**Preprints of the 19th World Congress**
**The International Federation of Automatic Control**
**Cape Town, South Africa. August 24-29, 2014**

# A Nonlinear Control Law for Hover to Level Flight for the Quad Tilt-rotor UAV

**Gerardo R. Flores-Colunga** * **Rogelio Lozano-Leal** **

* *Heudiasyc UMR CNRS 7253 Laboratory, University of Technology of Compiègne, Compiègne 60205, France (e-mail: gfloresc@hds.utc.fr).*
** *Heudiasyc UMR CNRS 7253 Laboratory, UTC, Compiègne 60205, France, and UMI-LAFMIA CNRS 3175, Cinvestav 07300, México (e-mail: rlozano@hds.utc.fr).*

**Abstract:** This paper presents recent advances in the project: development of a convertible unmanned aerial vehicle (UAV). This aircraft is able to change its flight configuration from hover to level flight and vice versa by means of a transition maneuver, while maintaining the aircraft in flight. For this purpose a nonlinear control strategy based on Lyapunov design is given. Numerical results are presented showing the effectiveness of the proposed approach.

*Keywords:* UAV, Quad Tilt-rotor, Lyapunov-based control, transition maneuver, nonlinear control, hover and level flight

## 1. INTRODUCTION

Recent literature has focused on the topic of controlling the so-called *convertible aerial vehicles*, which are aircrafts capable to fly in hover and level flight. The changing between these flight regimes is called *transition maneuver*. In this paper the transition maneuver form hover to level flight is investigated for a class of a convertible aircraft, the Quad-tilt rotor. This research is a continuation of our previous work presented in Flores and Lozano (2013b), where the mathematical model of the Quad-tilt rotor has been developed.

Several control strategies have been proposed to accomplish autonomous transition maneuvers by using the classical fixed-wing configuration. See Casau et al. (2013) and Adrian et al. (2007) as just two examples of such an approach. Another configuration well investigated in this context is the *tail-sitter aircraft*, see Stone (2004) for example. The tail-sitter has in common with the classical fixed-wing aircraft, that in order to accomplish the transition maneuver, the aircraft's body must rotate 90 degrees, having to deal with the singularity presented in the pitch dynamics. In Cetinsoy et al. (2012) the authors present a new unmanned aerial vehicle called SUAVI. Such aircraft can perform vertical takeoffs and landings like a classical helicopter; further is capable to accomplish long duration horizontal flight like an airplane. The authors present a hierarchical control system where a high level controller is responsible for generating references for low level controllers, which are responsible for attitude and altitude stabilization. However, the paper lacks of any analysis concerning the transition maneuver, which is the most interesting phenomena in this kind of vehicles. In Naldi and Marconi (2011) an optimal transition maneuver for the tail-sitter V/STOL aircraft is investigated. The authors present some numerical trajectories at simulation level showing the transition maneuver. However, such maneuver is only obtained by means of numerical computations. In most of the available work concerning convertible UAV's, see for example Stone (2004), Stone (2002), Green and Oh (2006), Escareno et al. (2006), Naldi et al. (2010), Cetinsoy et al. (2012) and Escareno et al. (2007), the control problem of the transition maneuver has not been addressed analytically, instead both dynamics (hover and level flight) are studied separately. Thus, the controllers for the hover and airplane modes are derived individually, using a switching condition but without developing any analysis between those flying regimes.

This paper complements our previous work developed in Flores and Lozano (2013b). We have included the pitch dynamics in the analysis by considering the presence of both actuators: difference between thrust and elevator's deflection, i.e. the actuators in hover and level flight mode. The goal is to develop a control strategy suitable for handling the transition maneuver, but excluding switching between both dynamics involved, but also investigating the system in a continuous manner. We propose a technique to perform the transition form hover to airplane mode, while maintaining a desired altitude. The proposed controllers are stable in the sense of Lyapunov for the transition maneuver, going from hovering flight to high speed flight.

The remainder of this work is organized as follows. Section 2 presents the system describing the longitudinal model of the Quad-Tilt rotor UAV as well as the problem statement. Section 3 details the nonlinear control strategy based on Lyapunov design. Section 4 shows numerical results obtained when applying the proposed controllers. Conclusions and future work are finally presented in section 5.

