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On-line Ampacity Monitoring from Phasor Measurements

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Abstract: The knowledge of transmission line parameters involving series impedance and shunt admittance is crucial to power system analysis and power system state estimation. Particularly in areas of power systems protection it is critical to have accurate values of these parameters to determine the line ampacity. The line parameters change according to the load level and weather conditions, so it is necessary to identify the line parameters in real time. The paper deals with the on-line parameter identification of transmission lines from synchronized measurements of current and voltage phasors provided by the phasor measurement units. The series and shunt line parameters are recursively estimated from the phasor measurements and weather conditions using the extended Kalman filter. Contrary to the other works, an actual line temperature and reference resistance are estimated, which makes possible to monitor the actual state of the transmission line in order to determine an actual reserve or prediction of future transmission capacity. A case study based on data measured from a 110kV overhead transmission line is presented to demonstrate the effectiveness of the proposed approach.

Keywords: System identification, Electrical networks, Energy distribution, Estimation parameters, Extended Kalman filter, Overload

1. INTRODUCTION

The growth in the amount of energy produced from renewable sources leads to an increase in load of transmission line because of the high variability of their performance and the need to transmit energy over long distances from the sources to the load centers. The need to transfer large amount of power causes that some lines can be operated close to their ampacity limit. Therefore it is necessary to monitor the state of that power lines.

The knowledge of transmission line parameters involving series impedance and shunt admittance is crucial to power system analysis. Particularly in areas of power systems protection it is critical to have accurate values of these parameters to determine the line ampacity, for appropriate setting of the protection relays, electricity network state estimation, calculating fault distance, and making a sound tripping decision, see Liao and Kezunovic (2009). For example as it was mentioned in Zarco and Gomez-Exposito (2000), errors of the line parameters can lead to permanent errors in the data provided by the state estimator for a long time without being detected.As the parameters change according to ambient conditions and load of the line, the actual values differ from the designed parameters. It is therefore necessary to determine the actual parameters to ensure effective and safe operation of the transmission line. The influence of changes of the transmission line resistance due to the weather on the state estimation performance has been studied e.g. in Bockarjova and Andersson (2007).

In the recent years, there were proposed several approaches to real-time monitoring of the power line ampacity. There are systems based on measuring the sag or tension of the conductor or weather-related magnitudes assuming that the right parameters of the lines are known, see e.g. Albizu et al. (2013), Olsen and Edwards (2002) and the references therein. On the contrary, the approach proposed in this paper will determine the state of the line from the synchronized voltage and current phasor measurements measured by the phasor measurement units (PMU) from the both ends of the line and ambient weather conditions.

In the past, several approaches for identification of the line parameters from the measured phasors have been discussed e.g. in Liao (2009). The weighted least squares (WLS) method is widely used for power system state and parameter estimation, where the weights are commonly set to be the inverse of the measurement error variances, Abur and Expósito (2004). At each time, the line parameters are estimated based on a redundant number of current and voltage measurements taken at different time instants. The number of measurements which can be used must be sufficiently large and is limited by numerical stability to yield reliable instantaneous estimates of time varying parameters, see Liao and Kezunovic (2009).

The paper proposes a transmission line model and a technique for on-line estimation of the series and shunt parameters of the transmission line on the basis of the synchronized phasor measurements. The proposed model of the line considers the parameter changes due to the variability of the line load level and ambient weather conditions. Further, the extended Kalamn filter (EKF) is used for the parameter estimation.

In Section 2, the problem of parameter identification of the overhead transmission lines based on the synchronized phasor measurements is stated. Section 3 describes the proposed model of changes of the transmission line parameters depending on the load level and ambient weather conditions. Section 4 introduces the EKF based parameter estimation method. The proposed approach is illustrated in a case study for identification of pa-

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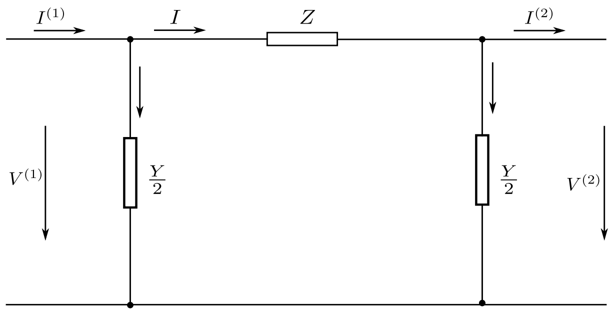

Fig. 1. Positive sequence equivalent two-port $\pi-$circuit of a transmission line.

rameters of an 110kV overhead transmission line in Section 5. Finally, the results are summarized in Section 6.

