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A fast decomposition approach for traffic control

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Abstract: Real-time road traffic control has been the subject of active research efforts for more than fifty years. In recent years, however, the convergence of ubiquitous sensing with seamless communication technologies has motivated the development of more computationally efficient control methods, able to operate in real-time in a live environment. In this work, we present a fast decomposition method for network optimization problems, with application to real-time traffic control. Our approach is based on a nonlinear programming formulation of the network control problem and consists of an alternating directions method using forward numerical simulation in place of one of the optimization subproblems. The method is scalable to realistic city-size road networks for real-time applications, and is shown to perform well on synthetic and real traffic networks.

1. INTRODUCTION

The theory of macroscopic traffic flow modeling is based on the *continuum approximation*, under which a large scale road network with millions of individual agents can be represented by a network of scalar hyperbolic conservation law. Foundational results on conservation law models of traffic flow date back to Lighthill and Whitham [1955] and Richards [1956], and more recently Lebacque [1996] for consistency results between mathematical and physical considerations, and Garavello and Piccoli [2006] for well-posed network models of such systems.

A discretized version of continuous first order traffic flow models, the *cell-transmission model* (CTM), was proposed by Daganzo [1994, 1995], and shown (see Lebacque [1996]) to correspond to a numerical discretization of the associated partial differential equation (PDE) using a Godunov scheme (see LeVeque [2002]), assuming piecewise affine dynamics.

For control applications, a number of results have focused on embedding a discretized first order macroscopic model in a *model predictive control* (MPC) framework (see Garcia et al. [1989]), with particular emphasis on the network formulation of such models, see for instance Hegyi et al. [2002, 2005], Lin et al. [2012]. Nonlinear programming formulations were then proposed to solve the resulting optimization problem (see also Gomes and Horowitz [2006], Jacquet et al. [2006], Papamichail et al. [2010], Lu et al. [2010]).

Recently, approaches focused on optimizing for the network model within a particular set of equilibria, such as uniform density equilibria in Pisarski and Canudas-de-Wit [2012], were proposed. Additionally, a variety of tractable results exists for the stabilization problem, both locally (see Blandin et al. [2010]) and globally in a distributed framework (see Wongpiromsarn et al. [2012]).

However, real-time resolution of the open-loop nonlinear traffic control problem remains largely open, notably due to the size of the state space considered when solving the continuum model using an explicit discretization method. In that framework, the *Courant-Friedrichs-Lewy* (CFL) condition (see LeVeque [2002]), required for stability of explicit discretization methods, requires very small time steps (order 1 second) on typical urban road networks, leading to a large state-space even for relatively small time horizons.

Alternative approaches have been investigated notably in Portilla et al. [2013], in which the authors propose a decentralized approach for the large-scale nonlinear programming problem. While their method offers a 50% computation time reduction with respect to the fully centralized problem, computation times illustrated remain long for real-time applications.

The main contribution of this work is a fast and highly scalable algorithm for the traffic network optimization problem. The algorithm relies on an effective decomposition along the lines of an alternating directions method, with two associated subproblems. The first subproblem is concerned with optimizing the control variables given fixed traffic states, whereas the second subproblem consists of optimizing the traffic states given fixed control variables. We treat the second problem using a forward simulation procedure, which leads to far more efficient computational results than methods based on decomposing the full nonlinear programming problem on a partition of the set of controllers.

The paper is organized as follows. Section 2 presents the traffic model. Section 3 introduces the full nonlinear control model, and section 4 details the proposed decomposition method. In section 5, we present numerical results on both a small synthetic network and on a real urban network. Section 6 concludes the paper with relevant directions for future research.

2. TRAFFIC MODEL

2.1 Link model

We define a graph as $G := (V, E)$, where V denotes a set of nodes, and E a set of directed links connecting the nodes of V. The graph G is assumed to be strongly connected. Each link $e \in E$ is directed from a tail node, $tail(e) \in V$, to a head node $head(e) \in V$. For each node $v \in V$, we define the sets of outgoing and incoming links from and to that node as: $O(v) := \{e \in E | tail(e) = v\}$ and $I(v) := \{e \in E | head(e) = v\}$. We further assume that a subset $\tilde{V} \subset V$ of the junctions are controllable. A subset of the links on the boundary of the network are designated as sources which serve to inject demand into the network and a subset are designated as sinks with unlimited capacity.

