{\lambda_{max}(Q_i)}{\lambda_{min}(Q_i)} < \frac{\lambda_{max}(P_i)}{\lambda_{min}(P_i)} e^{-\alpha_i(\tau_i-\tau_0)}\eta_i^2,\ i=1,2,\ldots,r. \quad (18)$$

And consider the linear system given by

$$\dot{y}(t)=(A_i+B_iK_i)y(t),\ y(0)=y_0,\ t\geq 0,\ i\in\mathcal{I}_r, \quad (19)$$

where $y(t) \in \mathbb{R}$, $t \geq 0$, is the system state, $\mathcal{I}_r = \{1,2,\ldots,r\}$ and A_i, B_i, K_i are defined as above. Here let $c_i = y_0^{\mathrm{T}} P_i y_0$. Hence, (18) can be rewritten as

$$\frac{\lambda_{max}(Q_i)}{\lambda_{min}(Q_i)} < \frac{c_i \frac{\lambda_{max}(P_i)}{\lambda_{min}(P_i)} e^{-\alpha_i(\tau_i-\tau_0)}\eta_i^2}{c_i},\ i=1,2,\ldots,r. \quad (20)$$

Then it follows from *Theorem 1* and conditions given by (11), (13)-(14), (16), (20) that

$$y^{\mathrm{T}}(\tau_i-\tau_0)P_i y(\tau_i-\tau_0) < c_i \frac{\lambda_{max}(P_i)}{\lambda_{min}(P_i)}\eta_i^2$$
$$\frac{y^{\mathrm{T}}(\tau_i-\tau_0)P_i y(\tau_i-\tau_0)}{y_0^{\mathrm{T}} P_i y_0} < \frac{\lambda_{max}(P_i)}{\lambda_{min}(P_i)}\eta_i^2,\ i\in\mathcal{I}_r. \quad (21)$$

Note that

$$\frac{\lambda_{min}(P_i) \| y(\tau_i-\tau_0) \|^2}{\lambda_{max}(P_i) \| y_0 \|^2} < \frac{y^{\mathrm{T}}(\tau_i-\tau_0)P_i y(\tau_i-\tau_0)}{y_0^{\mathrm{T}} P_i y_0},$$

then it follows from (21) that

$$\frac{\| y(\tau_i-\tau_0) \|}{\| y_0 \|} < \eta_i. \quad (22)$$

Then it follows from the result given by (22) and the inequities given by (12) and (17) that

$$\mathbb{E}(\| x(t_{m+1}+\tau) \|) \leq \sum_{i=r+1}^{N} p_i e^{(\tau_i-\tau_0)\lambda_{max}(A_i+B_iF_i)+\tau_0\varphi_i}$$
$$\times\mathbb{E}(\| x(t_m+\tau) \|) + \sum_{i=1}^{r} p_i e^{\tau_0\varphi_i} \times \mathbb{E}(\eta_i \| x(t_m+\tau) \|)$$
$$=(\sum_{i=1}^{r} p_i\eta_i e^{\tau_0\varphi_i} + \sum_{i=r+1}^{N} p_i e^{(\tau_i-\tau_0)\lambda_{max}(A_i+B_iF_i)+\tau_0\varphi_i})$$
$$\times\mathbb{E}(\| x(t_m+\tau) \|) \leq \phi\mathbb{E}(\| x(t_m+\tau) \|),\ m=0,1,2,\ldots, \quad (23)$$

which implies that

$$\mathbb{E}(\| x(t_{m+1}+\tau) \|) \leq \phi^m \mathbb{E}(\| x(t_1+\tau) \|),\ m=0,1,2,\ldots. \quad (24)$$

Since $0<\phi<1$, it follows from (24) that $\mathbb{E}(\| x(t_m+\tau) \|) \to 0$ as $m\to\infty$, which implies that $\mathbb{E}(\| x(t) \|) \to 0$ as $t\to\infty$.

This completes the proof.

4. ILLUSTRATIVE NUMERICAL EXAMPLE

In this section we present a numerical example to demonstrate the utility of the proposed nonsynchronized feedback controller. Specially, consider the linear switched systems consisting of controllable and uncontrollable subsystems with stochastic switch signal and uncertain time delay in detection given by (1)-(3) with

$$n=m=2=N,,\ r=1,\ p_1=0.26,\ p_2=0.74$$
$$x(t)=\begin{bmatrix} x_1(t)\\ x_2(t)\end{bmatrix},\ x_0=\begin{bmatrix} x_{10}\\ x_{20}\end{bmatrix}=\begin{bmatrix} 10\\ -6\end{bmatrix}$$
$$A_1=\begin{bmatrix} 0.1 & 0\\ 0 & 0.1\end{bmatrix},\ A_2=\begin{bmatrix} 0.1 & 0\\ 0 & 0.2\end{bmatrix}$$
$$B_1=\begin{bmatrix} 1 & 0\\ 0 & 0\end{bmatrix},\ B_2=\begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix}.$$

Obviously, since $rank[B_1,A_1B_1]=1$ and $rank[B_2,A_2B_2]=2$, $[A_1,B_1]$ is a uncontrollable subsystem and $[A_2,B_2]$ is a controllable subsystem. In addition, let $\tau_1=\tau_2=2$(s), $\tau_0=0.5$(s). By solving (11)-(16), we obtain a feasible solvation with control gains given by

$$K_1=\begin{bmatrix} 0.1 & 0\\ 0 & 0.1\end{bmatrix},\ K_2=\begin{bmatrix} -0.3 & 0\\ 0 & -0.4\end{bmatrix} \quad (25)$$

and other parameters and matrices given by

$$P_1=M_1=Q_1=\begin{bmatrix} 1 & 0\\ 0 & 1\end{bmatrix}=\tilde{Q}_1,$$
$$\eta_1=1.26,\ \phi=0.95,\ \alpha_1=0.3,$$
$$\lambda_{max}(Q_1)=\lambda_{max}(P_1)=\lambda_{min}(Q_1)=\lambda_{min}(P_1)=1.$$

