A fault and delay tolerant multi-sensor control scheme

Nikola Stanković *,** Sorin Olaru * Silviu-Iulian Niculescu **

 * Supélec System Sciences (E3S) - Automatic Control Department, Gif-sur-Yvette, France (e-mail: nikola.stankovic@supelec.fr).
 ** Laboratory of Signals and Systems (L2S, UMR CNRS 8506), CNRS-Supélec, Gif-sur-Yvette, France.

Abstract: The present article deals with a fault and delay tolerant multi-sensor control scheme. In particular, we consider a process affected by additive disturbance which is monitored by redundant sensors. The process is controlled by a digital controller which is connected with the sensors via shared network. It is assumed that sensors are subject to abrupt faults while the communication between the sensors and the controller is affected by time-varying delay. In order to tackle network-induced delay, we design a prediction-based controller with compensation. A fault and delay detection and identification mechanism is designed as well.

Keywords: Fault tolerant control; networked control systems; time-delay.

1. INTRODUCTION

Using redundant sensors in control is unavoidable in safety-critical applications (see e.g. Blanke [2003]). The main objective of the redundant observation is to provide resilience of the controlled system with respect to an eventual sensor malfunction. When a fault occurs in one or several sensors, the controller has to decide which received information, among the conflicting ones, is the most relevant for computing the control action. Multisensor realisation via shared communication network can make fault-tolerant control (FTC) strategy even more complicated due to the network-induced effects such as time-delay and packet dropouts. The network-induced delays are usually time-varying (except for some particular network protocols) (see e.g. Zhang et al. [2001]). One of the reasons for the delay variability is network congestion (see e.g. Hespanha et al. [2007]). Taking into account the redundant sensor architecture, i.e., an increased number of nodes, one observes that delays could be even larger due to an increased exploitation of the network resources (see e.g. Nilsson [1998]).

While sensor fusion mitigates measurement noises and some fault events, it also makes possible that an inappropriate choice at this critical stage can significantly affect the performance of the plant. In this study, we consider an *active* multi-sensor networked control system (NCS) which provides fault and delay tolerance with respect to abrupt sensor faults and time-varying delay. Namely, whenever an information provided by a faulty sensor is detected, it is discarded (see Seron et al. [2008]). On the other side, if an information is provided with a delay, a predictionbased control (see Montestruque and Antsaklis [2003]) is employed. Fault detection and isolation (FDI) is set-based and it is implemented through set-membership testings of appropriate residual signals. In other words, the realisation of the FDI is achieved through the separation of residual sets which bound residual signals for healthy and faulty

data transmission. Separation is enforced by a reference governor which is designed using the receding horizon optimisation framework. The origin for such a fault tolerant control approach can be found in Stoican et al. [2012]. However, in this article we consider a control architecture where communication between the sensors and the controller is carried out via shared communication network. Particularly, beside the abrupt faults, performance of the closed-loop dynamics can also be degraded by a delayed data transmission.

2. NOMINAL MULTI-SENSOR DYNAMICS

Let us consider a plant modelled by the following linear differential equation:

$$\dot{x}(t) = A_c x(t) + B_c u_j(t) + E_c \omega(t), \qquad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $u_j \in \mathbb{R}^m$ is the control signal which is computed based on information provided by the j^{th} sensor, $j \in \mathbb{Z}_{[1,M]}$, while $\omega \in \mathbb{R}^p$ is a bounded process disturbance (see Fig. 1). The process disturbance is bounded by a C-set $\mathcal{W} \subset \mathbb{R}^p$.

It is assumed that the output of the process is measured periodically each $T_s = t_{k+1} - t_k \in \mathbb{R}^+$. The measuring is performed by M redundant sensors, with each sensor providing state vector perturbed by a different measurement noise. All sensors are assumed to be static (or with very fast dynamics relative to the plant dynamics) and to satisfy, under nominal functioning, the observation equation:

$$y_j[t_k] = x[t_k] + \eta_j[t_k],$$
 (2)

where $\eta_j \in \mathbb{R}^n$ is a bounded measurement noise. For each $j \in \mathbb{Z}_{[1,M]}$, the measurement noise is bounded by a C-set $\mathcal{N}_j \subset \mathbb{R}^n$. In the case when the full state vector is not measurable, one can employ M state observers which are



Fig. 1. Multi-sensor control scheme

collocated with the sensors. Using such an estimated statefeedback does not considerably differ from the analysis and the control design which are provided further in this paper, thus, it is omitted.