The traffic state at time t at location x is characterized by $\rho(t, x)$, where ρ denotes the density of vehicles. The traffic flow q is given by an empirical concave function of density, the *fundamental diagram*: $q := Q(\rho)$, defined over the interval $[0, \rho_{\max}]$. The critical density ρ_c is the density at which the flux functions reaches its maximum $q_{\max} = Q(\rho_c)$. In the following, the quantities $\rho_{\max}, \rho_c, q_{\max}$, may be link dependent and indexed accordingly $\rho_{\max,e}, \rho_{c,e}, q_{\max,e}$.

The link model is given by a first order scalar conservation law for the vehicles, with the flux function $Q(\cdot)$:

$$\frac{\partial \rho}{\partial t}(t, x) + \frac{\partial Q(\rho)}{\partial x}(t, x) = 0.$$

Given a discretization grid defined by a time step Δt and a space step Δx, let ρ_i^n denote the value of the numerical solution to the LWR PDE given by the Godunov scheme (see LeVeque [2002]), a time step n, space cell i:

$$\rho_i^{n+1} = \rho_i^n + \frac{\Delta t}{\Delta x}\big(q_G(\rho_{i-1}^n, \rho_i^n) - q_G(\rho_i^n, \rho_{i+1}^n)\big), \qquad (1)$$

where the numerical Godunov flux $q_G(\rho_i, \rho_{i+1})$ reads:

$$q_G(\rho_i, \rho_{i+1}) = \begin{cases} Q(\rho_i) & \text{if } \rho_{i+1} \le \rho_i \le \rho_c \\ Q(\rho_{i+1}) & \text{if } \rho_c \le \rho_{i+1} \le \rho_i \\ q_{\max} & \text{if } \rho_{i+1} \le \rho_c \le \rho_i \\ \min\Big(Q(\rho_i), Q(\rho_{i+1})\Big) & \text{if } \rho_i \le \rho_{i+1}. \end{cases} \qquad (2)$$

In the following, we note C_e the maximal space step index on link e.

2.2 Junction model

At each node, the so-called junction model specifying the dynamics across the junction is defined as in Garavello and Piccoli [2006]. Let $B \in \mathbb{R}_+^{|O(v)| \times |I(v)|}$ denote the (node and time dependent in general) matrix of splitting rates. The possibly non-unique junction flow $q_{\text{in}}^n \in \mathbb{R}_+^{|I(v)|}$ into node v is given by the solution to:

$$\begin{aligned} \max_{q \in \mathbb{R}_+^{|I(v)|}} \quad & \mathbf{1}^T q \\ \text{s.t.} \quad & 0 \le q \le S(\rho_{C_e,e}^n), \; e \in I(v), \\ & Bq \le R(\rho_{0,f}^n), \quad\; f \in O(v), \end{aligned} \qquad (3)$$

where the sending function $S(\cdot)$, defined on $[0, \rho_{\max}]$ by

$$S(\rho) = \begin{cases} Q(\rho), & \text{if } \rho \in [0, \rho_c] \\ q_{\max}, & \text{if } \rho \in [\rho_c, \rho_{\max}] \end{cases} \qquad (4)$$

expresses the value of the maximal amount of flow that can be sent under a given density value, and the receiving function $R(\cdot)$, defined on $[0, \rho_{\max}]$ by:

$$R(\rho) = \begin{cases} q_{\max}, & \text{if } \rho \in [0, \rho_c] \\ Q(\rho), & \text{if } \rho \in [\rho_c, \rho_{\max}], \end{cases} \qquad (5)$$

defines the value of the maximal amount of flow that can be received under a given density value. In the following section we present the control formulation.