2. PROBLEM STATEMENT

Let us assume a three-phase transmission line having installed PMUs for synchronous measurement of the voltage and current phasors from both ends of the line. Moreover, the line is fully transposed and operating in the steady state under balanced conditions. Thus, all series or shunt devices are symmetrical in the three phases. Under these assumptions it is possible to model the line with a single phase positive sequence equivalent circuit, see e.g. Abur and Expósito (2004), Blackburn (1993). So, the transmission line is represented by an equivalent two-port $\pi-$circuit with a positive sequence impedance Z and a shunt admittance Y, which is shown in Figure 1.

Using the Kirchhoff's laws the line admittance Y can be calculated as

$$Y = 2\frac{I^{(1)} - I^{(2)}}{V^{(1)} + V^{(2)}} \tag{1}$$

and subsequently the line impedance Z is given as

$$Z = \frac{V^{(1)} - V^{(2)}}{I}. \tag{2}$$

It is possible to express the current I through the series impedance Z using $I^{(1)}$ and $V^{(1)}$ as

$$I = I^{(1)} - \frac{Y}{2}V^{(1)}, \tag{3}$$

or alternatively using $I^{(2)}$ and $V^{(2)}$ as

$$I = I^{(2)} + \frac{Y}{2}V^{(2)}. \tag{4}$$

PMUs synchronously measure magnitudes and angles of the voltage and current phasors at both ends of each phase with a given sampling period Δt, from which the positive sequence voltage and current phasors $V^{(n)} = v^{(n)}\angle\varphi^{(n)}$ and $I^{(n)} = i^{(n)}\angle\xi^{(n)}$, $n = 1, 2$, respectively, are determined subsequently. It is appropriate to consider that these measurements contain random errors. So let the relations between the measured and true voltage magnitude $v^{(n)}$ and angle $\varphi_v^{(n)}$ be defined as

$$\bar{v}^{(n)} = v^{(n)} + \tilde{v}^{(n)}, \tag{5}$$

$$\bar{\varphi}^{(n)} = \varphi^{(n)} + \tilde{\varphi}^{(n)}, \quad n = 1, 2, \tag{6}$$

where $\bar{v}^{(n)}$ is a measured magnitude of positive sequence voltage phasor and $\bar{\varphi}_v$ is a measured angle of the voltage phasor. Variables $\tilde{v}^{(n)}$ and $\tilde{\varphi}^{(n)}$ are independent random measurement errors with zero mean Gaussian distributions and variances σ_v^2 and σ_φ^2, respectively. These variances correspond mainly to the limited degree of accuracy of the used meters.

Similarly, current measurement equations describing dependence between the measured current phasor positive sequence magnitude and angle and true magnitude $i^{(n)}$ and angle $\xi^{(n)}$, respectively, are given as follows

$$\bar{i}^{(n)} = i^{(n)} + \tilde{i}^{(n)} \tag{7}$$

$$\bar{\xi}^{(n)} = \xi^{(n)} + \tilde{\xi}^{(n)}, \quad n = 1, 2, \tag{8}$$

where $\bar{i}^{(n)}$ is a measured magnitude of the positive sequence current phasor and $\bar{\xi}^{(n)}$ is a measured angle of the current phasor. Variables $\tilde{i}^{(n)}$ and $\tilde{\xi}^{(n)}$ are independent random measurement errors with zero mean Gaussian distributions and variances σ_i^2 and σ_ξ^2, respectively.

In addition to the phasor measurements, let us assume that meteorological data such as the ambient temperature, wind speed and direction and coverage rate of sky by clouds are measured along the line.

The objective is to estimate the conductor temperature T and positive sequence impedance $Z = R+jX$ and admittance $Y = G+jB$, where R is a series positive sequence resistance, X is a positive sequence reactance, G is a shunt conductance and B is a susceptance, from recursively obtained measurements of the positive sequence components of the phasors and the weather measurements.

3. MODEL OF THE LINE PARAMETER CHANGES

The line parameters change due to the line load level and ambient weather conditions. To obtain accurate parameter estimates, it is desirable to include the effect of these factors to the model.

At first, it should be remarked that in order to ensure better clarity of the text, all coefficients used below will be listed in table 1.

The dependence of series resistance R to average temperature T of the line can be denoted by

$$R(T) = R_{20}(1 + \alpha\,(T - 20)), \tag{9}$$

where R_{20} is a resistance of conductor at reference temperature $20°C$, T is an average line temperature expressing an average between a temperature in the core and on the surface of the line.

