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Controllers design for two interconnected systems via unbiased observers

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Abstract: This paper proposes functional observer based controller for each subsystem of two interconnected system. The new approach that we use to express the control error permits to use some results on large scale delay systems to solve the problem. The proposed controllers are free of the interconnection terms, and the observation errors are unbiased. The observers based controllers are designed via some matrices gains which are obtained via LMIs (i.e. via a tractable approach) from the Lyapunov stability results. An algorithm is given for the computation of the solutions. The result can be extended for the H_∞ control of linear and nonlinear large scale systems.

Keywords: Interconnected systems, Filtering techniques, LMI techniques, Observer based control

1. INTRODUCTION

The study of large-scale dynamical systems is of great importance during the last years. This is due to the fact that physical systems are generally interconnected in practice giving a large-scale systems. In fact many practical control applications can often include electrical power systems, nuclears reactors, chemical process control systems, transportation systems, computer communication, economic systems and so on (Mahmoud [1997], Nguyen and Bagajewicz [2008]). Several dynamics subsystems can be distinguished (Chen and Lee [2009]) and delays generally arise in the processing of information transmission. This can cause instabilities and oscillations in these systems. For these reasons, many studies have been devoted to the analysis of stability of these systems (see (Siljak [1978], Lyou and al. [1984], Hmamed. [1986], Huang and al. [1995]) and the references therein).

The stability of such systems is generally based on checking the sign of the eigenvalues of the stability matrix (Chen and Lee [2009]). This is not a very tractable method, especially for the synthesis step. In (Chen. [2006]), a more tractable LMI approach has been proposed for the stability analysis.

Notice that in most practical situations, the complete state measurements are not available for each subsystem. But we know that for the control purpose, the availability of the states of a functional of the states of each subsystem is required (Dhbaibi and al. [2008]). Many works treat this problem by two ways generally. The first one is by designing an observer containing the interconnection term (see Sundareshan and Huang. [1984], Siljak [1991] and the references therein). the second method proposes an observer without interconnection term (see Pagilla and Zhu [2005], Dhbaibi and al. [2006]and the references therein). In (Trinh and Aldeen [1997]) a reduced order observer is proposed which dynamics are given from the set of unstable and/or poorly damped eigenvalues of the considered Linear Time Invariant system. In this paper, we propose a functional observer based controller to estimate directly a stabilizing control for a wide class of large scale systems, constituted by subsystems with interconnection terms between each other. Our proposed functional filter is without interconnection terms which makes its implementation more easy.

2. PROBLEM FORMULATION

Let us consider a large-scale time-delay system S which is described as an interconnection of n subsystems S_1, S_2... S_N, that are represented by (see Chen and Lee [2009])

$$S_i : \dot{x}_i(t) = A_i x_i(t) + A_{ii} x_i(t - d_{ii}) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij} x_j(t - d_{ij}) + B_i u_i(t) \tag{1a}$$

$$y_i(t) = C_i x_i(t) \tag{1b}$$

for $i, j = 1...N; i \neq j$ and where $x_i(t) \in \mathbb{R}^{n_i}$ is the state vector for the i^{th} system, $y_i(t) \in \mathbb{R}^{p_i}$ is its measured output and $u_i(t) \in \mathbb{R}^{n_i}$ is the control input. A_i, A_{ij}, B_i and C_i are constant matrices with appropriate dimensions and d_{ij} denotes the physical communication delays in the interconnections of the large-scale system S.
In our Purpose, we suppose that all the B_i matrices are equal to the identity matrix: $B_i = I_n$.

Our objective is to design an observer-based controller with the following structure

$$\dot{\eta}_i(t) = N_i \eta_i(t) + N_{ri} \eta_i(t - d_{ii}) + J_i y_i(t) + J_{ri} y_i(t - d_{ii}) \tag{2a}$$

$$\hat{u}_i(t) = \eta_i(t) + E_i y_i(t) \tag{2b}$$

where $\eta_i(t) \in \mathbb{R}^{n_i}$ is the state vector for the i^{th} system.
It must be noticed that both the filter and the controller

do not contain an interconnection term, which makes it more easy to be designed for each subsystem.

