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On Satisfaction of the Persistent Excitation Condition for the Zone MPC: Numerical Approach

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Abstract: In recent years, advanced control techniques such as Model Predictive Control based on optimization and making use of a model providing the predictions of the future behavior of the controlled system have been massively developed. These model-based controllers rely heavily on the accuracy of the available model (predictor of the controlled system behavior) which is crucial for their proper functioning. However, as the current operating conditions can be shifted away from those under which the model has been identified, the model sometimes happens to lose its prediction properties and needs to be re-identified. Unlike the theoretical assumptions, the data from the real operation suffer from undesired phenomena accompanying the closed-loop data. In the current paper, we focus on developing an algorithm which would serve as an alternative to the (often costly or even unrealizable) open loop excitation experiment. The requirements such an algorithm should meet are: low computational complexity, low level of original MPC performance degradation and ability to provide sufficiently informative data when necessary. Unlike to the currently available approaches which solve this problem for the classical MPC formulation (tracking error penalization), in this paper we propose an algorithm which works well also for the zone MPC formulation (penalization of output zone violation), however, it is versatile enough and can be extended considering wider variety of the optimization formulations.

Keywords: Predictive control; closed loop identification; system identification.

1. INTRODUCTION

Modern control methods such as Model Predictive Controller (MPC) [1] have become very popular among the academic community during the last years and they are able to provide undisputable potential to be actively used in various branches of industry as well. Great development in the area of numerical optimizations has enabled these advanced methods to be applied also to highly complex systems with sampling periods in orders of milliseconds using low-performance processors with minimal computational power and price [2,3]. Even though, the idea of MPC is more popular rather among the academicians than among the process engineers. One of the reasons can be the fact that besides numerous benefits and vast potential, the MPC brings also several drawbacks. The most crucial of them is the fact that for its proper functioning, it needs a mathematical model of the controlled system. While model creation is mentioned only marginally in majority of the academical works dealing with the MPC (they usually assume that the model is either perfectly known or found in the literature), the task is more complicated and time-consuming in case of real application [4].

One of the very typical and common problems when considering an MPC controlling a real system is such situation when model the controller (MPC) uses becomes unusable. Such model can degrade the controller performance badly and it is usually necessary to re-identify the model.

It is often hardly possible to execute a statistically rigorous identification experiment either because of the operating or economical reasons and therefore, it is necessary to identify only from the data which are available—closed-loop data. These data use to suffer from several undesired phenomena such as insufficient excitation, correlation between certain inputs or input-disturbance correlation [5] causing that even the well-designed identification methods can fail. Thus, it is of key importance to pay attention also to this fact and take it *directly* into account when designing the identification procedure [5–8]. Even though there is a wide spectrum of the methods dealing with the so-called closed loop identification, their performance is guaranteed only if the controller introducing the feed-back into the system is sufficiently simple and linear. As the MPC brings a piecewise affine feedback into the system, even the use of the mentioned methods of the closed-loop identification might not bring the desired results [9].

All the above mentioned examples show the importance of taking the fact that the derived model might need to be re-identified into account directly when designing any advanced controller including the MPC. Thus, such methods should be searched which enable to both satisfy the pre-defined controller performance and provide sufficiently "rich" data containing enough information for the potential re-identification procedure. The significance of effort to solve this problem is demonstrated by the intensity of the discussion in the available literature.

Here, let us mention that the majority of the presented approaches considers the "classical" formulation of the MPC in which the deviation from the required reference trajectory is penalized [1] as well as the norm of the control effort. However, in many industrial applications, the reference tracking (in sense of set-point) is not particularly reasonable. A typical example is the control either of various chemical processes (temperature control in the depropanizer column [10]) or in the above-mentioned building climate control area (e.g. [11–13])—when controlling the zone temperature, it is not necessary to track certain exact temperature profile and keeping the temperature within a pre-defined range is sufficient.

For this special class of the predictive controllers, the currently available algorithms of simultaneous excitation and predictive control developed for the classical MPC penalizing the reference deviation can not be used. Up to date, there is only a tiny number of works trying to formulate and solve this task for the zone MPC [10].