⋆ This work was supported in part by the Mexican National Council for Science and Technology.

## 2. SYSTEM DESCRIPTION

In this section, the system which represents the dynamics of the Quad-tilt rotor UAV and the problem statement are presented. Details on the mathematical model and the nomenclature used in this paper can be seen in our previous work Flores and Lozano (2013b).

### *2.1 Flight envelope description*

The flight envelope of the vehicle encompasses three different flight conditions achieved by means of the collective angular displacement of the rotors (Fig. 1). Indeed, tilting the four rotors forward will produce the transition from helicopter mode to airplane mode. These flight conditions are explained below:

(1) Hover flight mode (*HF*). During the *HF* mode the vehicle's motion relies only on the rotors. Within this phase the vehicle features VTOL flight profile. The controller for this regime disregards the aerodynamic terms due to the negligible translational speed.
(2) Fast forward flight mode (*FFF*). At this flight regime the aircraft has gained enough speed to generate aerodynamic forces to lift and control the vehicle motion. In this mode the vehicle behaves like a common airplane.
(3) Transition flight mode (*TF*). It is possible to distinguish an intermediate operation mode, the transition maneuver *TF*, which links the two flight conditions, *HF* and *FFF*.

The mechanism that allows the transition maneuver is composed of two servomechanisms responsible to switch the flight configuration from hover to level flight and vice versa. The position of such mechanism is represented by $\gamma$. Therefore, $\gamma$ represents the tilting angle of the rotors (see Fig. 2).

### *2.2 Dynamical model of the Quad-Tilt rotor UAV*

The moment exerted about the $CG$ can be written as

$$J\ddot{\theta} = \tau^b \tag{1}$$

where $J$ represents system's inertia while $\tau^b$ is the torque input obtained by controlling the differential of thrusts for the hover mode, and by controlling the aircraft elevator for the airplane mode. Thus, two inputs are presented in the pitch dynamics given by

$$\tau^b = \tau^b_T + \tau^b_M \tag{2}$$

where $\tau^b_M$ is the airfoil's pitching moment and $\tau^b_T$ is the induced moment due the difference of thrust between $T_{3,4}$ and $T_{1,2}$, (see Fig 2). Therefore, $\tau^b_T$ is model as follows

$$\tau^b_T = l_1(-T_{3,4}\cos\gamma + T_{1,2}\cos\gamma) \tag{3}$$

where $l_1$ is the distance from the $CG$ to the rotors shown in Fig. 2. It follows that the action of the input $\tau^b_M$ is a function of airspeed $V_t$ and angle of attack $\alpha$. By consequence, as long as $\gamma$ increases, the elevator deflection induces a largest pitching moment on the aircraft. As we will see in Section 3, a desired velocity $\dot{x}_s$ (and therefore its corresponding airspeed $V_t$) is achieved by increasing the tilting angle $\gamma$ from 0 to 90 degrees. Furthermore, since the main contribution to $M$ is provided by the elevator deflection $\delta$, which is an input variable, we represent the pitching moment as $M = C_{m_\alpha}(\alpha)\delta$, please see Flores et al. (2012) for details. Thus, by the aforementioned discussion we proceed to model the airfoil's pitching moment by

$$\tau^b_M = \sin\gamma C_{m_\alpha}(\alpha)\delta. \tag{4}$$

Following a similar approach as in Oishi and Tomlin (2000), we introduce the additional states $(\gamma, \dot{\gamma})$, which model the dynamics of the tilting mechanism as follows

$$\ddot{\gamma} = u_\gamma \tag{5}$$

where we have introduce a control variable $u_\gamma$.

With the aforementioned discussion, the system modelling presented in Flores and Lozano (2013b), can be completed as follows

$$\begin{aligned}
\ddot{x} &= T\sin(\theta+\gamma) - D\cos(\theta-\alpha) \\
\ddot{z} &= T\cos(\theta+\gamma) + L\cos(\theta-\alpha) - mg \\
\ddot{\theta} &= \frac{l_1 T_d\cos\gamma + \sin\gamma C_{m_\alpha}(\alpha)\delta}{J} \\
\ddot{\gamma} &= u_\gamma
\end{aligned} \tag{6}$$

where $T = T_{3,4} + T_{1,2}$ is the total thrust and the difference of these thrusts is $T_d = T_{1,2} - T_{3,4}$.