Problem 1. The objective is to establish a functional observer (6a) and a control law (6b)such that:
(i) $\lim_{t\to\infty} u_i(t) - K_i x_i(t) = 0$
(ii) the resulting closed-loop system (1)-(6) is asymptotically stable where the observer (6) is unbiased i.e. its dynamics does not depend explicitly on the subsystem state $x_i(t)$.

The approach used in this paper is to design the controller into two steps. The first one consists to use the item (i) of problem 1 i.e. to search for a state feedback gain K_i verifying for subsystem (1). Once this gain is found we then search in a second step, an unbiased controller via observer (6) permitting to construct this gain K_i.

3. DESIGN OF THE STATE FEEDBACK GAIN

Here,we suppose that we have access to all the state vector, $y_i(t) = x_i(t)$, such that $u_i(t) = K_i x_i(t)$.

By replacing the control input $u_i(t)$ by $K_i x_i(t)$ in each subsystem (1), then we obtain the following closed loop subsystems:

$$\begin{aligned} S_i : \dot{x}_i(t) &= A_i x_i(t) + A_{ii} x_i(t-d_{ii}) \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij} x_j(t-d_{ij}) + K_i x_i(t) \\ &= (A_i + K_i) x_i(t) + A_{ii} x_i(t-d_{ii}) \\ &+ \sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij} x_j(t-d_{ij}) \end{aligned} \tag{3}$$

where $K_i(t) \in \mathbb{R}^{n_i \times n_i}$ Using the results of Chen. [2006], the first condition to stabilize $x(t)$ is to get the matrix $(A_i + K_i)$ Hurwitz. The gain matrix K_i can be obtained by resolving the following simple LMI:

$$(A_i + K_i)^T P_i + P_i(A_i + K_i) < 0, P_i > 0 \tag{4}$$

By choosing: $Y_i = P_i K_i$, we solve:

$$A_i^T P_i + Y_i + P_i A_i + Y_i^T < 0, P_i > 0 \tag{5}$$

and we can obtain : $K_i = X_i{}^{-1} Y_i$

So, after this first step, we have K_i such that 3 must be stable with $A_i + K_i$ Hurwitz.

4. OBSERVER BASED CONTROLLER DESIGN

Now, we design an observer:

$$\begin{aligned} \dot{\eta}_i(t) &= N_i \eta_i(t) + N_{ri} \eta_i(t-d_{ii}) \\ &+ J_i y_i(t) + J_{ri} y_i(t-d_{ii}) \qquad (6a) \\ \hat{u}_i(t) &= \eta_i(t) + E_i y_i(t) \qquad (6b) \end{aligned}$$

which estimates the control input.

Remark 1. Notice that for simplicity purpose, we consider in this paper only two interconnected systems. Nevertheless, our results can be obviously extended to a large-scale system composed with N interconnected systems.

The estimation error is defined as:

$$\begin{aligned} e_i(t) &= u_i(t) - \hat{u}_i(t) \\ &= K_i x_i(t) - \eta_i(t) - E_i y_i(t) \\ &= \psi_i x_i(t) - \eta_i(t) \end{aligned} \tag{7}$$

where $\psi_i = K_i - E_i C_i$. then

$$\dot{e}_i(t) = \psi_i \dot{x}_i(t) - \dot{\eta}_i(t) \tag{8}$$

By replacing $\dot{x}_i(t)$ and $\dot{\eta}_i(t)$ from (3) and (6) respectively, we obtain:

$$\begin{aligned} \dot{e}_i(t) = \psi_i[(A_i + K_i)x_i(t) + A_{ii}x_i(t-d_{ii}) + A_{ij}x_j(t-d_{ij})] \\ - N_i\eta_i(t) - N_{ri}\eta_i(t-d_{ii}) - J_iC_ix_i(t) - J_{ri}C_ix_i(t-d_{ii}) \end{aligned}$$

Using the fact that : $\eta_i(t) = \psi_i x_i(t) - e_i(t)$, the error dynamic can be written as:

$$\begin{aligned} \dot{e}_i(t) &= \psi_i[(A_i + K_i)x_i(t) + A_{ii}x_i(t-d_{ij}) + A_{ij}x_j(t-d_{ij})] \\ &\quad -J_iC_ix_i(t) - J_{ri}C_ix_i(t-d_{ii}) - N_i[\psi_ix_i(t) - e_i(t)] \\ &\quad -N_{ri}[\psi_ix_i(t-d_{ii}) - e_i(t-d_{ii})] \\ &= [\psi_i(A_i + K_i) - N_i\psi_i - J_iC_i]x_i(t) + \psi_iA_{ij}x_j(t-d_{ij}) \\ &\quad +[\psi_iA_{ii} - J_{ri}C_i - N_{ri}\psi_i]x_i(t-d_{ii}) \\ &\quad +N_ie_i(t) + N_{ri}e_i(t-d_{ii}) \end{aligned} \tag{9}$$