The aim of this paper is to provide such algorithm which (apart from the fulfillment of the control requirements) would be able to ensure sufficient excitation of the system for the needs of the system re-identification. As the task of sufficient excitation is strongly practically motivated, the algorithm developed to solve it should be versatile and simply extendable for various classes of predictive controllers. The time and computational resources consumption should be kept at minimum and the algorithm should be able to use the potential of the parallel calculations. Last but not least, it should be implementationally non-demanding so that it could be added to already operating control system easily without any massive re-implementation of the existing controller code.

This paper is structured as follows: Sec. 2 introduces the formulation of the problem. In Sec. 3, overview of the possible ways of solving this problem is presented and several approaches are discussed. The newly proposed two stage algorithm is described and explained in Sec. 4. The performance of the proposed algorithm is tested considering a simple case study presented in Sec. 5. Sec. 6 concludes the paper.

2. PROBLEM FORMULATION

In this Section, the necessary background is provided.

2.1 Model under investigation

In this paper, a simple linear time-invariant (LTI) model is considered. Such model can be described by the well-known classical ARX structure [14] as

$$y_k = Z_k^T \theta + \varepsilon_k, \tag{1}$$

where y_k and u_k are the system output and input sequences and ε_k is zero-mean white noise. The vector of parameters θ is considered in the following form:

$$\theta = \left[b_{n_d} \dots b_{n_b} - a_1 \dots - a_{n_a}\right]^T \tag{2}$$

while $Z_k = \left[u_{k-n_d} \cdots u_{k-n_b} y_{k-1} \cdots y_{k-n_a}\right]^T$ is the regressor. Parameters of structure n_a, n_b, n_k specifies numbers of lagged inputs and outputs, respectively a relative delay of the outputs to the inputs.

2.2 Model predictive control

Besides the energy supplied into the system, the most common MPC formulation penalizes also deviation from a pre-defined reference—the tracking error. As it has been already emphasized, such formulation might not be desirable in some cases. In many industrial branches, it is more convenient to penalize violation of a pre-defined range of values instead of direct tracking error penalization. As a typical example, predictive controller trying to both minimize the energy consumed for the heating/cooling of a building and keep the room temperature(s) within the thermal comfort zone can be chosen. In such situation, the strategy of tracking the reference temperature can be inconvenient due to possible aggressive control performance and it is more suitable to hold the zone temperature within the admissible range. The control requirements can be formulated into the following cost function:

$$\begin{aligned}
\min: \quad & J_{ZMPC,k} = \sum_{i=1}^{P} W_1 \|u_{k+i}\| p + \sum_{i=1}^{P} W_2 \|av_{k+i}\| p \\
\text{s.t.}: \quad & \text{linear dynamics (1)} \\
& u_{k+i}^{min} \leq u_{k+i} \leq u_{k+i}^{max}, \quad i = 1, \dots P \\
& \hat{y}_{k+i|k} \leq y_{k+i}^{min} - av_{k+i}.
\end{aligned} \tag{3}$$

Here, y^{min} is the minimal allowed value of output and u^{max} and u_{min} are input constraints. Weighting matrices are denoted as W_1, W_2 and P specifies the prediction horizon. Symbol av represents the auxiliary variables used in order to relax constrains on y_{min} and p denotes the norm of the weighting of the particular term in the cost function. Afterwards, (3) can be rewritten into the quadratic programming problem:

$$\min E^T H E + j^T E \tag{4}$$

s.t.

$$\text{linear dynamics (1), and}$$

$$\begin{bmatrix} -I_{2P} \\ I_{2P} \\ [\mathbb{B} \;\; I_P] \end{bmatrix} E \leq \begin{bmatrix} U^{min} \\ \mathbf{0} \\ U^{max} \\ AV^{max} \\ \mathbb{A}x - Y^{min} \end{bmatrix} \tag{5}$$

where $E = \left[\mathbf{U} \;\; AV\right]^T$ is the vector of the optimized variables. Although such controller possesses many favorable properties, its potential and utilization crucially depend on the availability of a high accuracy mathematical model with good prediction behavior. In the real-life operation, it oftentimes happens that a model that used to work properly and reliably looses its accuracy and ability to provide good predictions and then, it is inevitable to obtain a new one. However, the data which are at disposal come from the closed-loop operation. This illustrates the need for designing such controllers that are able to generate data which are sufficiently rich and contain enough information that can enable the occasional re-identification. Still, the overall control performance must not be significantly degraded.