### *2.3 Problem Statement*

The main goal is to perform a transition maneuver from *HF* to *FFF* mode, by tilting the rotors from 0 to $\frac{\pi}{2}$, such that the translational speed $\dot{x}(t)$ varies from 0 to some value $\dot{x}_s(t)$, while the altitude $z(t)$ remains equal to a desired value $z_d(t)$. Therefore, the problem under study can be formulated as follows:

**Problem formulation** *Consider the Quad-Tilt rotor model with longitudinal equations of motion given by (6) and a time $t_f > 0$ which is the time where the transition maneuver is completed. Derive feedback control laws for thrusts $T(t) : [0, t_f] \to \Re$, $T_d(t) : [0, t_f] \to \Re$, elevator deflection $\delta(t) : [0, t_f] \to \Re$ and torque $u_\gamma(t) : [0, t_f] \to \Re$ so that the state boundary conditions*

$$\begin{aligned}
&z(0) = z_d, \quad x(0) = \dot{x}(0) = \dot{z}(0) = 0 \\
&\theta(0) = \dot{\theta}(0) = \gamma(0) = \dot{\gamma}(0) = 0 \\
&z(t_f) = z_d, \quad \dot{z}(t_f) = 0, \quad \dot{x}(t_f) = \dot{x}_s \\
&\theta(t_f) = \dot{\theta}(t_f) = \dot{\gamma}(t_f) = 0, \quad \gamma(t_f) = \frac{\pi}{2}
\end{aligned} \tag{7}$$

*are fulfilled and the system states converge to their desired values as follows $z(t) \to z_d$, $\dot{z}(t) \to 0$, $\dot{x}(t) \to \dot{x}_s$, $\gamma(t) \to \pi/2$, $\theta(t) \to 0$.*

## 3. CONTROL STRATEGY

In this section we present a control strategy for the transition maneuver of the Quad-Tilt rotor aircraft considering the problem statement given in previous section. The controller will be developed in two parts. The first part will be responsible to stabilize the attitude dynamics of the aircraft in the entire transition regime, as well as the dynamics of the tilting rotor, by means of the control variables $T_d(t)$, $\delta(t)$ and $u_\gamma(t)$. The second part involves the altitude stabilization to $z_d$ as well as the velocity

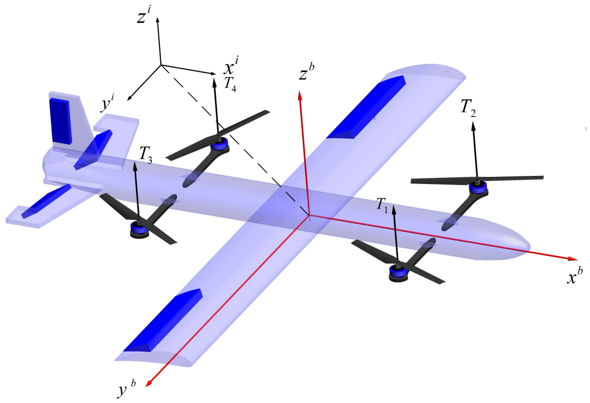

Fig. 1. The Quad-tilt rotor UAV.

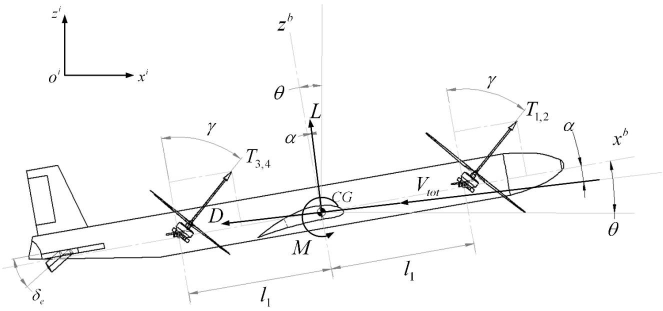

Fig. 2. Free-body scheme showing the forces acting on the Quad-Tilt rotor UAV.