If we want to obtain the estimation error dynamics independant from x_i, we consider the following unbiasedness conditions:

$$\psi_i(A_i + K_i) - N_i\psi_i - J_iC_i = 0 \tag{10a}$$
$$\psi_iA_{ii} - J_{ri}C_i - N_{ri}\psi_i = 0 \tag{10b}$$

then

$$\dot{e}_i(t) = N_ie_i(t) + N_{ri}e_i(t-d_{ii}) + \psi_iA_{ij}x_j(t-d_{ij}) \tag{11}$$

By taking $\mathbb{B}_i = A_i + K_i$, the unbiasedness condition for the free of delay part is given by:

$$\psi_i\mathbb{B}_i - N_i\psi_i - J_iC_i = 0 \tag{12}$$

which can be written as:

$$[N_i \;\; L_i \;\; E_i] \begin{bmatrix} K_i \\ C_i \\ C_i\mathbb{B}_i \end{bmatrix} = K_i\mathbb{B}_i \tag{13}$$

where $L_i = J_i - N_iE_i$.
In the same way, the unbiasedness condition for the delayed part can be written in the following compact form:

$$[N_{ri} \;\; L_{ri} \;\; E_i] \begin{bmatrix} K_i \\ C_i \\ C_iA_{ii} \end{bmatrix} = K_iA_{ii} \tag{14}$$

by taking $L_{ri} = J_{ri} - N_{ri}E_i$.

These two conditions can be rewritten in the following form:

$$[N_i \ N_{ri} \ L_i \ L_{ri} \ E_i] \begin{bmatrix} K_i & 0 \\ 0 & K_i \\ C_i & 0 \\ 0 & C_i \\ C_i\mathbb{B}_i & C_iA_{ii} \end{bmatrix} = [K_i\mathbb{B}_i \ K_iA_{ii}] \quad (15)$$

or in the form:

$$\chi_i\Gamma_i = \Phi_i \quad (16)$$

with:

$$\chi_i = [N_i \ N_{ri} \ L_i \ L_{ri} \ E_i] \quad (17)$$

$$\Gamma_i = \begin{bmatrix} K_i & 0 \\ 0 & K_i \\ C_i & 0 \\ 0 & C_i \\ C_i\mathbb{B}_i & C_iA_{ii} \end{bmatrix} \quad (18)$$

$$\Phi_i = [K_i\mathbb{B}_i \ K_iA_{ii}] \quad (19)$$

The matrix equation (16) has a solution χ_i if and only if:

$$rank \begin{bmatrix} \Gamma_i \\ \Phi_i \end{bmatrix} = rank\Gamma_i \quad (20)$$

then the solution is given by:

$$\chi_i = \Phi_i\Gamma_i^+ - Z_i(I - \Gamma_i\Gamma_i^+) \quad (21)$$

where Γ_i^+ is a generalized inverse and Z_i is an arbitrary matrix.
Now we can obtain the different above matrices such that:

$$N_i = \chi_i \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = N_i^1 - Z_iN_i^2 \quad (22)$$

where

$$N_i^1 = \Phi_i\Gamma_i^+ \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, and: N_i^2 = (I - \Gamma_i\Gamma_i^+) \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (23)$$

$$N_{ri} = \chi_i \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix} = N_{ri}^1 - Z_iN_{ri}^2 \quad (24)$$

where

$$N_{ri}^1 = \Phi_i\Gamma_i^+ \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix}, and: N_{ri}^2 = (I - \Gamma_i\Gamma_i^+) \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (25)$$

$$L_i = \chi_i \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix} = L_i^1 - Z_iL_i^2 \quad (26)$$

where

$$L_i^1 = \Phi_i\Gamma_i^+ \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix}, and: L_i^2 = (I - \Gamma_i\Gamma_i^+) \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix} \quad (27)$$