The first straightforward question before formulating the problem itself is how the data "informativeness" should be evaluated. One way is to quantify the information content of the data set based on the so-called information matrix [15] and the persistent excitation condition.

2.3 Persistent excitation condition

Let us consider ARX model structure (1). Then, the matrix ΔI_k^{k+M} defined as

$$\Delta I_k^{k+M} = \sum_{t=k+1}^{k+M} Z_t Z_t^{\mathrm{T}}. \tag{6}$$

represents the increment of the information matrix from the time k to the time $k + M$. Knowing this matrix, the persistent excitation condition can be formulated as:

$$\Delta I_k^{k+M} \succeq \gamma I \succ \mathbf{0}, \tag{7}$$

where γ is a scalar specifying the level of the required excitation and I is a unit matrix of corresponding dimension.

3. MPC WITH GUARANTEED PERSISTENT EXCITATION CONDITION

As already mentioned, the goal of this paper is to develop such algorithm for the zone MPC which will be able to not only satisfy the control requirements formulated into the cost function but also to provide sufficiently excited data making the re-identification easier. As the proposed algorithm is partially based on the algorithm of the authors of this paper which has been shown to work properly with the classical formulation of the MPC, a brief overview of the approaches is provided.

The first and perhaps the most straightforward way of tackling the issue of simultaneous MPC and identification is to incorporate the persistent excitation condition (7) into the optimization task directly as an additional constraint. This approach, however, suffers from several drawbacks, e.g. (7) comprises the output predictions which are problematic to include into the control problem formulation. Replacing (7) by

$$\sum_{t=k+1}^{k+m} \psi_t \psi_t^T \succeq \gamma I \succ \mathbf{0} \tag{8}$$

with $\psi_t = \left[u_{t-1} \;\cdots\; u_{t-n_b}\right]^{\mathrm{T}}$ solves the problem. This approximation, unfortunately, does not ensure the PE in every direction and leads to a biased estimate of parameters $a_1, \ldots, a_{n_a}$ in θ (see [16]). Note that (8) introduces a quadratic matrix inequality which can be transformed into a linear matrix inequality [17] and then, a semi-definite programming task can be solved. The next alternative solution to this problem has been offered in [18] where the receding horizon principle was utilized which brought a significant simplification of the originally non-convex problem. The bottleneck common for all the mentioned approaches is the choice of the excitation level γ as it is not clear how to choose the value of γ so that the data are excited enough and the optimization problem remains feasible. The procedure of choosing γ is not intuitive whatsoever and the alternative option will be focused on.

This alternative is the approach which has been lately published in [16,19]. This works provided a solution based on the maximization of the information matrix. The objective was to not deteriorate the original control behavior defined by the MPC cost function by more than a chosen value (being the tuning parameter of the algorithm). Such algorithm works in two stages – in the first one, the original MPC task is solved and then, the maximization of the information matrix is performed in the second step.

$$\begin{aligned} U^* &= \arg\max_U \gamma \\ \text{s.t.:} & \sum_{t=k+1}^{k+M} Z_t Z_t^T \succeq \gamma I, \\ & J_{MPC,k}(U) \leq J^*_{MPC,k} + \Delta J, \\ & u^{min}_{k+i} \leq u_{k+i} \leq u^{max}_{k+i}, i = 1, \ldots, P \end{aligned} \tag{9}$$

Here, ΔJ specifies the maximum allowed increment of the original MPC cost function $J^*_{MPC,k}$. The first advantage of such a formulation is that the complete information matrix is used for the optimization instead of its approximation (8). Then, the usually used excitation level γ is replaced by the maximal allowed perturbation ΔJ which specifies the balance between the excitation and the control performance degradation in more intuitive way. Making use of W_1 and W_2, this increase can be simply transformed into the control cost increase and/or the reference deviation which is advantageous especially in practical applications.

One potential complication comes from the fact that the optimization task is non-convex and very difficult to solve in general. The authors of the currently developed approaches use the elliptical approximation which can decrease the computational complexity of the task, however, it works reliably only for simple low-order systems.