A multi-sensor system is in its nominal operational mode whenever the control action is computed based on measurements provided by a functional sensor without delays. Let us assume that the q^{th} sensor is functional and it transmits data instantaneously to the controller. Then, the discrete-time representation of (1) is given by:

$$x[t_{k+1}] = Ax[t_k] + Bu_q[t_k] + E\omega[t_k], \qquad (3)$$

where A, B and E are the corresponding discrete-time state, input and disturbance matrices, respectively. One can notice that, in the nominal operational mode, the control action $u_q[t_k]$ is active and constant over the entire sampling interval, i.e., $\forall t \in \mathbb{R}_{[t_k, t_{k+1})}$. It is also noticeable that we assumed that the process disturbance signal in (3) is constant between two consecutive samplings. However, as long as ω is bounded, a continuously varying process disturbance can be also tackled by the control strategy which is proposed in this study.

The following hypothesis holds throughout this paper. Hypothesis 1. The pair (A, B) is controllable.

In the nominal operational mode, digital controller, which is collocated with the actuator, provides the following control signal:

$$u_{q}[t_{k}] = u_{ref}[t_{k}] - K \left(y_{q}[t_{k}] - x_{ref}[t_{k}] \right), \qquad (4)$$

where K is a stabilising control gain and $u_{ref} = f(x_{ref})$ is a reference control signal.

The control objective is for the state of the plant (3) to track the following reference dynamics:

$$x_{ref}[t_{k+1}] = Ax_{ref}[t_k] + Bu_{ref}[t_k].$$
 (5)

Performance of the controller is determined by the tracking error $z = x - x_{ref}$. The dynamics of this error is obtained from (3), (4) and (5) as:

$$z[t_{k+1}] = (A - BK) z[t_k] - BK\eta_q[t_k] + E\omega[t_k].$$
(6)

Notice that, for a Schur matrix A - BK, $\omega \in W$ and $\eta_q \in \mathcal{N}_q$, the tracking error dynamics (6) admits a robust positively invariant set (RPI) (see e.g. Blanchini and Miani [2008]).

3. MULTI-SENSOR NCS

Let us now consider the case where the communication between the sensors and the controller is affected by networkinduced delay. Namely, at $t = t_k$, $k \in \mathbb{Z}_+$, redundant sensors measure the relevant plant parameters. Instantaneously after, they require from the network permission in order to transmit the acquired data to the controller. If the network is idle, then one node obtains permission for the transmission. If not, all transmissions are postponed until the network is available. A time lag that is induced in this way is often referred to as *access-time* delay and it is identified as the main source of delays in networked control systems (see Lin and Antsaklis [2005]). Delay can also arise due to time it takes to transfer data through the communication channel.

In the presence of a time-varying delay, using clockdriven sensors and an event-driven controller leads to a time-variant closed-loop representation (see Cloosterman et al. [2009]). A simplified model, which considers delay realisation on a finite grid, was proposed by Lin and Antsaklis [2005]. According to this result, a receiving buffer is associated to each sensor. Buffers are positioned at the controller site. Such buffers are read periodically at a frequency T, where $T_s = NT$ and N is a sufficiently large positive integer. For the inter-sampling period T, the corresponding difference equation is given by:

$$x[t_k + T] = \tilde{A}x[t_k] + \tilde{B}u_q[t_k] + \tilde{E}\omega[t_k], \qquad (7)$$

Let $\beta_j[t_k + iT]$, $j \in \mathbb{Z}_{[1,M]}$, denote a value of the j^{th} buffer at $t = t_k + iT$, where $i \in \mathbb{Z}_{[0,N-1]}$. Regarding stored information at each inter-sampling instant, the buffers can be classified into one of the following groups:

• $\mathcal{I}^D[t_k + iT] = \left\{ j \in \mathbb{Z}_{[1,M]} : \beta_j[t_k + iT] = y_j[t_{k-1}] \right\};$ • $\mathcal{I}^F[t_k + iT] = \left\{ j \in \mathbb{Z}_{[i_k,j_k]} : \beta_i[t_k + iT] = n^F \in \mathcal{N}^F \right\};$

•
$$\mathcal{I}^F[t_k+iT] = \left\{ j \in \mathbb{Z}_{[1,M]} : \beta_j[t_k+iT] = \eta_j^F \in \mathcal{N}_j^F \right\}$$