$$L_{ri} = \chi_i \begin{bmatrix} 0 \\ 0 \\ 0 \\ I \\ 0 \end{bmatrix} = L_{ri}^1 - Z_iL_{ri}^2 \quad (28)$$

where

$$L_{ri}^1 = \Phi_i\Gamma_i^+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ I \\ 0 \end{bmatrix}, and: L_{ri}^2 = (I - \Gamma_i\Gamma_i^+) \begin{bmatrix} 0 \\ 0 \\ 0 \\ I \\ 0 \end{bmatrix} \quad (29)$$

$$E_i = \chi_i \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I \end{bmatrix} = E_i^1 - Z_iE_i^2 \quad (30)$$

where

$$E_i^1 = \Phi_i\Gamma_i^+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I \end{bmatrix}, and: E_i^2 = (I - \Gamma_i\Gamma_i^+) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I \end{bmatrix} \quad (31)$$

Once L_i, N_i and E_i determinated, we compute J_i using:

$$J_i = L_i + N_iE_i \quad (32)$$

Similarely we get J_{ri} using L_{ri}, N_{ri} and E_i as:

$$J_{ri} = L_{ri} + N_{ri}E_i \quad (33)$$

then, with some algebraic manipulations, the error dynamic can be rewritten as:

$$\begin{aligned} \dot{e}_i(t) = &(N_i^1 - Z_iN_i^2)e_i(t) + (N_{ri}^1 - Z_iN_{ri}^2)e_i(t - d_{ii}) \\ &+(Q_i^1 + Z_iQ_i^2)x_j(t - d_{ij}) \end{aligned} \quad (34)$$

where:

$$Q_i^1 = \mu_i^1A_{ij} \quad (35)$$

$$Q_i^2 = \mu_i^2A_{ij} \quad (36)$$

with:

$$\mu_i^1 = K_i - E_i^1C_i \quad (37)$$

$$\mu_i^2 = E_i^2C_i \quad (38)$$

In order to study the stability of the error, we introduce the following new augmented state :

$$\bar{e}_i = \begin{bmatrix} x_i \\ e_i \end{bmatrix} \quad (39)$$

In fact that we don't have access to all state vector, the applied control $u_i(t)$ of (1a) is replaced by $\hat{u}_i(t)$ such that

$$\begin{aligned} S_i : \dot{x}_i(t) = &A_ix_i(t) + A_{ii}x_i(t - d_{ii}) \\ &+ A_{ij}x_j(t - d_{ij}) + \hat{u}_i(t) \end{aligned} \quad (40)$$

Since $e_i(t) = u_i(t) - \hat{u}_i(t)$, so we can replace $\hat{u}_i(t)$ in (40) by:

$$\hat{u}_i(t) = K_ix_i(t) - e_i(t) \quad (41)$$

and we obtain:

$$S_i : \dot{x}_i(t) = (A_i + K_i)x_i(t) + A_{ii}x_i(t - d_{ii}) + A_{ij}x_j(t - d_{ij}) - e_i(t) \tag{42}$$

Now, let us consider dynamics of the augmented state $\bar{e}_i$, then we have:

$$\begin{aligned}\dot{\bar{e}}_i = \begin{bmatrix}\dot{x}_i \\ \dot{e}_i\end{bmatrix} &= \begin{bmatrix}A_i + K_i & -I \\ 0 & N_i^1 - Z_i N_i^2\end{bmatrix}\begin{bmatrix}x_i(t) \\ e_i(t)\end{bmatrix} \\ &+ \begin{bmatrix}A_{ii} & 0 \\ 0 & N_{ri}^1 - Z_i N_{ri}^2\end{bmatrix}\begin{bmatrix}x_i(t - d_{ii}) \\ e_i(t - d_{ii})\end{bmatrix} \\ &+ \begin{bmatrix}A_{ij} & 0 \\ Q_i^1 + Z_i Q_i^2 & 0\end{bmatrix}\begin{bmatrix}x_j(t - d_{ij}) \\ e_j(t - d_{ij})\end{bmatrix}\end{aligned} \tag{43}$$

Then the error dynamics can be rewritten in a compact form as (for $i, j = 1, 2; i \neq j$)

$$\dot{\bar{e}}_i = \mathcal{A}_i \bar{e}_i + \mathcal{A}_{ii}\bar{e}_i(t - d_{ii}) + \mathcal{A}_{ij}\bar{e}_j(t - d_{ij}) \tag{44}$$

The expressions of matrices $\mathcal{A}_i$, $\mathcal{A}_{ii}$ and $\mathcal{A}_{ij}$ are directly deduced from equation (43).