Significant complexity reduction was achieved and presented by the authors of this paper in [20] where similarly to the approaches presented in [18], the fact that the majority of the industrial MPCs works with receding horizon was exploited. In every time step k (in which the optimal control sequence for $P-$step ahead is computed) of the optimization task (3), only the first element u_k, of the computed input sequence U is used. Therefore, it is not necessary to re-calculate the whole U in the second step of algorithm (3), just the first input sample u_k is optimized which results in the reduction of an M-dimensional optimization task to a one-dimensional one.

This approach was further developed and the results were presented in [21] where the extension for the class of the zone MPC was introduced. While considering the classical MPC formulation and utilizing the receding horizon principle, the complex task was reduced into a search through the set of all the admissible inputs u_k bounded from above and below by values that could be calculated analytically thanks to the simplicity of the cost function of the classical MPC, the situation became more involved in the case of the zone MPC. The bounds for the admissible set of u_k couldn't be found analytically and certain approximations had to be applied which increased the computational complexity of the task. Moreover, further inaccuracies were introduced into the whole process.

In the following text, a fresh new algorithm for the zone MPC with guaranteed persistent excitation is provided. The two-stage procedure is solved in the original unsimplified form without any approximations which can aggravate the overall performance of the algorithm.

4. PROPOSED NUMERICAL ALGORITHM

In this section, the new algorithm for the zone MPC with guaranteed persistent excitation is proposed.

4.1 First step

In the first step of the algorithm, the optimization task formulated by (3) and supplied by the corresponding box-constraints on inputs and lower bound for the predicted output is solved. Performing this, the optimal input sequence $U^*_{ZMPC} = [u_{k+i}],\ i = 1, 2, \ldots, P$, and the corresponding cost function value $J_{ZMPC,k}(U^*_{ZMPC})$ are obtained. The sequence U^*_{ZMPC} is used for the initialization of the second stage of the algorithm while the cost function value $J_{ZMPC,k}(U^*_{ZMPC})$ is used as the constraint.

4.2 Second step

The performance criterion for this stage is defined as:

$$\mathcal{J}(U) = \max\left(\min \operatorname{eig}(\Delta I_k^{k+M})\right) \tag{10}$$

where ΔI_k^{k+M} corresponds to (6). The choice of the optimization criterion being the minimal eigenvalue of the information matrix comes from the fact that we are trying to excite also the least informative directions from which the least information arrives (this usually corresponds to the most difficultly identifiable model parameters).

Then, the constraints can be summarized as:

$$\begin{aligned} u^{min}_{k+i} &\leq u_{k+i} \leq u^{max}_{k+i}, \\ J_{ZMPC,k}(U) &\leq J^*_{ZMPC,k} + \Delta J,\ i = 1, 2, \ldots, P. \end{aligned} \tag{11}$$

The first M samples of U^*_{ZMPC} calculated in the previous step are used as the initial guess U^0 of the profile which is optimized iteratively following the direction of the increase of the cost function (10), $U^{l+1} = U^l + \beta G^l$, where G^l is the search direction for the l-th iteration of the gradient search and β is the length of the step.

Here, let us note that not the whole sequence from the previous step is optimized. The reason is very pragmatical—as the predictive controller (being the first part of this two-stage algorithm) is re-calculated at each sampling instant and "new" optimal profile is obtained depending on the current measurements/disturbance predictions, not too much should be cared about the data excitation for the times close to the end of the prediction horizon. On the other hand, more than just one input sample shall by optimized in the sense of data excitation as with particular input sample, only a single direction corresponding to particular estimated parameter can be excited. The more parameters are to be identified, the more input samples should be taken into account.

The numerical gradient of the criterion (10) is calculated using the following procedure: one by one, all samples of U^l are gradually perturbed with chosen Δu. Performing this, a set of M perturbed input vectors is obtained,

$$\mathcal{U} = \{U_i = [u_1, u_2, \ldots, u_i + \Delta u, u_{i+1}, \ldots, u_M,], i = 1, 2, \ldots, M\}.$$

Then, evaluating the cost criterion for the second stage defined by (10) for each of the perturbed input profiles and comparing the values with the current criterion value $\mathcal{J}^c$, the vector of numerical gradients G can be obtained,

$$G = \left[\frac{\Delta\mathcal{J}_1}{\Delta u}, \frac{\Delta\mathcal{J}_2}{\Delta u}, \ldots, \frac{\Delta\mathcal{J}_i}{\Delta u}, \ldots, \frac{\Delta\mathcal{J}_M}{\Delta u}\right].$$

Here, $\Delta\mathcal{J}_i = \mathcal{J}(U_i) - \mathcal{J}^c$.