•
$$\mathcal{I}^{H}[t_{k}+iT] = \left\{ j \in \mathbb{Z}_{[1,M]} : \beta_{j}[t_{k}+iT] = y_{j}[t_{k}] \land j \notin \mathcal{I}^{F}[t_{k}+sT], \forall s \in \mathbb{Z}_{[0,i)} \right\};$$

• $\mathcal{I}^{R}[t_{k}+iT] = \left\{ j \in \mathbb{Z}_{[1,M]} : \beta_{j}[t_{k}+iT] = y_{j}[t_{k}] \land j \in \mathcal{I}^{F}[t_{k}+sT], s \in \mathbb{Z}_{[0,i)} \right\},$

where $\mathcal{N}_{j}^{F} \subset \mathbb{R}^{n}$ is a *C*-set. Sets of indices \mathcal{I}^{D} , \mathcal{I}^{F} and \mathcal{I}^{H} denote buffers with *outdated*, *faulty* and *healthy* information. On the other side, the set \mathcal{I}^{R} corresponds to sensors which are under recovery.

Hypothesis 2. Let $j \in \mathcal{I}^H[t_k + sT]$, $s \in \mathbb{Z}_{[0,N-1]}$. If the switching mechanism selects β_j at $t = t_k + sT$, then the same buffer is also used at $t = t_k + iT \quad \forall i \in \mathbb{Z}_{[s,N-1]}$.

3.1 Fault scenario

Any sensor is prone to faults which can be either temporary (e.g. change in operating conditions) or permanent (e.g. physical damage of the component). A fault can be defined formally as an instantaneous transition between the *healthy mode* of functioning (as given by (2)) and a *faulty mode* of functioning. Depending on the sense of the switch we have a failure or a recovery event (see e.g. Stoican et al. [2012]). For the simplicity of this paper, we consider only the following total outage of a sensor:

$$y_j[t_k] = x[t_k] + \eta_j[t_k] \underbrace{\xrightarrow{FAULT}}_{RECOVERY} y_j[t_k] = \eta_j^F[t_k]. \quad (8)$$

Briefly, the fault scenario is outlined as follows:

- Sensors are prone to abrupt faults as in (8).
- Data transmission from sensors to the controller is subject to random and time-varying delay which is less than the sampling period.
- Sets $\mathcal{I}^{D}[t_{k} + iT] \neq \emptyset$ $\forall i \in \mathbb{Z}_{[0,N-\mu)}$ and $\mathcal{I}^{H}[t_{k} + (N-\mu)T] \neq \emptyset$, where $\mu \in \mathbb{Z}^{+}$ is the controllability index of the pair (\tilde{A}, \tilde{B}) .

Remark 1. The abruptness hypothesis can be discarded in favour of faults which describe a gradual output decay. However, none of these elements are conceptually different from the scenario described in (8). In the sense that, no new insight in the treatment of the FTC mechanisms can be gained by using the more complex cases. Also, the outlined fault scenario can be relaxed with respect to allowable delay range. The only requirement is that delay would have to be bounded. This will, of course, impose a more "aggressive" control compensation (see the following section).

3.2 Control design for multi-sensor NCS

Let us introduce the following hypothesis:

Hypothesis 3. The pair (\tilde{A}, \tilde{B}) is controllable.

Let $\tau_j[t_k] \in [0, T_s)$. Because the receiving buffers are read at the higher frequency, the delay parameter is restricted on the grid of the interval $[0, T_s)$ determined by T, i.e.,

$$\tau_j[t_k] = \tau_{is}^j[t_k]T, \text{ where } \tau_{is}^j = \left\lceil \frac{\tau_j N}{T_s} \right\rceil.$$
(9)

Assume that
$$\tau_{is}^{j}[t_{k}] = \min_{i} \left\{ \tau_{is}^{i}[t_{k}] : i \in \mathbb{Z}_{[1,M]} \right\}.$$