For this purpose and similarly to (Chen. [2006]), let us consider the following subsystem (for $i, j \in \bar{N} = \{1, 2, ..., N\}$)

$$\dot{x}_i(t) = F_i x_i(t) + \sum_{i=1}^{N} F_{ij} x_j(t - h_{ij}),\ t \geqslant 0 \tag{45}$$

The stability condition of system (45) is given through the following lemma.

Lemma 1.(Chen. [2006]) The system (45) is asymptotically stable with $h_{ij} \in \mathbb{R}^+$ provided that F_i is Hurwitz, and there exist matrices P_i and V_{ij} such that the following LMI condition holds.

$$\begin{bmatrix}\Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & -\Xi_{22}\end{bmatrix} < 0 \tag{46}$$

where

$$\Xi_{11} = bdiag\left[\Phi_1, \Phi_2, \cdots, \Phi_N\right] \tag{47}$$

with

$$\Phi_i = F_i^T P_i + P_i F_i + \sum_{j=1}^{N} V_{ij} \tag{48}$$

$$\Xi_{12} = bdiag\left[\wedge_1, \wedge_2, \cdots, \wedge_N\right] \tag{49}$$

$$\Xi_{22} = bdiag\left[\widehat{\wedge}_1, \widehat{\wedge}_2, \cdots, \widehat{\wedge}_N\right] \tag{50}$$

$$\wedge_i = \left[F_{1i}^T P_1 \ \cdots \ F_{Ni}^T P_N\right] \tag{51}$$

$$\widehat{\wedge}_i = bdiag\left[V_{1i}, V_{2i}, \cdots, V_{Ni}\right] \tag{52}$$

Proof. The proof is given in (Chen. [2006]), Theorem 1, by using Lyapunov method.

The idea is to apply Lemma1 to system (44) in order to obtain the stability condition, in closed loop, of the two interconnected systems (1). The result is given in the following lemma

Lemma 2. The new error state is asymptotically stable with $d_{ij} \in \mathbb{R}^+$ provided that $\mathcal{A}_1$, $\mathcal{A}_2$ are Hurwitz, and there exist matrices $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{W}_{11}$, $\mathcal{W}_{12}$, $\mathcal{W}_{21}$ and $\mathcal{W}_{22}$ such that the following LMI condition holds.

$$\begin{bmatrix}(1,1) & 0 & \mathcal{A}_{11}^T\mathcal{X}_1 & \mathcal{A}_{21}^T\mathcal{X}_2 & 0 & 0 \\ 0 & (2,2) & 0 & 0 & \mathcal{A}_{12}^T\mathcal{X}_1 & \mathcal{A}_{22}^T\mathcal{X}_2 \\ \mathcal{X}_1\mathcal{A}_{11} & 0 & -\mathcal{W}_{11} & 0 & 0 & 0 \\ \mathcal{X}_2\mathcal{A}_{21} & 0 & 0 & -\mathcal{W}_{21} & 0 & 0 \\ 0 & \mathcal{X}_1\mathcal{A}_{12} & 0 & 0 & -\mathcal{W}_{12} & 0 \\ 0 & \mathcal{X}_2\mathcal{A}_{22} & 0 & 0 & 0 & -\mathcal{W}_{22}\end{bmatrix} < 0 \tag{53}$$

with

$$(1,1) = \mathcal{A}_1^T\mathcal{X}_1 + \mathcal{X}_1\mathcal{A}_1 + \mathcal{W}_{11} + \mathcal{W}_{12} \tag{54}$$

$$(2,2) = \mathcal{A}_2^T\mathcal{X}_2 + \mathcal{X}_2\mathcal{A}_2 + \mathcal{W}_{21} + \mathcal{W}_{22} \tag{55}$$

Now, we can state the following theorem witch gives in terms of LMI the stability condition of the error system of the two interconnected systems.