The box-constraints for the values of the particular input samples are satisfied performing a simple projection on

the admissible input interval $\langle u^{min}, u^{max}\rangle$. The iterative search is terminated if the improvement of the criterion (10) is less than a chosen threshold or if the degradation of the original MPC performance is worse than the maximal allowed perturbation ΔJ. If the second situation occurs, the last input vector which does not exceed the allowed ΔJ is returned by the gradient algorithm. Finally, the first input sample is applied to the system and the whole two-stage procedure is repeated with the new measurements.
Here, let us mention that the calculations performed in the second stage are highly parallelizable and the evaluation of the cost function criterion and the MPC cost function violation can be done simultaneously for all input samples. As a result, the overall effectiveness of this approach increases as the latest programming techniques enabling the parallel calculations can be exploited.

5. CASE STUDY

In this case study, the proposed sufficient excitation algorithm for the zone MPC is tested. As the bench-mark, a simple system with the ARX structure is considered and particular settings of the algorithm are inspected. The simplicity of the bench-mark is intentional to clearly demonstrate good theoretical properties of this algorithm.

5.1 Description

Let us consider the following simple system with single input and single output with the ARX structure (1) with $\theta = \begin{bmatrix}0.002 & 0.001 & 0.002 & 0.966 & -0.5 & 0.49\end{bmatrix}^{\mathrm{T}}$ and with the noise variance $\sigma_\varepsilon^2 = 0.08$. The system is controlled by the zone MPC (4) with constraints (5) and $u^{max} = 20$, $u^{min} = 0$, $av^{max} = 2$, $P = 70$ while y^{min} is generated according to the following rules

$$y_k^{min} = \begin{cases} 13 & 10^3q+1 \le k < 10^3(q+1), q \text{ is even} \\ 10 & 10^3q+1 \le k < 10^3(q+1), q \text{ is odd.} \end{cases} \quad (12)$$

The weighting terms 6000/0.01 and 100/500000 have been used for penalization of the violation of the required reference y^{min}. The value before the slash represents a quadratic weighting while the value after it represents a linear weighting. Such settings of the MPC controller have been chosen to obtain satisfactory MPC performance. With this tuning, a simulation with the length of $N = 10000$ samples. Similarly, the simulations have been run also for our new algorithm with several tuning parameter settings: $M \in \{6, 7, \ldots, 10\}$ a $\Delta J \in \{60\times10^3, 80\times10^3, 100\times10^3\}$. Here, a question why such settings were used could arise. As the increment of the information matrix (whose least eigenvalue is optimized in the second step of our algorithm) is a sum of M matrices ZZ^{T}, its rank is M and therefore, it has M nonzero eigenvalues. If the optimization were performed for $M < n_a+n_b$ (which in our case is $3+3 = 6$), the least eigenvalue of the increment of the information matrix would be 0 and its maximization would lack any sense. In case that the trace or the determinant of the increase of the information matrix was optimized, then also $M < 6$ would be reasonable. Note that the same model (without any adjustments or re-identification) identified from the excited data has been used over the whole simulation.

5.2 Results

Two viewpoints are considered when evaluating the results from the above mentioned procedures, namely i) possibility of system re-identification (parameter adjustment), and ii) the quality requirements and restrictions imposed by the constraints of the MPC problem formulation.
As the goal is to re-identify the model parameters for the MPC from the closed-loop data, first comparison will be focused on the amount of information contained in the data quantified by the minimal eigenvalue of information matrix increase during the whole simulation period $\lambda_{min}(\Delta I_1^N)$. In order to demonstrate that the higher information content leads to the better identifiability of the parameters of the model (which is our primary goal), 100 models were identified (each out of 700 samples) for each setting of our algorithm and also for the original MPC. Then, the following statistics can be introduced:

$$q_E = (E(\hat{\Theta}) - \theta_0^{\mathrm{T}})S(E(\hat{\Theta}) - \theta_0^{\mathrm{T}})^{\mathrm{T}}, \quad (13)$$