Without having the up-to-date measurements available, the switching mechanism selects one of the buffers with outdated information. Let $i_1 \in \mathbb{Z}_{[0,N-1]}$ such that $i_1 < \tau_{is}^j[t_k]$. Assume that $\beta_{s_1}[t_k + iT]$, where $s_1 \in \mathcal{I}^D$, is selected during the inter-sampling intervals determined by $i \in \mathbb{Z}_{[0,i_1]}$. Since $\beta_{s_1}[t_k + iT] = y_{s_1}[t_{k-1}] \quad \forall i \in \mathbb{Z}_{[0,i_1]}$, the following state prediction can be computed by using the available mathematical model of the plant:

$$\theta_{t_k|t_{k-1}}^{(s_1)} = Ay_{s_1}[t_{k-1}] + Bu_{ref}[t_{k-1}] - BK\left(y_{s_1}[t_{k-1}] - x_{ref}[t_{k-1}]\right).$$
(10)

By using (10) in control (4) one obtains:

$$x[t_{k} + (i_{1} + 1)T] = \tilde{A}^{i_{1}+1}x[t_{k}] + \sum_{i=0}^{i_{1}} \tilde{A}^{i}\tilde{E}\omega[t_{k}] + \sum_{i=0}^{i_{1}} \tilde{A}^{i}\tilde{B}\left(u_{ref}[t_{k}] - K\left(\theta_{t_{k}|t_{k-1}}^{(s_{1})} - x_{ref}[t_{k}]\right)\right).$$
(11)

Next, assume the s_1^{th} buffer is updated at $t = t_k + (i_1+1)T$, but the transmitted measurements are provided by the faulty sensor, i.e., $\beta_{s_1}[t_k + (i_1 + 1)T] = \eta_{s_1}^F[t_k]$. Consequently, the switching mechanism selects another buffer, say β_{s_2} , where $s_2 \in \mathcal{I}^D[t_k + iT]$, $\forall i \in \mathbb{Z}_{[0,i_2]}$ and $i_2 < \tau_{is}^j[t_k]$. The corresponding discrete-time representation at $t = t_k + (i_2 + 1)T$ is:

$$x[t_{k} + (i_{2} + 1)T] = \tilde{A}^{i_{2}+1}x[t_{k}] + \sum_{i=0}^{i_{2}} \tilde{A}^{i}\tilde{E}\omega[t_{k}]$$

+
$$\sum_{i=i_{2}-i_{1}}^{i_{2}} \tilde{A}^{i}\tilde{B}\left(u_{ref}[t_{k}] - K\left(\theta_{t_{k}|t_{k-1}}^{(s_{1})} - x_{ref}[t_{k}]\right)\right)$$
(12)
+
$$\sum_{i=0}^{i_{2}-i_{1}-1} \tilde{A}^{i}\tilde{B}\left(u_{ref}[t_{k}] - K\left(\theta_{t_{k}|t_{k-1}}^{(s_{2})} - x_{ref}[t_{k}]\right)\right),$$

where $\theta^{(s_2)}$ is computed in the similar way as in (10).

Due to different measurement noises, switching among the buffers with obsolete data introduces a *switching error* which we denote by γ :

$$\gamma^{(s_2 \leftarrow s_1)}[t_k] = \theta^{(s_2)}_{t_k | t_{k-1}} - \theta^{(s_1)}_{t_k | t_{k-1}}.$$
 (13)

By using (13) in (12), one obtains:

$$x[t_{k} + (i_{2}+1)T] = \tilde{A}^{i_{2}+1}x[t_{k}] + \sum_{i=i_{2}-i_{1}}^{i_{2}} \tilde{A}^{i}\tilde{B}K\gamma^{(s_{2}\leftarrow s_{1})}[t_{k}]$$

$$\sum_{i=0}^{i_{2}}\tilde{A}^{i}\tilde{B}\left(u_{ref}[t_{k}] - K\left(\theta_{t_{k}|t_{k-1}}^{(s_{2})} - x_{ref}[t_{k}]\right)\right)$$

$$+ \sum_{i=0}^{i_{2}}\tilde{A}^{i}\tilde{E}\omega[t_{k}].$$
(14)

In the similar way, the switching mechanism selects another buffer with the outdated measurements when the