Theorem 1. The new system error is asymptotically stable if there exist matrices $X_{11} = X_{11}^T > 0$, $X_{12} = X_{12}^T > 0$, $X_{21} = X_{21}^T > 0$, $X_{22} = X_{22}^T > 0$, Y_{12}, Y_{22}, X_{22}, $W_{111} = W_{111}^T > 0$, W_{112}, $W_{113} = W_{113}^T > 0$, $W_{121} = W_{121}^T > 0$, W_{122}, $W_{123} = W_{123}^T > 0$, $W_{211} = W_{211}^T > 0$, W_{212}, $W_{213} = W_{213}^T > 0$, $W_{221} = W_{221}^T > 0$, W_{222}, $W_{223} = W_{223}^T > 0$ such that

$$\begin{bmatrix}\mathbb{B}_1^T X_{11} + X_{11}\mathbb{B}_1 & \mathbb{B}_1^T K_1^T X_{12} + K_1^T X_{12} N_1^1 + K_1^T Y_{12} N_1^2 \\ X_{12}K_1\mathbb{B}_1 + N_1^{1T} X_{12} K_1 + N_1^{2T} Y_{12}^T K_1 & N_1^{1T} X_{12} + X_{12} N_1^1 + N_1^{2T} Y_{12}^T + Y_{12} N_1^2\end{bmatrix} < 0 \tag{56}$$

$$\begin{bmatrix}\mathbb{B}_2^T X_{21} + X_{21}\mathbb{B}_2 & \mathbb{B}_2^T K_2^T X_2^2 + K_2^T X_{22} N_2^1 + K_2^T Y_{22} N_2^2 \\ X_{22}K_2\mathbb{B}_2 + N_2^{1T} X_{22} L_2 + N_2^{2T} Y_{22}^T K_2 & N_2^{2T} Y_{22}^T + X_{22} N_1^2 + N_2^{1T} X_{22} + Y_{22} N_2^2\end{bmatrix} < 0 \tag{57}$$

$$[(k, \ell)] < 0 \tag{58}$$

for $1 \leq k, \ell \leq 12$ such that

$$(1,1) = \mathbb{B}_1^T X_{11} + X_{11}\mathbb{B}_1 + W_{111} + W_{121}$$

$$(1,2) = (2,1)^T = \mathbb{B}_1^T K_1^T X_{12} + K_1^T X_{12} N_1^1 + K_1^T Y_{12} N_1^2 + W_{112} + W_{122}$$

$$(1,5) = (5,1)^T = A_{11}^T X_{11} + X_{11} A_{11} + W_{111} + W_{121}$$

$$(1,6) = (6,1)^T = A_{11}^T K_1^T X_{12} + K_1^T X_{12} N_{r1}^1 + K_1^T Y_{12} N_{r1}^2 + W_{112} + W_{122}$$

$$(1,7) = (7,1)^T = A_{21}^T X_{21} + Q_2^{1T} X_{22} K_2 + Q_2^{2T} Y_{22}^T K_2$$

$$(1,8) = (8,1)^T = A_{21}^T K_2^T X_{22} + Q_2^{1T} X_{22} + Q_2^{2T} Y_{22}^T$$

$$(2,2) = N_1^{1T} X_{12} + X_{12} N_1^1 + N_1^{2T} Y_{12}^T + Y_{12} N_1^2 + W_{113} + W_{123}$$

$$(3,3) = \mathbb{B}_2^T X_{21} + X_{21}\mathbb{B}_2 + W_{211} + W_{221}$$

$$(3,4) = (4,3)^T = \mathbb{B}_2^T K_2^T X_{22} + K_2^T X_{22} N_2^1 + K_2^T Y_{22} N_2^2 + W_{212} + W_{222}$$