with $S = \frac{1}{n-1}(\hat{\Theta} - E(\hat{\Theta}))^{\mathrm{T}}(\hat{\Theta} - E(\hat{\Theta}))$, being a sample covariance matrix. Here, $\hat{\Theta} = \begin{bmatrix}\hat{\theta}_1 & \ldots & \hat{\theta}_n\end{bmatrix}^{\mathrm{T}}$. θ_i specify the parameters identified from the i-th set of data and n is the number of the identified models. Freely spoken, the parameter q_e specifies the inaccuracy of the parameter estimates and the lower it is, the closer are the identified parameters to the real ones.
Besides the ability to re-identify the model, it is of high interest how well does the designed controller satisfy the original MPC requirements. To investigate the control performance, two factors are compared. First of all, the ability to satisfy the required output range was investigated. This was quantified by the average low reference violation $e_y^+ = \|\max((Y^{min} - Y), 0)\|$. Last but not least, it was necessary to evaluate the energy consumption of the compared algorithms. As the objective was to develop an algorithm able to not only satisfy the control requirements but also provide sufficiently informative data, the "price increase" related to these informative data needs to be known. This increase in the case of our algorithm with various settings compared to the original zone MPC is defined as $I_E = \sum_1^N u_M^2 / \sum_1^N u_{ZMPC}^2 (\%)$ where u_M specifies input generated by our algorithm for the specific M and u_{ZMPC} refers to input generated by the original MPC.
The summary of the results is provided in Tab 1-3. The first thing which is obvious from the provided tables is that for any setting of our algorithm, the generated data are much more informative which corresponds to $\lambda_{min}(\Delta I_1^N)$ being much higher than for the original zone MPC. Let us remind that the smallest eigenvalue of the increase of the information matrix is quite nonintuitive to determine which value is sufficiently large and which is not. Still, it provides a good relative comparison of the approaches.
The fact that the data generated by our algorithm bring much better possibility to obtain a high-quality model by the re-identification than those generated by the classical zone MPC is indisputable. The values of the parameter q_e are several hundred times lower for each setting of our algorithm which implies higher accuracy of the parameter estimates. The improvement in the ability to estimate model parameters is illustrated by Fig. 2 showing the step responses of the identified models Particular subplots containing the step responses for various M correspond to different values of ΔJ. It is obvious that the green responses (the responses of models identified from the data provided by the classical zone MPC) are far away from the real response (blue) which is not the case of our algorithm for which the step responses of the models very accurately reproduce the real one.
Regarding the output zone satisfaction, the results for particular settings of our algorithm are comparable to the classical zone MPC results and the deviations in the average output tracking error e_y^+ are negligible. This can be explained such that as the zone satisfaction is required instead of exact zone tracking, the fact that the output flutters a little bit might not mean that the required zone is violated.
A very important is the comparison of the consumed energy. It can be seen that in case of our algorithm, the increase of about 2.5 – 9% compared to the original zone MPC occurs depending on the setting of our algorithm. Here, it should be realized that the objective was to provide an alternative for the economically, operationally and time demanding open-loop identification experiment – it is not required (and not even desired) that this

algorithm operates non-stop. It shall be employed only in the situation when the current model used by MPC is not suitable any more due to its inaccuracy. Therefore, the energy consumption increase in the order of percents is only temporary and lasts only over the time necessary for the re-identification of the model. In order to illustrate the energy consumption increase, graphical comparison of the energy consumption is presented in Fig. 1. Data used for this comparison (10^4 samples) were split into 10 equal sectors and for each sector, the average energy consumption increase per one sample was evaluated for all settings of our algorithm and for the classical zone MPC.