Fig. 2. Model-based controller

previously used buffer is updated by a faulty information. Assume that the s_r^{th} buffer is selected at $t = t_k + iT$, $\forall i \in \mathbb{Z}_{[i_{r-1}+1,\tau_{is}^j]}$. Therefore, one can get:

$$\begin{aligned} x[t_{k} + \tau_{is}^{j}[t_{k}]T] &= \tilde{A}^{\tau_{is}^{j}[t_{k}]}x[t_{k}] + \sum_{i=0}^{\tau_{is}^{j}[t_{k}]-1} \tilde{A}^{i}\tilde{E}\omega[t_{k}] \\ &+ \sum_{i=0}^{\tau_{is}^{j}[t_{k}]-1} \tilde{A}^{i}\tilde{B}\Big(u_{ref}[t_{k}] - K\left(\theta_{t_{k}|t_{k-1}}^{(s_{r})} - x_{ref}[t_{k}]\right)\Big) \\ &+ \sum_{i=\tau_{is}^{j}[t_{k}]-1}^{\tau_{is}^{j}[t_{k}]-1} \tilde{A}^{i}\tilde{B}K\gamma^{(s_{r}\leftarrow s_{r-1})}[t_{k}] + \dots \end{aligned}$$
(15)
$$+ \sum_{i=\tau_{is}^{j}[t_{k}]-1}^{\tau_{is}^{j}[t_{k}]-1} \tilde{A}^{i}\tilde{B}K\gamma^{(s_{2}\leftarrow s_{1})}[t_{k}], \end{aligned}$$

where $\gamma^{(s_r \leftarrow s_{r-1})}$, $\gamma^{(s_{r-1} \leftarrow s_{r-2})}$, ..., $\gamma^{(s_2 \leftarrow s_1)}$ are the switching errors computed according to (13).

Since $j \in \mathcal{I}^H[t_k + \bar{\tau}_{is}^j T]$, the switching mechanism selects $\beta_j[t_k + \bar{\tau}_{is}^j T] = y_j[t_k]$. According to Hypothesis 2, once the buffer with the healthy information is selected, it is employed until the end of the sampling period. The corresponding state vector is given as:

$$\begin{aligned} x[t_{k+1}] &= Ax[t_k] + E\omega[t_k] \\ &+ \sum_{i=N-\tau_{is}^j[t_k]}^{N-1} \tilde{A}^i \tilde{B} \Big(u_{ref}[t_k] - K \left(\theta_{t_k|t_{k-1}}^{(s_r)} - x_{ref}[t_k] \right) \Big) \\ &+ \sum_{i=N-1-i_{r-1}}^{N-1} \tilde{A}^i \tilde{B} K \gamma^{(s_r \leftarrow s_{r-1})}[t_k] + \dots \\ &+ \sum_{i=N-1-i_1}^{N-1} \tilde{A}^i \tilde{B} K \gamma^{(s_2 \leftarrow s_1)}[t_k] + \sigma[t_k + (N-1-i)T] \Big) \\ &+ \sum_{i=0}^{N-\tau_{is}^j[t_k]-1} \tilde{A}^i \tilde{B} \Big(u_{ref}[t_k] - K \Big(y_j[t_k] - x_{ref}[t_k] \Big), \end{aligned}$$
(16)

where σ is the compensation term which is introduced when the healthy information is available to the controller. The model-based controller scheme with compensation is shown on Fig. 2. Let $\varepsilon_j \in \mathbb{R}^n$ denote the prediction error with respect to the j^{th} buffer which is defined by the following difference:

$$\varepsilon_j[t_k] = y_j[t_k] - \theta_{t_k|t_{k-1}}^{(j)}.$$
(17)

This information becomes available immediately when the up-to-date information is provided by one of the sensors. By using (17) and

$$\theta_{t_k|t_{k-1}}^{(s_r)} = \theta_{t_k|t_{k-1}}^{(j)} - \gamma^{(j \leftarrow s_r)}[t_k]$$

in (16), one obtains:

$$\begin{aligned} x[t_{k+1}] &= \underbrace{Ax[t_k] + Bu_j[t_k] + E\omega[t_k]}_{\text{Nominal dynamics}} \\ &+ \sum_{i=N-\tau_{is}^j[t_k]}^{N-1} \tilde{A}^i \tilde{B} K \varepsilon_j[t_k] + \sum_{i=N-\tau_{is}^j[t_k]}^{N-1} \tilde{A}^i \tilde{B} K \gamma^{(j \leftarrow s_r)}[t_k] \\ &+ \sum_{i=N-1-i_{r-1}}^{N-1} \tilde{A}^i \tilde{B} K \gamma^{(s_r \leftarrow s_{r-1})}[t_k] + \dots \\ &+ \sum_{i=N-1-i_1}^{N-1} \tilde{A}^i \tilde{B} K \gamma^{(s_2 \leftarrow s_1)}[t_k] \\ &+ \sum_{i=N-1-i_1}^{N-\tau_{is}^j[t_k] - 1} \tilde{A}^i \tilde{B} \left(\sigma[t_k + (N-1-i)T] \right), \end{aligned}$$
(18)

where $\sigma_k = \left[\sigma[t_k + \tau_{is}^j[t_k]T]^T \dots \sigma[t_k + (N-1)T]^T\right]$ is the compensation vector.