$$(3,9) = (9,3)^T = A_{12}^T X_{11} + Q_1^{1T} X_{12} K_1 + Q_1^{2T} Y_{12}^T K_1$$

$$(3,10) = (10,3)^T = A_{12}^T K_1^T X_{12} + Q_1^{1T} X_{12} + Q_1^{2T} Y_{12}^T$$

$$(3,11) = (11,3)^T = A_{22}^T X_{21} + X_{21} A_{22} + W_{211} + W_{221}$$

$$(3,12) = (12,3)^T = A_{22}^T K_2^T X_{22} + K_2^T X_{22} N_{r2}^1 + K_2^T Y_{22} N_{r2}^2 + W_{212} + W_{222}$$

$$(4,4) = N_2^{1T} X_{22} + X_{22} N_2^1 + N_2^{2T} Y_{22}^T + Y_{22} N_2^2 + W_{213} + W_{223}$$

$$
\begin{aligned}
(5,5) &= -W_{111} \\
(5,6) = (6,5)^T &= -W_{112} \\
(6,6) &= -W_{113} \\
(7,7) &= -W_{211} \\
(7,8) = (8,7)^T &= -W_{212} \\
(8,8) &= -W_{213} \\
(9,9) &= -W_{121} \\
(9,10) = (10,9)^T &= -W_{122} \\
(10,10) &= -W_{123} \\
(11,11) &= -W_{221} \\
(11,12) = (12,11)^T &= -W_{222} \\
(12,12) &= -W_{223}
\end{aligned}
$$

Any other block of the above matrix is equal to 0.
Once the LMIs are verified, the matrix gain Z_1 and Z_2 are given by

$$Z_1 = X_{12}^{-1} Y_{12} \tag{59}$$

and

$$Z_2 = X_{22}^{-1} Y_{22} \tag{60}$$

Proof. We just give a sketch of proof due to lack of place; in fact the whole proof is derived from that of (Chen. [2006]) and is too long. Notice that we express the Hurwitzness of matrices $\mathcal{A}_1$ and $\mathcal{A}_2$ as required by Lemma 2 by using Lyapunov approach which give the LMI (56) and (57).

The LMI (58) is obtained from inequality (53) of Lemma 2 where matrices $\mathcal{A}_1$, $\mathcal{A}_2$, $\mathcal{A}_{12}$ and $\mathcal{A}_{21}$ are given by (44).

Then matrices $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{W}_{11}$, $\mathcal{W}_{12}$, $\mathcal{W}_{21}$ and $\mathcal{W}_{22}$ of Lemma 2 are chosen as follows

$$\mathcal{X}_1 = \begin{bmatrix} X_{11} & K_1^T X_{12} \\ X_{12} K_1 & X_{12} \end{bmatrix} \qquad \mathcal{X}_2 = \begin{bmatrix} X_{21} & K_2^T X_{22} \\ X_{22} K_2 & X_{22} \end{bmatrix} \tag{61}$$

$$\mathcal{W}_{11} = \begin{bmatrix} W_{111} & W_{112} \\ W_{112}^T & W_{113} \end{bmatrix} \qquad \mathcal{W}_{12} = \begin{bmatrix} W_{121} & W_{122} \\ W_{122}^T & W_{123} \end{bmatrix} \tag{62}$$

$$\mathcal{W}_{21} = \begin{bmatrix} W_{211} & W_{212} \\ W_{212}^T & W_{213} \end{bmatrix} \qquad \mathcal{W}_{22} = \begin{bmatrix} W_{221} & W_{222} \\ W_{222}^T & W_{223} \end{bmatrix} \tag{63}$$

Notice that instead of taking diagonal lyapunov matrices $\mathcal{X}_1$ and $\mathcal{X}_2$ to make the inequality be LMI, we consider a more general class as shown by (61).

So it suffices to take $Y_{12} = X_{12} Z_1$ and $Y_{22} = X_{22} Z_2$ to obtain LMI (58).

The observers (6) are easily reconstructed using the gain Z_i for $i = 1, 2$ using equations (22) to (33). The following algorithm permits to compute the observers easily.

The observer design algorithm

The different steps of the functional filter design are summarized as follows :

(1) Step 1 : Compute Γ_i and verify that rank condition is satisfied ;
(2) Step 2 : Compute matrices N_i^1 and N_i^2, E_i^1 and E_i^2, Q_i^1 and Q_i^2 for $i = 1, 2$
(3) Step 3 : Obtain matrices X_{12}, X_{22}, Y_{12}, Y_{22} through resolution of LMIs.
(4) Step 4 : Determine matrix gains Z_1 and Z_2.
(5) Step 5 : Compute the filter matrices N_i, E_i, J_i.

5. CONCLUSION

This paper presented a computationally tractable solution via LMI to the observation based control problem of interconnected systems. A new method is proposed which permits to first write the functional control error, and after that, using results on the stability of large scale time delayed systems, a functional filter is easily designed for each interconnected subsystem. Notice that the proposed filters are free of interconnection terms which make the design easy. The future works will concern the design of functional H_∞ filters based controllers of more general classes of interconnected systems.

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