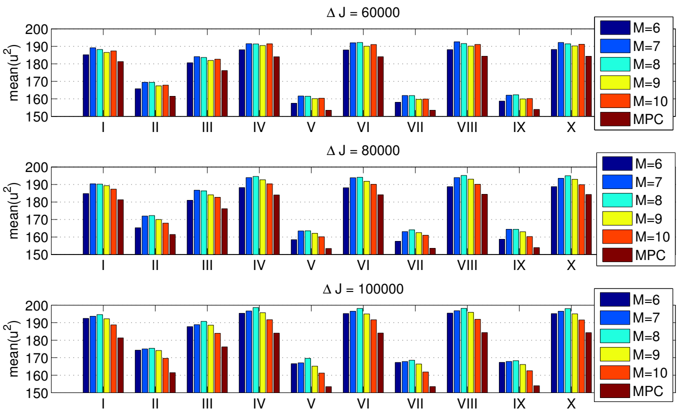

Fig. 1. Energy consumption comparison

Further insight can be obtained inspecting the dependence of the obtained results on the values of the tuning parameters of the algorithm. When inspecting the dependence on ΔJ, it is quite expectable that for higher ΔJ, the controller consumes more energy and on the other hand also brings more informative data which causes more accurate estimates. This is quite natural because allowing higher perturbation ΔJ of the original cost function, the performance evaluated by the MPC cost function will be degraded, however, more space for the data excitation will be achieved. Still, it is important to mention that even for the least chosen $\Delta J = 60000$, significant changes in the accuracy of the estimated parameters can be observed. This is very illustratively presented by the Fig. 2 where the changes of the estimate accuracy are quite low for different settings of the algorithm, however, they are huge compared to the estimate accuracy in case of the classical MPC. One could wonder what is the best and most proper choice of ΔJ. Here, the answer is that the choice of ΔJ is highly individual and it strongly depends on the application and also on how much one can afford to aggravate the performance of the original controller.

Being interested in the choice of the parameter M, there is no clear relation between the value of M and the performance of the algorithm. However, it appears that the best performance can be achieved for the M lying somewhere in the middle of the interval which was chosen in this paper. As already mentioned, it does not make sense to choose $M < n_a + n_b$ (due to the regularity of the problem). Also, in general it is not very advantageous to choose a too high value of M because the excitation is then optimized over longer horizon and for too long horizons, undesired uncertainties can be introduced resulting from the multi-step predictions of the model being less accurate. Moreover, as the industrial MPCs work with the receding horizon, it is not necessary to pick up such high values.

The ultimate goal of this work was to keep the complexity of the algorithm reasonably low. The average duration of one run of the complete algorithm (zone MPC calculation + excitation of the data) over all the considered settings was $1.1\,\mathrm{s}$ while in the case of the zone MPC only (without excitation) it was $0.3\,\mathrm{s}$.

As our algorithm was intended to be a more versatile alternative for the algorithm described in [21], let us show a brief comparison of the newly proposed algorithm and the one presented in [21]. To avoid too lengthy comparisons, the algorithm presented in [21] was tested for just $M = 7$ and under the conditions described in Sec. 5.1. For the algorithm from [21], $\Delta J = 30000, 40000, 50000$ were chosen. Let us note that in case of the algorithm presented in [21], the choice of ΔJ has different meaning—in the case of the current algorithm, ΔJ represents the perturbation caused by the M input samples while in the case of the other algorithm, all the perturbation is caused by just single input sample. In Tab 4, the values of the evaluative factors $I_E(\%), e_y^+, q_e, \lambda_{min}(\Delta I_1^N)$ for the "older" algorithm are presented. It can be seen that the performance of that algorithm is very similar to the performance of the new one considering the presented statistics. At the expense of the $5-9\%$ energy consumption increase, the algorithm presented in [21] provides much better estimates of the parameters of the model than the classical zone MPC. Inspecting Tab. 1-3, it could even appear that with certain settings, the algorithm presented in [21] is able to identify the parameters even more accurately than the new one. However, here it should be realized that the higher minimal eigenvalue of the information matrix increment does not necessarily mean that the resulting model is much better than the other. The smallest eigenvalue specifies the direction from which smallest amount of information has been obtained. However, it is not related to the significance of the corresponding parameter. It is clear that although improving the estimate of particular parameter of the model, the resulting prediction performance might change negligibly—this happens when the parameter whose estimate has been improved is not significant enough. Returning back to the comparison with the elder algorithm, taken relatively to the classical zone MPC, the differences in performance are almost negligible and thus it can be concluded that the current algorithm and the one presented in [21] are equivalent. However, the new algorithm has one superior property being the versatility and the fact that it can be very simply extended for much more general class of the optimal controllers.