convergence to a desired value $\dot{x}_s$. For this purpose, the angle $\gamma$ will be taken as a virtual control, while the control variable $T(t)$ will be derived to achieve $\dot{x}(t) \to \dot{x}_s$. Before developing the controllers, we take into account the following assumptions:

(1) **A1**. Small angles of attack will be considered, i.e. $|\alpha| \leq \overline{\alpha}$, where $\overline{\alpha} \approx 0.2$ rad which is a reasonably approximation in order to accomplish the expressions: $C_l \approx C_{l_\alpha}\alpha$ and $C_m \approx C_{m_\alpha}\alpha$, see Flores and Lozano (2013b) for details.
(2) **A2**. From $L = \frac{1}{2}C_l\rho V_t^2 S$ and $M = \frac{1}{2}C_m\rho V_t^2 S\bar{c}$ and assumption A1 we approximate the lift and drag forces as well as the pitching moment as: $L \approx l\dot{x}^2$, $D \approx d\dot{x}^2$ and $M \approx m_\theta \dot{x}^2$, respectively, where $l = 0.5C_l\rho S$, $d = 0.5C_d\rho S$ and $m_\theta = 0.5C_m\rho S\bar{c}$.

### *3.1 Control Strategy for the Attitude Dynamics and the Tilting Mechanism*

In this subsection, a control law is derived for the purpose of stabilizing the attitude dynamics, as well as the dynamics of the tilting mechanism. This algorithm exploits the backstepping procedure. For this purpose, let us rewrite the last two equations of system (6) in the following form

$$\begin{aligned}\dot{\theta}_1 &= \theta_2\\ \dot{\theta}_2 &= \frac{l_1 T_d\cos\gamma + \sin\gamma C_{m_\alpha}(\alpha)\delta}{J}\\ \dot{\gamma}_1 &= \gamma_2\\ \dot{\gamma}_2 &= u_\gamma\end{aligned} \tag{8}$$

where $\theta = \theta_1$, $\dot{\theta} = \theta_2$, $\gamma = \gamma_1$ and $\dot{\gamma} = \gamma_2$. Note that system (8) is not in a pure strict-feedback form and the controls $T_d(t)$ and $\delta(t)$ have different relative degree w.r.t. $\theta_1(t)$. Therefore, dynamically extending the state to include $T_d$, $\dot{T}_d$, $\delta$ and $\dot{\delta}$, the new inputs for the extended system result in

$$\begin{aligned}\tilde{T}_d &= \ddot{T}_d\\ \tilde{\delta} &= \ddot{\delta}.\end{aligned} \tag{9}$$

Now, we define a transformation which results in algebraically less complicated equations as follows

$$\begin{aligned}X &:= bT_d\cos\gamma + a\sin\gamma\delta\\ \dot{X} &:= a\delta\gamma_2\cos\gamma_1 + a\dot{\delta}\sin\gamma_1 - bT_2\gamma_2\sin\gamma_1 + b\dot{T}_d\cos\gamma_1\\ \ddot{X} &:= 2a\dot{\delta}\gamma_2\cos\gamma_1 + a\delta u_\gamma\cos\gamma_1 - a\delta\gamma_2^2\sin\gamma_1\\ &\quad + a\tilde{\delta}\sin\gamma_1 - 2b\dot{T}_2\gamma_2\sin\gamma_1 - bT_d u_\gamma\sin\gamma_1\\ &\quad - bT_2\gamma_2^2\cos\gamma_1 + b\tilde{T}_d\cos\gamma_1\end{aligned} \tag{10}$$

where $a = \frac{C_{m_\alpha}(\alpha)}{J}$ and $b = \frac{l_1}{J}$. Recalling (10), we reformat (8) to

$$\begin{aligned}\dot{\theta}_1 &= \theta_2\\ \dot{\theta}_2 &= X\\ \dot{X} &= Y\\ \dot{Y} &= Z\end{aligned} \tag{11}$$

where $\ddot{X} = \dot{Y}$. Now, the new controls $(\tilde{T}_d, \tilde{\delta})$ have a relative degree equal to four w.r.t the angle $\theta_1(t)$. Let $(\hat{\theta}_1, \hat{\gamma}_1) \in \Re^2$ be the desired trajectories such that the feedback controllers $(\tilde{T}_d, \tilde{\delta})$ track the error $e(t) := (\theta_1(t) - \hat{\theta}_1(t), \gamma_1(t) - \hat{\gamma}_1(t))$ to zero. It is important to note that the desired trajectory $\hat{\theta}_1$ must be zero in order to maintain a horizontal stabilization.