Regarding the discrete-time system (18), one can notice that $\gamma^{(j \leftarrow s_r)}[t_k], \ldots \gamma^{(s_2 \leftarrow s_1)}[t_k]$ and $\varepsilon_j[t_k]$ are known parameters when up-to-date information is provided to the controller. In order to be able to compute the compensation vector, one has to determine τ_{is}^j . This value can be assessed by the controller either by using time-stamps (see e.g. Nilsson [1998]) or by the FDDI mechanism which is described in the subsequent section.

In order to compensate the tracking error caused by delay in the sensor-to-controller loop, σ_k is determined such that:

$$\sum_{i=0}^{N-\tau_{is}^{j}[t_{k}]-1} \tilde{A}^{i}\tilde{B}\sigma[t_{k} + (N-1-i)T]$$

$$= -\sum_{i=N-\tau_{is}^{j}[t_{k}]}^{N-1} \tilde{A}^{i}\tilde{B}K\left(\varepsilon_{j}[t_{k}] + \gamma^{(j\leftarrow s_{r})}[t_{k}]\right)$$

$$-\sum_{i=N-1-i_{r-1}}^{N-1} \tilde{A}^{i}\tilde{B}K\gamma^{(s_{r}\leftarrow s_{r-1})}[t_{k}] - \dots$$

$$-\sum_{i=N-1-i_{1}}^{N-1} \tilde{A}^{i}\tilde{B}K\gamma^{(s_{2}\leftarrow s_{1})}[t_{k}].$$
(19)

The existence of the compensation vector is characterised by the following results. Theorem 2. Let $\mu \in \mathbb{Z}^+$ denote the controllability index of the pair (\tilde{A}, \tilde{B}) . Then, the linear equations (19) is consistent if $\tau_{is}^j[t_k] \leq N - \mu$.

Remark 3. Computation of σ_k is carried out on-line, i.e., the linear equation (19) needs to be solved each time when up-to-date measurements are transmitted to the controller. Nevertheless, numerical complexity of solving such an equation is low. Also, by computing a compensation vector that drives the closed-loop system in a C-set around the reference signal, instead to the reference itself, better control performance can be achieved.

4. FAULT DETECTION AND ISOLATION IN THE PRESENCE OF DELAY

In order to provide detection of an abrupt sensor fault or an outdated information, one has to define a residual signal and the corresponding *thresholds* that characterise the nominal functioning and data transmission for that sensor. In this paper, we use information from the buffers in combination with the reference dynamics:

$$r_j = \beta_j - x_{ref}, \quad j \in \mathbb{Z}_{[1,M]}. \tag{20}$$

In order to perform fault or delay detection, residual signals are compared with their pre-defined thresholds at each inter-sampling instant. Due to the process and the measurement noises, thresholds are characterised by sets. We differ the following residual signals.

Healthy residual: when stored data are *up-to-date* and provided by the functional sensor;

Faulty residual: when stored data are provided by the faulty sensor;

Delayed residual: when stored data are *outdated* and provided by the functional sensor.

While delayed and faulty residuals determine buffers with outdated and faulty measurements, it is important to notice that the healthy residual signals may determine buffers with healthy information but also they may correspond to sensors that are under recovery. In order to discern between these cases, one has to keep track of the recovery process for each sensor that used to be faulty.