3. CONTROL MODEL

We consider a MPC formulation for the discretized dynamics (1)-(2)-(3)-(4)-(5), with traffic lights modeled by the maximal flow $q_{\max}$ on links incoming to the controlled junction, and splitting rates B at the controlled junction.

3.1 Actuators model

For each junction, we consider that traffic lights timings can be controlled, and specifically that the control variables are the green times allocated to the signal phases. The set of phases as well as the cycle lengths are assumed to be fixed. Let us note H the set of phases for a given junction, and t_j the green time (intersection and time dependent in general) allocated to a phase $j \in H$. The phase green times t_j impact both the maximal sending flow on the incoming links, and the splitting rates.

For a given incoming link e, we note $g_e \in [0; 1]$ the reduction from the total design capacity, due to the presence of the traffic light and phase green times t_j. This proportion is given by the ratio of the maximal number of vehicles sent across the junction in the presence of the traffic light, compared to the number of vehicles sent across the junction at capacity without a traffic light, which reads:

$$g_e = \frac{1}{L} \sum_{j \in H} t_j \sum_{f \in j(1): e \in j(0)} \frac{\eta_{ef}}{q_{\max,e}}, \qquad (6)$$

where L is the cycle length, $j(0)$ denotes the origin link of each movement of the phase j in H and $j(1)$ a destination link of each movement, and η_{ef} the capacity of the movement ef.

Under the action of the traffic light, the capacity reduction g_e is modeled as a change in the maximal capacity of the link e. The corresponding flux function $Q_{C_e}^c(\cdot)$ for the most downstream cell C_e on link e, reads:

$$\begin{cases} Q_{C_e}^c(\rho) = Q_{C_e}(\rho) & \text{if} \quad Q_{C_e}(\rho) \le g_e\, q_{\max,C_e} \\ Q_{C_e}^c(\rho) = g_e\, q_{\max,C_e}(\rho) & \text{if} \quad Q_{C_e}(\rho) \ge g_e\, q_{\max,C_e} \end{cases} \qquad (7)$$

A second impact of the phrase green time t_j lies in the change in the matrix B of splitting rates at the controlled junction. The matrix of controlled splitting rates B^c, can be defined in terms of t_j for controllable junctions. Let e, f denote an incoming, respectively outgoing, link at a controlled junction, and p_{ef} denote the element of B^c corresponding to the pair (e, f), characterizing the proportion of the flow on link e aiming for link f. The splitting rate p^c_{ef} for an adjacent pair of links (e, f) reads:

$$p^c_{ef} = \frac{\sum_{j \in H:(e,f)\in j} t_j\, \eta_{ef}}{\sum_{j \in H: e \in j(0)} t_j\, \eta_{ef}}. \tag{8}$$

In the following section we outline how the actuators model impacts the traffic model introduced previously. For simplicity of notation, we omit the superscript c indicating that the quantities are controlled.

3.2 Traffic dynamics resulting from control model

In the rest of the article, we assume that the flux function $Q(\cdot)$ is a piecewise affine function with two components. Using the actuators model from equation (7), it follows that under the proposed control, the flux function (and consequently the sending and receiving functions), is either triangular or trapezoidal.

Allowing for a reduction of $q_{\max}$ yields a range of values for the critical density, $\rho_c \in [\rho_{c_1}, \rho_{c_2}]$. The sending function reads:

$$S(\rho) = \begin{cases} \nu\rho & \text{if} \quad \rho \in [0, \rho_{c_1}] \\ q_{\max}\, g & \text{if} \quad \rho \in [\rho_{c_1}, \rho_{\max}] \end{cases} \tag{9}$$

and the receiving function reads:

$$R(\rho) = \begin{cases} q_{\max} & \text{if} \quad \rho \in [0, \rho_{c_2}] \\ q_{\max} + \omega(\rho - \rho_{c_2}) & \text{if} \quad \rho \in [\rho_{c_2}, \rho_{\max}] \end{cases} \tag{10}$$

where

$$\rho_{c_1} = \frac{q_{\max}\, g}{\nu}, \qquad \rho_{c_2} = \rho_{\max} + \frac{q_{\max}}{\omega},$$

and

$$\omega = \frac{q_{\max}}{\rho_{c_2} - \rho_{\max}}.$$

In the following section we describe the proposed optimization formulation and decomposition approach for the nonlinear control model.