The system (11) is in the form of a chain of four integrators, and then we can apply a standard backstepping methodology to guarantee exponential convergence of the tracking error $e(t)$ to zero. The backstepping procedure results in the following control

$$\begin{aligned}Z &= -k_\theta(\theta_1 - \hat{\theta}_1) - k_{\dot{\theta}}(\theta_2 - \hat{\theta}_2) - k_X(X) - k_Y(Y)\\ u_\gamma &= -k_{\gamma_1}(\gamma_1 - \hat{\gamma}_1) - k_{\gamma_2}(\gamma_2 - \dot{\hat{\gamma}}_1)\end{aligned} \tag{12}$$

where $k_\theta$, $k_{\dot{\theta}}$, $k_X$, $k_Y$, $k_{\gamma_1}$ and $k_{\gamma_2}$ are positive real numbers. It is important to note that the inputs $(Z, u_\gamma)$ are functions of known variables. Now, it remains to express the control inputs $(\tilde{T}_d, \tilde{\delta})$ according to the known variables $(Z, u_\gamma)$. From the last equation of (10), it follows that

$$\begin{aligned}&a\delta u_\gamma\cos\gamma_1 + a\tilde{\delta}\sin\gamma_1 - bT_d u_\gamma\sin\gamma_1 + b\tilde{T}_d\cos\gamma_1 =\\ &\quad Z - 2a\dot{\delta}\gamma_2\cos\gamma_1 + a\delta\gamma_2^2\sin\gamma_1 + 2b\dot{T}_2\gamma_2\sin\gamma_1\\ &\quad + bT_2\gamma_2^2\cos\gamma_1 := \tilde{Z}\end{aligned} \tag{13}$$

where the term $\tilde{Z}$ is an auxiliary variable which is a function of known signals, and can be calculated from (12) and (13).

Note that by using (9) and (10) the original control inputs $(T_d, \delta)$ can be recovered.

### *3.2 Altitude and Forward Velocity Control Strategy*

In this subsection, the altitude and $x$-velocity control algorithm will be developed to make the transition maneuver from *HF* to *FFF* mode, in such a way that the aircraft altitude remains constant.

Motion in the $x$-$z$ plane is accomplished by orientating the vehicle's thrust vector towards direction of the desired displacement, i.e. by increasing the angle $\gamma$. As a consequence, $\gamma$ acts as a virtual controller for the altitude dynamics. In order to stabilize the altitude $z$, with a bounded control input, we will use the nested saturation control approach with the following control

$$u_z = -\epsilon\sigma_1(\dot{z} + \sigma_2(z + \dot{z} - z_d)) \quad (14)$$

where $\epsilon$ is the maximum amplitude of the control input $u_z$ and $\sigma_i(\cdot)$ is a saturation function such that $|\sigma_i(\cdot)| \leq M_i$ for $i = 1, 2$, where $M_i$ are positive real numbers. The stability analysis of (14) can be found in Flores and Lozano (2013b). The above control input is such that $z$ converges to $z_d$.

Knowing that there exists a fast response in the attitude dynamics compared to the position dynamics of the UAV (see Flores and Lozano (2013a), for example), we assume in the following that the pitch angle remains stabilized by means of the controller developed in subsection 3.1, i.e. $\theta \to 0$. Therefore, the convergence of $z$ to the desired value $z_d$ will occur at a slow rate, but the altitude dynamics will remain asymptotically stable. Thus, from second equation of (6), control (14) and assumption (A1) it follows that

$$\cos(\gamma) = \frac{mg - l\dot{x}^2 + u_z}{T}. \quad (15)$$

The angle $\gamma$ in (15) will be the desired angle for the controller (12), i.e. $\hat{\gamma}_1 = \arccos\left(\frac{mg - l\dot{x}^2 + u_z}{T}\right)$. The thrust should satisfy

$$T(0) \approx mg \quad (16)$$

due to the TF mode begins in HF mode. Let us now compute the control velocity corresponding to the dynamics of $x$ in (6). Introducing (15) into the first equation of (6) we get

$$\ddot{x} = (T^2 - (mg - l\dot{x}^2)^2)^{\frac{1}{2}} - d\dot{x}^2. \quad (17)$$