Depending on a network-induced delay for each sensorto-controller link, information that is stored in the corresponding buffer can be updated at any moment. However, such an update is noted only at the inter-sampling instants. Let us assume that $j \in \mathcal{I}^H[t_k + iT]$. Then, the corresponding healthy residual signal is determined by:

$$r_j^H[t_k + iT] = \beta_j[t_k + iT] - x_{ref}[t_k] = z[t_k] + \eta_j[t_k].$$
(21)

In the same manner, let $j \in \mathcal{I}^F[t_k+iT]$. The corresponding faulty residual signal is obtained as:

$$r_{j}^{F}[t_{k}+iT] = \beta_{j}[t_{k}+iT] - x_{ref}[t_{k}] = \eta_{j}^{F}[t_{k}] - x_{ref}[t_{k}].$$
(22)

The third case concerns functional sensors with data transmission affected by time-varying delay, i.e., $j \in \mathcal{I}^D[t_k + iT]$. The corresponding delayed residual signal is determined by:



Fig. 3. Admissible domain of reference state

$$r_j^D[t_k + iT] = \beta_j[t_k + iT] - x_{ref}[t_k] = = z[t_{k-1}] + \eta_j[t_{k-1}] + x_{ref}[t_{k-1}] - x_{ref}[t_k].$$
(23)

Let \mathcal{Z} denote a RPI set for (6). For each $z[t_k] \in \mathcal{Z}$, the nominal tracking error dynamics (6) stays within the same set. Therefore, the bounding set for the healthy residual signals is constructed as:

$$\mathcal{R}_j^H = \mathcal{Z} \oplus \mathcal{N}_j. \tag{24}$$

When measurements are provided by a faulty sensor i.e., $\beta_j[t_k + iT] = \eta_j^F[t_k]$ (or $\beta_j[t_k + iT] = \eta_j^F[t_{k-1}]$), the corresponding bounding set is determined as:

$$\mathcal{R}_j^F(x_{ref}) = \{-x_{ref}[t_k]\} \oplus \mathcal{N}_j^F.$$
(25)

Finally, the bounding set for the delayed residual signals is determined by:

$$\mathcal{R}_j^D(x_{ref}) = \{x_{ref}[t_{k-1}] - x_{ref}[t_k]\} \oplus \mathcal{Z} \oplus \mathcal{N}_j.$$
(26)

By verifying if $r_j[t_k+iT]$, $i \in \mathbb{Z}_{[0,N-1]}$, resides in one of the sets (24)-(26), one can affirm that the j^{th} residual signal is healthy, faulty or delayed at $t = t_k + iT$. This provides an unequivocal fault/delay detection and isolation as long as the residual sets are piecewise-disjoint.

Taking into consideration (24), (25) and (26), the separation condition can be directly used in order to define the admissible domain of reference state x_{ref} , which allows exact fault detection and isolation:

$$\mathcal{D}_{x_{ref}} = \{x_{ref}[t_k], x_{ref}[t_{k-1}] : \mathcal{R}_j^H \cap \mathcal{R}_j^F(x_{ref}) = \emptyset, \\ \mathcal{R}_j^H \cap \mathcal{R}_j^D(x_{ref}) = \emptyset, \ \forall j \in \mathcal{I}\}.$$

$$(27)$$

Notice that $\mathcal{R}_j^H \cap \mathcal{R}_j^F(x_{ref}) = \emptyset$ and $\mathcal{R}_j^D \cap \mathcal{R}_j^F(x_{ref}) = \emptyset$ are equivalent.

As long as the invariant set \mathcal{Z} is defined offline, the previous sets can also be described offline and the actual FDDI is a fast online set membership evaluation which differentiates between the healthy, faulty and outdated measurements transmitted through each feedback channel.

The FDDI mechanism discussed here deals with acknowledging the transition from healthy to faulty or healthy to delayed residual signals. In other words, the inclusion of the residual signal in one of the sets (24), (25) or (26) makes sense only if the tracking error is in its own bounding sets. Moreover, employing a sensor that is faulty during one sampling interval and which becomes healthy during the subsequent sampling period, introduces a tracking error that cannot be compensated by control action. Therefore, we assume that if a sensor is faulty, it cannot be used immediately after when it admit a healthy residual signal. In other words, a recovery confirmation is required in order to employ that sensor again. In the case of the state feedback, the recovery of a faulty sensor is rather simple. Namely, since residuals (21), (22) and (23) do not have transient behaviour, a faulty sensor can be employed again one sampling period after it admits a healthy behaviour. The reason for this will be clarified in the subsequent section.

Remark 4. Recovery process is far more complex when controller's input is provided by a state observer. In particular, beside verifying the inclusion of the tracking error, one also has to verify the inclusion of the estimation tracking error.