4. MATHEMATICAL PROGRAMMING FORMULATION

4.1 Nonlinear control model

The nonlinear control model is formulated as the optimization of an objective function defined over the network over a finite horizon $N\,\Delta t$. In the rest of the article we use a proxy for the total distance travelled as the objective function, which is expressed as a weighted sum of flows on the link. The weighting factor between the flow on the most downstream cell of the link and the others cells is given by $0 \leq \zeta_{ef} \ll 1$, which prioritizes the most downstream cell.

We remind the reader that $x_{C_e,e}$ generically denotes the value of the quantity x (or function $x(\cdot)$) in the most downstream cell of link e, that the value of the quantity x into the others cells of the link e is labelled $x_{i,e}$ for $i = 0 \dots C_e - 1$, and that x denotes the concatenation of the $x_{i,e}, i = 0 \dots C_e, e \in E$.

The equality constraint (11b) describes flow propagation between cells corresponding to the link model (1), with the Godunov flux defined according to equation (2), for a flux function mapping the density to the flow (11c). The constraints (11d)-(11e)-(11f) express a relaxation of a discretization of the junction model (3)-(4)-(5).

Constraint (11g) defines the effect of the control at a junction in terms of the upstream link and associated capacity reduction. Constraints (11h) states that the control decision variables t^n may be updated on the time scale $\Delta\tau$ and not in between.

Constraints (11i) and (11j) correspond to the fact that the green times for each phase should sum to the cycle length, which is constant, and should stay within certain bounds.

$$\max_{\rho,q,t} \sum_{n=0}^{N} \left(\sum_{e \in E} g^n_e q^n_{C_e,e} + \sum_{i=1}^{C_e - 1} \zeta_{e,i} q^n_{i,e} \right) \tag{11a}$$

$$\text{s.t.} \quad \rho^{n+1}_{i,e} = \rho^n_{i,e} + \frac{\Delta t}{\Delta x}\left(q_G(\rho^n_{i-1,e}, \rho^n_{i,e}) - q_G(\rho^n_{i,e}, \rho^n_{i+1,e})\right),$$
$$n = 0 \dots N, e \in E, i = 1 \dots C_e - 1 \tag{11b}$$

$$q^n_{i,e} = Q_{i,e}(\rho^n),$$
$$n = 0 \dots N, e \in E, i = 1 \dots C_e - 1, \tag{11c}$$

$$\sum_{e \in I(v)} B_v q^{n+1}_{C_e,e} \leq R^n_{0,f}, f \in O(v),$$
$$n = 0 \dots N, e \in E, v = head(e), \tag{11d}$$

$$0 \leq q^{n+1}_{C_e,e} \leq S^n_{C_e,e}, n = 0 \dots N, e \in E \tag{11e}$$

$$q^{n+1}_{0,e} = \sum_{i \in I(tail(e))} p_{i,e} q^{n+1}_{C_i,i},$$
$$n = 0 \dots N, e \in E \tag{11f}$$

$$g^n_e = \frac{1}{L_v} \sum_{j \in H_v} t^n_{v,j} \sum_{f \in j(1): e \in j(0)} \frac{\eta_{ef}}{q_{\max,C_e,e}},$$
$$n = 0 \dots N, e \in E \tag{11g}$$

$$g^{n+1}_e = g^n_e, \qquad (n+1)\tau \neq 0 \mod \Delta,$$
$$n = 0 \dots N, e \in E \tag{11h}$$

$$\sum_{i \in H_v} t^n_{v,i} = L_v, n = 0 \dots N, v \in \tilde{V}, \tag{11i}$$

$$l_{v,i} \leq t^n_{v,i} \leq u_{v,i}, n = 0 \dots N, v \in \tilde{V}, i \in H_v, \tag{11j}$$

$$q^n_{i,e} \geq 0, n = 0 \dots N, e \in E, i = 0, \dots C_e. \tag{11k}$$

Constraint (11k) characterizes the domain of definition of the traffic state variables q and $\rho^n_i \in [0, \rho_{\max,e}]$ for $i = 0, \dots C_e, e \in E$ and for all $n = 1, \dots N$.