Notice that in view of (16), $T > (mg - l\dot{x}^2)$ holds. Notice also that the velocity subsystem above is stable. The steady state velocity is reached when the RHS is zero, i.e. when

$$T^2 - (mg - l\dot{x}^2)^2 = (d\dot{x}^2)^2$$

or

$$(d^2 + l^2)\dot{x}^4 - 2mgl\dot{x}^2 - (T^2 - m^2g^2) = 0$$

and the only solution is

$$\dot{x}_s^2 = \frac{2mgl + \sqrt{(2mgl)^2 + 4(d^2 + l^2)(T^2 - m^2g^2)}}{2(d^2 + l^2)}. \quad (18)$$

Notice that the RHS of (15) varies from a value close to 1 at the begining of the transition to a value close to 0 at the end of the transition, which means that the tilting angle $\gamma$, varies from 0 to $\frac{\pi}{2}$.

The stability of (17) can be studied using the candidate Lyapunov function $W = \frac{1}{2}(\dot{x} - \dot{x}_s)^2$, whose derivative with respect to (17) is given by $\dot{W} = (\dot{x} - \dot{x}_s)((T^2 - (mg - l\dot{x}^2)^2)^{\frac{1}{2}} - d\dot{x}^2)$. By choosing a control input $T$ as follows

$$T = \sqrt{(-k_x(\dot{x} - \dot{x}_s) + d\dot{x}^2)^2 + (mg - l\dot{x}^2)^2}$$

we conclude that in the neighborhood of $\dot{x} = \dot{x}_s$, $\dot{W} = (\dot{x} - \dot{x}_s)^2 < 0$ holds, which proves the (local) stability of the horizontal velocity subsystem (17).

## 4. NUMERICAL RESULTS

In this section some simulations showing the performance of the algorithm proposed in Section 3 are presented.

The following numerical results have been obtained by using the following parameters: $m = 1.1$kg; $g = 9.8m/s^2$; $l = 1$; $d = 0.1$; $z_d = 15$m; under the initial conditions: $x = \dot{x} = 0$; $z = 15$ and $\dot{z} = 0$. The lift and drag coefficients correspond to the typical values for a miniature aircraft.

The evolution of the dynamic states of the nonlinear system is shown in Fig. 3. The transition begins in $t = 0$ supposing a stabilized position carried out by the helicopter mode. The input control $u_z$ is the responsible for begin the transition maneuver. When the transition begins, there exists an increment in the velocity $\dot{x}$ due to the tilting of the rotors. Such increase continues growing until the vehicle achieves the desired velocity $\dot{x}_s = 10$; at this point the transition maneuver has finished. The angle $\gamma$ achieves the value of 90 degrees, as we can see at the bottom of Fig. 3, indicating the end of the transition maneuver.

The simulations results demonstrate the stability of the closed loop system with the transition maneuver where the autopilot changes the tilting angle $\gamma$ from 0 to 90 degrees.