The change of the average line temperature T in terms of the weather conditions and the load level can be addressed according to the IEEE standard IEEE (2007) by the following heat balance differential equation

$$\tau\frac{dT}{dt} = \frac{1}{l}R(T)|I(t)|^2 - q_c(T) - q_r(T) + q_s(t), \tag{10}$$

where $|I(t)|$ is an absolute value of the current through the series impedance at the time t, $q_c(T)$ is a convected heat loss rate per unit length, $q_r(T)$ denotes a radiated heat loss rate per unit length and $q_s(t)$ is a solar heat gain rate per unit length.

The impact of the ambient weather conditions on the heat losses and gain are calculated using the relations adopted from IEEE (2007), which are mentioned below.

The solar heat gain:

$$q_s(t) = aQ_{se}(t)\sin\theta(t)A, \tag{11}$$

where $\theta = \arccos\left[\cos(H_c)\cos(Z_c(t) - Z_1)\right]$ is effective angle of incidence of the sun rays.

The radiated heat loss:

Table 1. Description of symbols

Symbol	Description	Units
α	temperature coefficient of resistance	$1/°C$
τ	total heat capacity of conductor	$J/(m \cdot °C)$
l	length of the transmission line	m
a	solar absorptivity	–
$H_c(t)$	altitude of the sun at the time t	degrees
$Z_c(t)$	azimuth of the sun at the time t	degrees
Z_1	azimuth of the line	degrees
A	projected area of conductor	m^2/m
$Q_{se}(t)$	total solar and sky radiated heat flux at the time t – elevation corrected	W/m^2
D	conductor diameter	mm
ε	emissivity	–
T_a	ambient temperature	$°C.$
K_a	wind direction factor	–
μ_f	dynamic viscosity of air	$Pa \cdot s$
ρ_f	density of air	kg/m^3
k_f	thermal conductivity of air	$W/(m \cdot °C)$
V_w	wind speed at conductor	m/s

$$q_r(T) = 0.0178 D\varepsilon \left[\left(\frac{T + 273}{100} \right)^4 - \left(\frac{T_a + 273}{100} \right)^4 \right]. \quad (12)$$

The convected heat loss is determined according to the IEEE standard IEEE (2007) as the maximum from convected heat losses computed for natural convection q_{cn}, low wind speed q_{c1} and high wind speed q_{c2}:

$$q_c(T) = \max(q_{c1}, q_{c2}, q_{cn}), \quad (13)$$

where

$$q_{c1}(T) = \left[1.01 + 0.0372 \left(\frac{D\rho_f V_w}{\mu_f} \right)^{0.52} \right] k_f K_a (T - T_a),$$
$$q_{c2}(T) = \left[0.0119 \left(\frac{D\rho_f V_w}{\mu_f} \right)^{0.6} \right] k_f K_a (T - T_a),$$
$$q_{cn}(T) = 0.0205 \rho_f^{0.5} D^{0.75} (T - T_a)^{1.25}. \quad (14)$$

The main advantage of identification of the line resistance through estimation R_{20} and average temperature of the conductor T is that it allows to take into account the whole measurement history for estimation of the actual resistance R. So the obtained estimates are becoming less affected by random measurement errors when the new measurements come.

The influence of the line temperature on its reactance X was modeled in Bockarjova and Andersson (2007) by a relation analogous to (9) The reactance changes through the variation of the line inductance due to the thermal expansion of the conductor. This impact is nevertheless negligible and thus the reactance X will be assumed to be constant herein.

Shunt parameters G and B obviously also depend on weather conditions as e.g. ambient humidity. However, the description of the dependence based on physical laws would be troublesome. Thus, their variability will be modeled by a random walk as it was done e.g. in Slutsker et al. (1996).

The next section focuses on description of a technique of identification of the line parameters using the EKF.

4. PARAMETER IDENTIFICATION BY EKF

From the state estimation theory point of view, the transmission line is a system whose parameters represent the system state which has to be estimated. Generally, the state estimation using the EKF is based on recursive alternating the measurement update and the time update step at discrete time instants k, $k = 1, 2, \ldots$. So it is necessary to derive the discrete time state and measurement equations of the system. In order to distinguish the time dependence, the subscript k is added to the variables in this section.