One may note that the optimization problem is nonconvex. First the constraint corresponding to the numerical flux (2) is nonconvex due to the min function. Second, constraint (11c) is an equality constraint for a concave flux function. Finally, the objective function (11a) is bilinear.

The control variables t appear implicitly in several constraints. Upstream of a controlled junction, the flow value and the numerical flux are expressed as a function of the traffic control variable g_e, using equation (11c) with the expression (7), and resulting equation (2), respectively. Additionally, the sending and receiving functions across the junction, which appear in constraints (11d) and (11e) are impacted by the control actions through the modification of the flux function as detailed in equations (9) and (10).

The model given by (11a)-(11k) is therefore a continuous optimization problem with a nonconvex, nonlinear objective function and a nonconvex feasible region.

4.2 Fast decomposition approach

The very large size of the formulation (11a)-(11k) for networks of even moderate size means that many techniques typically used for nonlinear programming models are not effective in this case, and do not permit real-time optimization of the traffic control problem.

For that reason we develop a decomposition method inspired from an alternating directions approach. Here we consider two subproblems, the first problem being a feasibility problem consisting of finding traffic state variables compatible with a set of control variables. The second problem is an optimization problem for the control variables given fixed traffic state variables.

Solving the discretized network traffic model for a fixed set of control variables can be done very efficiently in a *forward simulation* framework. Conversely, given a set of traffic state variables, the control variables can be updated by solving a *linear program* in far fewer variables and constraints.

The forward simulation sub-problem consists of iteratively running the link model and the junction model defined in section 2.1 and 2.2. The link model is articulated around the numerical scheme from (1)-(2) with the controlled flux function from (7). The junction model is based on solving the linear program (3) with the controlled sending (9) and receiving (10) functions.

The linear program sub-problem consists of the optimization problem (11a)-(11k) for fixed values of the traffic state variables ρ and q.

5. NUMERICAL EXPERIMENTS

In this section we illustrate the performance of the decomposition approach introduced in this article in terms of runtime and optimality, as compared to the original nonlinear programming formulation. We consider a small synthetic network shown in figure 1 consisting of 5 links and 6 nodes, of which only 2 nodes are controlled by actuators (the other nodes are either source nodes or sink nodes), as well as a a real city-size network, shown in figure 2

Additional details about the numerical experiments can be found in table 1, including the scenario (i.e. which type of experiment), the method used (the proposed decomposition approach or the original nonlinear programming formulation), which network was used, the number of actuators, links, and nodes as well as the number of source nodes in parentheses, the time horizon of the forward simulation, and when in brackets the range of time horizons tested, the initial condition (demand) or range thereof, and similarly for the boundary condition, and the number of randomly-sampled experiments generated,

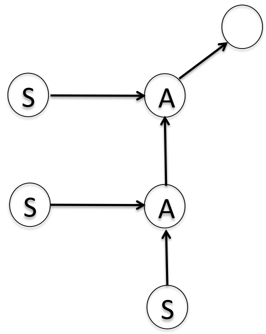

Fig. 1. **Synthetic network**: the marker A represents the actuators, the marker S denotes the source nodes, and the sink node is unmarked.

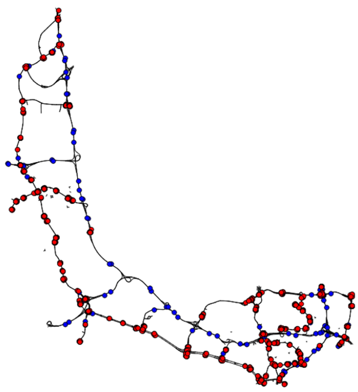

Fig. 2. **Real urban network**: the actuators are indicated by red and blue markers. Red markers correspond to arterial actuators whereas blue markers correspond to freeway actuators.