Hence, it is straightforward that

$$A_1+B_1K_1=\begin{bmatrix} 0.2 & 0\\ 0 & 0.1\end{bmatrix},\ A_2+B_2K_2=\begin{bmatrix} -0.2 & 0\\ 0 & -0.2\end{bmatrix},$$
$$A_1+B_1K_2=\begin{bmatrix} -0.2 & 0\\ 0 & 0.1\end{bmatrix},\ A_2+B_2K_1=\begin{bmatrix} 0.2 & 0\\ 0 & 0.3\end{bmatrix},$$
$$\lambda_{max}(A_1+B_1K_1)=-0.2,\ \varphi_1=0.3,\ \varphi_2=0.1.$$

And these parameters satisfy our conditions.

Here we simulate influence of our controller on the given switched system via the widely-used software MATLAB™. In particular, let $\tau = rand(1,1)$, $h_m = 1.8+0.2*rand(1,1)$, $m=0,1,2,\ldots$, $\beta=\lfloor 1.74+rand(1,1)\rfloor$ where $rand(1,1)$ is a pseudorandom value drawn from the standard uniform distribution on the open interval $(0,1)$. It is obvious that $h_m>\tau$, $m=0,1,2,\ldots$ and *Assumption 1* is satisfied.

The system states trajectories versus time with our nonsynchronized feedback controller are shown in Fig.1. Only

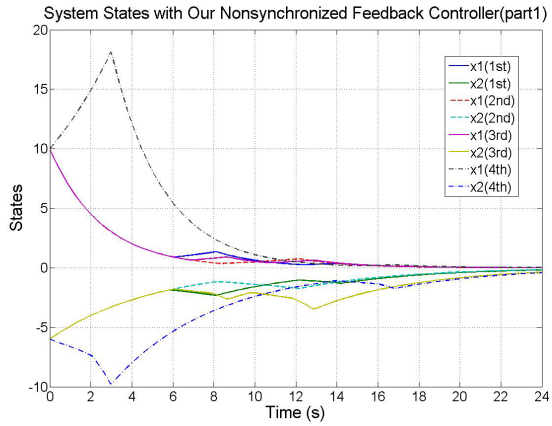

Fig. 1. States trajectories versus time with our nonsynchronized feedback controller (1st-4th simulation)

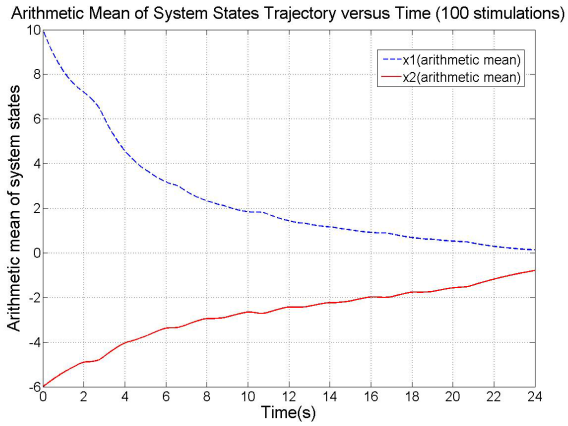

Fig. 2. Arithmetic mean of system states trajectories versus time with our nonsynchronized feedback controller (100 simulations)

4 simulation results are presented here due to the space limitation. And the arithmetic mean of system states trajectories versus time, which presents the *average trajectories* of state trajectories in 100 stimulations, are shown in Figure.2. It is shown obviously that as a result of the stochastic switch signals and uncertain time delay, the system states trajectories versus time are very different with each other. It is noted that the system may switch to any different subsystems or do not switch and stay in the same subsystems after getting the switch signals and the same system also get switch signals at different time in different simulation. In addition, the influence of time delay in detection can also be seen in the figures. For example, as shown in Fig.1, during $t = 7$(s) to $t = 9$(s), the system switches to the controllable subsystem from the uncontrollable subsystem in the 1st and 3rd simulation and switches to the uncontrollable subsystem from the controllable subsystem in the 2nd simulation, while it does not switch in the 4th simulation after getting the switch signals.

Furthermore, in Fig.1, it is shown clearly that all the trajectories are convergent. And in Fig.2, the arithmetic mean of system states trajectories versus time convergent to 0. Specially, the arithmetic means of $x_1(t),\ t \geq 0$ and $x_2(t),\ t \geq 0$ are both very close to 0 at $t = 24$(s). Hence, this example shows the effectiveness of our synchronized feedback controller for the linear switched systems consisting of controllable and uncontrollable subsystems with stochastic switch signal and uncertain time delay in detection.

5. CONCLUSION

In this paper, we develop a nonsynchronized feedback controller for linear switched systems consisting of controllable and uncontrollable subsystems with stochastic switch signal and uncertain time delay in detection and show its effectiveness by analyzing the expectation of system state norm. In addition, an illustrative numerical example is also presented to demonstrate the utility of the proposed controller.

As mentioned above, it is noted that design of our nonsynchronized feedback controller may be complex in calculation sometimes. Hence, a low computation cost way to design our feedback controller is included in our future work. Furthermore, output feedback controller for such switched systems is worth investigating.

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