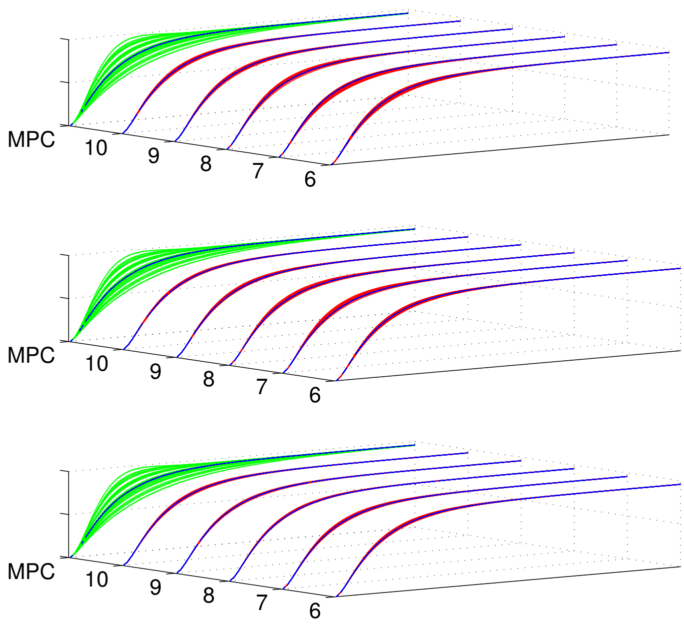

Fig. 2. Step responses ($\Delta J = 60 \times 10^3$ - top, $\Delta J = 80 \times 10^3$ - middle, $\Delta J = 100 \times 10^3$ - bottom).

Table 1. Results comparison $\Delta J = 60000$

	$I_E(\%)$	e_y^+	q_e	$\lambda_{min}(\Delta I_1^N)$
$M = 6$	2.40	0.01	8.5×10^{-8}	5.58
$M = 7$	4.67	0.01	4.69×10^{-8}	6.77
$M = 8$	4.47	0.01	4.26×10^{-8}	7.23
$M = 9$	3.47	0.01	5.85×10^{-8}	7.11
$M = 10$	4.53	0.01	4.33×10^{-8}	7.27
MPC	0.00	0.01	7.33×10^{-4}	2.75

Table 2. Results comparison $\Delta J = 80000$

	$I_E(\%)$	e_y^+	q_e	$\lambda_{min}(\Delta I_1^N)$
$M = 6$	2.50	0.01	7.65×10^{-8}	5.82
$M = 7$	4.91	0.01	2.31×10^{-8}	7.66
$M = 8$	5.99	0.01	1.39×10^{-8}	8.67
$M = 9$	4.94	0.01	1.40×10^{-8}	8.67
$M = 10$	3.68	0.02	3.38×10^{-8}	7.53
MPC	0.00	0.01	7.33×10^{-4}	2.75

Table 3. Results comparison $\Delta J = 100000$

	$I_E(\%)$	e_y^+	q_e	$\lambda_{min}(\Delta I_1^N)$
$M = 6$	5.50	0.01	0.91×10^{-8}	7.92
$M = 7$	7.58	0.01	0.18×10^{-8}	10.81
$M = 8$	8.37	0.01	0.49×10^{-8}	8.72
$M = 9$	6.87	0.02	0.84×10^{-8}	8.55
$M = 10$	4.53	0.02	0.33×10^{-8}	10.61
MPC	0.00	0.01	7.33×10^{-4}	2.75

Table 4. Results of another currently available algorithm, $M = 7$

	$I_E(\%)$	e_y^+	q_e	$\lambda_{min}(\Delta I_1^N)$
$\Delta J = 30000$	5.20	0.0	8.67×10^{-8}	4.51
$\Delta J = 40000$	5.27	0.0	5.52×10^{-8}	5.45
$\Delta J = 50000$	9.67	0.0	4.69×10^{-8}	8.56
MPC	0.00	0.01	7.33×10^{-4}	2.75

6. CONCLUSION

In this paper, a new algorithm ensuring sufficient excitation for the class of zone MPC was presented. The shown results clearly demonstrate that this newly proposed algorithm possesses not only good theoretical properties but it is also able to provide data with rich information content (which helps the potential re-identification of the model) at only a negligible increase of the energy consumption and hardly detectable aggravation of the control performance in the sense of zone satisfaction. The combination of its attractive performance (in the sense of ability of providing sufficiently excited data), low computational complexity and high versatility makes it a good candidate for the real-life application. The potential of the algorithm can be used with advantage in such processes where the open-loop excitation experiment is inadmissible either from the operational or the economical reasons.

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