5.1 Runtime and algorithmic performance

In order to the compare runtime of the proposed decomposition approach with the original nonlinear programming formulation, we consider an experimental setup where the state space is of constant size for all instances of the problem. An instance of the problem is characterized by its prediction horizon N, spanning the range $\{360, \ldots, 7200\}$. We use IBM ILOG CPLEX Optimizer v.12.5 to solve the relaxed formulation of (11a)–(11k). We terminate the original nonlinear programming control problem when primal feasibility is achieved, and we stop the decomposition method whenever the relative improvement in objective is below a small threshold (10^{-6} in the experiments).

As illustrated in figure 3, the proposed decomposition method is able to handle problems of large size much more

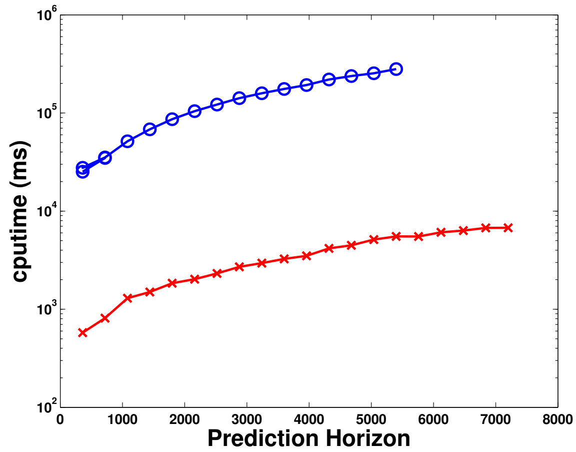

Fig. 3. **Runtime**: comparison of scalability between the nonlinear programming control formulation (blue circles) and our proposed decomposition method (red crosses).

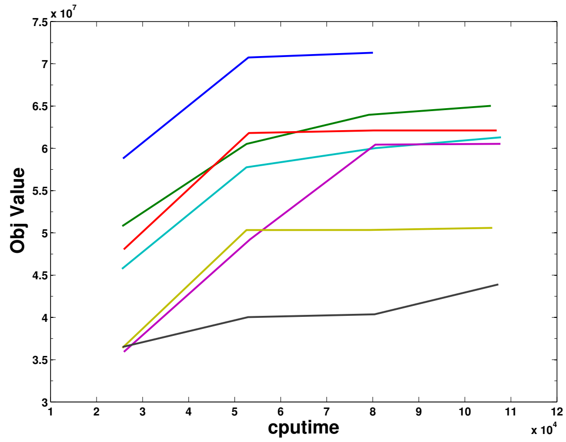

Fig. 4. **Improvement**: for a real traffic network with different starting points (differentiated by colors), and fixed initial/boundary conditions.

efficiently than the original nonlinear programming control formulation of (11a)-(11k), and leads to an improvement of about two orders of magnitude.

One may note that the complexity of the forward simulation which serves as the first subproblem in our decomposition approach is linear in both space dimension time dimension. The second subproblem used in our decomposition approach is a linear program and is furthermore of much smaller size than the original nonlinear formulation.

The algorithmic performance of the decomposition method introduced in this article can be further illustrated by considering the improvement in the objective function as a function of the number of iterations of the optimization method. In figure 4 we display the relation between the CPU time of the decomposition approach, and the objective value obtained, for an objective function which is maximized.

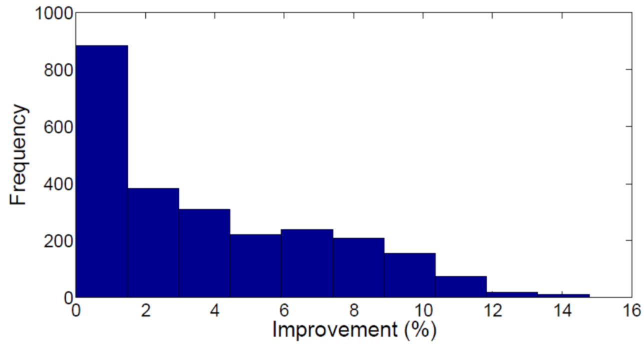

Fig. 5. **Improvement on synthetic network**: for the network-wide distance travelled improvement (relative improvement after control optimization in proportion of value before optimization).