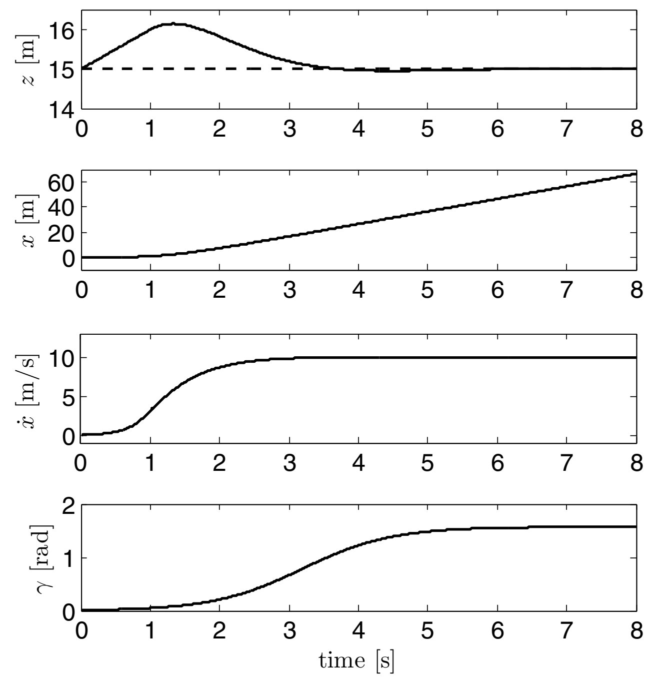


Fig. 3. The state $z$ converges to the desired value $z_d$ when the velocity $\dot{x}$ achieves the desired value $\dot{x}_s$. The angle $\gamma$ behaviour and convergence to $\pi/2$ is shown in the bottom.

## 5. CONCLUSION

The mathematical model initially presented in our previous work Flores and Lozano (2013b) has been completed by adding the pitch dynamics. A control strategy is proposed to perform a transition maneuver form hover to level flight, which feature two nonlinear control algorithms used to stabilize the systems dynamics during the transition maneuver. As future work we intend to propose a control maneuver from level to hover mode. Furthermore an experimental platform is under development in order to prove the proposed controllers.

## REFERENCES

Adrian, F., McGrew, J.S., Valenti, M., Levine, D., and How, J.P. (2007). Hover, transition, and level flight control design for a single-propeller indoor airplane. In *Proc. AIAA Guidance, Navigation Control Conference*, 1–43. Hilton Head, South Carolina.

Casau, P., Cabecinhas, D., and Silvestre, C. (2013). Hybrid control strategy for the autonomous transition flight of a fixed-wing aircraft. *IEEE Transactions on Control Systems Technology*, 21(6), 2194–2211.

Cetinsoy, E., Dikyar, S., Hancer, C., Oner, K., Sirimoglu, E., Unel, M., and Aksit, M. (2012). Design and construction of a novel quad tilt-wing uav. *Mechatronics*, 22(6), 723745.

Escareno, J., Salazar, S., and Lozano, R. (2006). Modeling and control of a convertible vtol aircraft. In *45th IEEE Conference on Decision and Control*.

Escareno, J., Stone, H., Sanchez, A., and Lozano, R. (2007). Modeling and control strategyfor the transition of a convertible uav. In *European Control Conference (ECC07)*.

Flores, G., Escareno, J., Lozano, R., and Salazar, S. (2012). Quad-tilting rotor convertible mav: Modeling and real-time hover flight control. *Journal of Intelligent and Robotic Systems*, 65(1-4), 457–471.

Flores, G. and Lozano, R. (2013a). Lyapunov-based controller using singular perturbation theory: An application on a mini-uav. In *Proc. IEEE American Control Conference (ACC'2013)*, 1599–1604. Washington, DC.

Flores, G. and Lozano, R. (2013b). Transition flight control of the quad-tilting rotor convertible mav. In *2013 International Conference on Unmanned Aircraft Systems (ICUAS)*, 789–794. Grand Hyatt Atlanta, Atlanta, GA.

Green, W. and Oh, P. (2006). Autonomous hovering of a fixed-wing micro air vehicle. In *International Conference on Robotics and Automation*.

Naldi, R., Gentili, L., Marconi, L., and Sala, A. (2010). Design and experimental validation of a nonlinear control law for a ducted-fan miniature aerial vehicle. *Control Engineering and Practice*, 18(7), 747–760.

Naldi, R. and Marconi, L. (2011). Optimal transition maneuvers for a class of v/stol aircraft. *Automatica*, 47(5), 870–879.

Oishi, M. and Tomlin, C. (2000). Switching in nonminimum phase systems: Application to vstol aircraft. In *Proc. IEEE American Control Conference (ACC'2000)*, 838–843. Chicago, IL.

Stengel, R. (2004). *Flight Dynamics*. Princeton University Press.

Stone, H. (2004). Control architecture for a tail-sitter unmanned air vehicle. In *5th Asian Control Conference*.

Stone, R.H. (2002). Aerodynamic modelling of a wing-in-slipstream tail-sitter uav. In *AIAA Biennial International Powered Lift Conference and Exhibit*. Williamsburg.