5. REFERENCE GOVERNOR DESIGN AND DELAY IDENTIFICATION

Design of the reference governor is carried out by using model-based receding horizon optimisation framework. The objective of the optimisation problem is to design a reference control input u_{ref} which provides minimal tracking mismatch between an ideal state reference trajectory to be followed x_r , and the real reference state x_{ref} , under imposed constraints. The implementation of the reference governor is carried out through the optimisation over a finite horizon:

$$u_{[0,s-1]}^{*} = \underset{u_{ref}}{\operatorname{arg\,min}} (\sum_{i=1}^{s} \| (x_{r}[t_{k+i}] - x_{ref}[t_{k+i}]) \|_{Q}^{2} + \sum_{i=0}^{s-1} \| (u_{r}[t_{k+i}] - u_{ref}[t_{k+i}]) \|_{P}^{2})$$

$$(28)$$

subject to:

$$\begin{aligned} x_{ref}[t_{k+i+1}] &= Ax_{ref}[t_k+i] + Bu_{ref}[t_k+i] \\ x_{ref}[t_k+i] &\in \mathcal{D}_{x_{ref}}, \quad \forall k \in \mathbb{Z}_+, \quad i \in \mathbb{Z}_{[0,s-1]}, \end{aligned}$$
(29)

In (28) $s \in \mathbb{Z}^+$ is a prediction horizon and $Q \succ 0$ and $P \succ 0$ are weighting matrices. The reference control action is set to $u_{ref}[t_k] = u_{[0]}^*$ which is the first component in the optimal sequence (28). Then, the optimisation is reiterated by receding the reference window.

Remark 5. Since $\mathcal{D}_{x_{ref}}$ is non-convex set, solving the optimisation problem (28)-(29) requires the mixed-integer programming.

5.1 Delay identification

By delay identification we mean evaluation of the minimal delay that is induced by the redundant feedback channels. The main interest of applying the fastest transmitted measurements is the fact that the resulting correction action is distributed over a larger compensation horizon. One can think of such a configuration as competition between redundant sensors where the priority is given to the one with the fastest transmission. Of course, at this stage information provided by faulty or sensors under recovery is discarded by the FDDI mechanism and not taken into account since it is not relevant for the computing of the control signal.

Delay parameter can be evaluated based on knowledge of T_s , N and membership testing of residual signals. Therefore, it is indirectly calculated when a buffer with healthy information is detected by the controller.

6. CONCLUSION

A redundant sensors architecture subject to abrupt faults and delays induced by the shared network has been considered for the control design. A unified fault detection and isolation algorithm for substantially different fault scenarios such as sensor outage and communication delays has been presented by exploiting set membership testing. We considered information provided by a dysfunctional sensor as worthless for computing the control action, and, consequently, we discarded them from the control loop. On the other side, information that is provided with delay by a functional sensor was considered still useful and handled by a model-based controller with delay compensation.

REFERENCES

- F. Blanchini and S. Miani. Set-Theoretic Methods in Control. Springer, 2008.
- M. Blanke. Diagnosis and fault-tolerant control. Springer Verlag, 2003.
- M.B.G. Cloosterman, N. Van de Wouw, W.P.M.H. Heemels, and Henk. Nijmeijer. Stability of networked control systems with uncertain time-varying delays. *IEEE Transactions on Automatic Control*, 54(7):1575– 1580, 2009.
- J.P. Hespanha, P. Naghshtabrizi, and Y. Xu. A survey of recent results in networked control systems. *Proceedings* of the IEEE, 95(1):138–162, 2007.
- H. Lin and P.J. Antsaklis. Stability and persistent disturbance attenuation properties for a class of networked control systems: switched system approach. *International Journal of Control*, 78(18):1447–1458, 2005.
- L.A. Montestruque and P.J. Antsaklis. On the modelbased control of networked systems. *Automatica*, 39(10): 1837–1843, 2003.
- J. Nilsson. *Real-time control systems with delays*. PhD thesis, Lund Institute of Technology, 1998.
- M.M. Seron, X.W. Zhuo, J.A. De Doná, and J.J. Martínez. Multisensor switching control strategy with fault tolerance guarantees. *Automatica*, 44(1):88–97, 2008.
- F. Stoican, S. Olaru, M.M Seron, and J. De Doná. Reference governor design for tracking problems with fault detection guarantees. *Journal of Process Control*, 22(5): 829–836, 2012.
- W. Zhang, M.S. Branicky, and S.M. Phillips. Stability of networked control systems. *IEEE Control Systems Magazine*, 21(1):84–99, 2001.