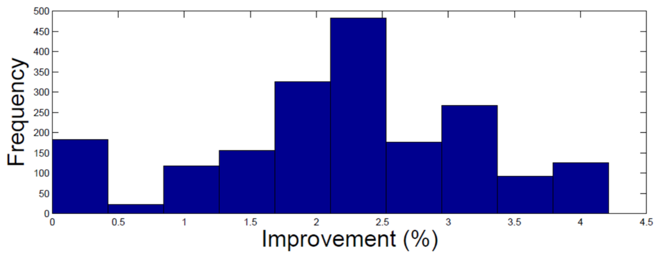

Fig. 6. **Improvement on real network**: for the network-wide distance travelled improvement on real 3000-link traffic network (relative improvement after control optimization in proportion of value before optimization).

5.2 Control performance in non incident situation

In this section we quantify numerically the performance of the decomposition approach proposed in terms of objective function improvement. We consider an instance of the control problem with the total distance travelled as an objective function, computed as the sum of flow overall discretized time and space steps. We measure the improvement by comparing the throughput after control optimization with respect to the initial value, estimated using a single forward simulation of the discretized network traffic model.

In order to assess how the decomposition approach performs under a variety of different real-world settings, we randomly sample a large number of initial and boundary conditions and measure the relative improvement for each sample. The results are illustrated in figure 5 for the synthetic network and in figure 6 for the real network.

It can be seen from the figures that although the improvement level of the control objective varies, due to the nonconvex nature of the problem, the decomposition method is always able to improve the objective, regardless of where the optimization process was initialized.

5.3 Control performance in incident situations

In this section we consider the case of an incident happening on the road network, and analyze how the decomposition approach can improve the traffic conditions once the incident occurs compared to the optimal situation before the incident. Incidents are modeled as reductions of the

Scenario	Method	Network	Actuators	Links	Nodes	Horizon	Initial	Boundary	Samples
Runtime	Decomp	Synth.	2	5	6 (3)	[360,7200]	4000	0	-
Runtime	Global	Synth.	2	5	6 (3)	[360,7200]	4000	0	-
Improvement	Decomp	Real	85	2925	2441 (44)	4320	1000	65	7
Histogram	Decomp	Real	85	2925	2441 (44)	4320	[0,2000]	[0, 130]	1048
Histogram	Decomp	Synth.	2	5	6 (3)	2160	[0,1100]	[0, 130]	33222
Incidents	Decomp	Real	85	2925	2441 (44)	4320	1000	65	1

Table 1. **Experimental setup**: traffic network specifications and experiment settings. In the Nodes column, numbers inside the () denote the number of source nodes.

link capacity $q_{\max}$, for 4 different road links on the freeway network. The initial and boundary conditions and network specifications are presented in Table 1.

We first apply the decomposition method to the network with no incidents. A 2.58% improvement on average on the total distance travelled is observed compared to the total distance travelled using a randomly generated control plan. After the incident occurrence, the network suffers from a 2.70% throughput reduction on average. Re-optimization using the decomposition method enables throughput increase equivalent to 98% of its previous optimal value before the incident.

6. CONCLUSION

We provide a fast decomposition approach for the optimal traffic control problem. The approach is based on the nonlinear programming formulation of the discretized control problem and is inspired by an alternating directions method using numerical simulation in place of an optimization formulation of the flow as a function of fixed controls. Numerical results demonstrate both the scalability of the proposed approach as well as the solution quality in terms of network throughput. Interesting avenues for further work involve assessing the error bounds of the proposed approach and explicitly taking into account uncertainty in the model parameters.

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