4.1 Discretized model for parameter identification

Let $\mathbf{p}_k$ be a vector of line parameters at time sample k, which are unknown and will be estimated, defined as

$$\mathbf{p}_k = \begin{bmatrix} T_k \\ R_{20,k} \\ X_k \\ G_k \\ B_k \end{bmatrix}. \quad (15)$$

Further the temporal change of the parameters is modeled as follows

$$\mathbf{p}_{k+1} = \mathbf{f}(\mathbf{p}_k) + \mathbf{w}_k = \begin{bmatrix} \mathrm{f}_1(\mathbf{p}_k) \\ \vdots \\ \mathrm{f}_5(\mathbf{p}_k) \end{bmatrix} + \begin{bmatrix} w_{1,k} \\ \vdots \\ w_{5,k} \end{bmatrix}, \quad (16)$$

where

$$\mathrm{f}_1 = T_k + \frac{\Delta t}{c_p} \left(\frac{1}{l} R_k |I_k|^2 + q_{s,k} - q_{c,k}(T_k) - q_{r,k}(T_k) \right) \quad (17)$$
$$\mathrm{f}_2 = R_{20,k}, \quad (18)$$
$$\mathrm{f}_3 = X_k, \quad (19)$$
$$\mathrm{f}_4 = G_k, \quad (20)$$
$$\mathrm{f}_5 = B_k, \quad (21)$$

and

$$R_k = R_{20,k}(1 + \alpha(T_k - 20)). \quad (22)$$

Equation (17) is obtained by discretization of (10) with sampling period Δt.

The noise $\mathbf{w}_k$ is a zero mean Gaussian white noise with covariance matrix $\mathbf{Q} = \mathrm{diag}(\sigma_{w_i}^2)$, $i = \{1, \ldots, 5\}$, where the variances $\sigma_{w_i}^2$ generally make it possible to represent uncertainty in the development of the parameter values.

As the identified line parameters represent real and imaginary parts of complex numbers, it is advantageous to identify them in the Cartesian coordinates system. However the measurements are reported in the polar coordinates. So, there are two ways how this can be handled. One way is to convert the polar measurements to the Cartesian coordinates. Nevertheless, the measurement errors in the converted measurements are correlated. Another way is to use polar coordinates system in the measurement equations and the Cartesian coordinates in state equations. So, the resulting EKF will work with mixed coordinate system. When using polar measurements, it is necessary to reflect the periodicity of angular measurements in the EKF, see Kurz et al. (2013). Hence conversion of measurements to the Cartesian coordinates according to Lerro and Bar-Shalom (1993), Ristic et al. (2004) will be preferred in the paper.

Let us consider a phasor $C = c\angle\theta$. Further, let the relation between the measured magnitude $\bar{c}$ and angle $\bar{\theta}$ and the true magnitude c and angle θ, respectively, be defined as

$$\bar{c} = c + \tilde{c}, \tag{23}$$

$$\bar{\theta} = \theta + \tilde{\theta}, \tag{24}$$

where both measurements contain random measurement errors $\tilde{c}$ and $\tilde{\theta}$ having zero mean Gaussian distribution with standard deviation σ_c and σ_θ, respectively. Using the polar-to-Cartesian unbiased consistent conversion presented in Lerro and Bar-Shalom (1993), the phasor can be converted to the Cartesian coordinates with correction for the average bias. Converted phasor is then given as

$$\bar{C} = \begin{bmatrix} \bar{C}_{\Re} \\ \bar{C}_{\Im} \end{bmatrix} = \begin{bmatrix} \bar{c}\cos\bar{\theta} \\ \bar{c}\sin\bar{\theta} \end{bmatrix} - \mu, \tag{25}$$

where

$$\mu = \begin{bmatrix} \bar{c}\cos\bar{\theta}\left(e^{-\sigma_\theta^2} - e^{-\sigma_\theta^2/2}\right) \\ \bar{c}\sin\bar{\theta}\left(e^{-\sigma_\theta^2} - e^{-\sigma_\theta^2/2}\right) \end{bmatrix} \tag{26}$$

represents average bias of the conversion.

Since the accuracy of the angle measurement is typically of the order of tenths of a degree, the term μ is negligible and can be omitted. Consequently, it makes possible, in accordance with Lerro and Bar-Shalom (1993), to approximate the elements of the covariance matrix Σ_c of the Cartesian coordinate errors as follows

$$\Sigma_c(1,1) = \bar{c}^2\sigma_\theta^2\sin^2\bar{\theta} + \sigma_c^2\cos^2\bar{\theta}, \tag{27}$$

$$\Sigma_c(2,2) = \bar{c}^2\sigma_\theta^2\cos^2\bar{\theta} + \sigma_c^2\sin^2\bar{\theta}, \tag{28}$$

$$\Sigma_c(1,2) = (\sigma_c^2 - \bar{c}^2\sigma_\theta^2)\sin\bar{\theta}\cos\bar{\theta}. \tag{29}$$

The measurements of the voltage phasors $\bar{U}^{(n)} = \bar{u}^{(n)}\angle\bar{\varphi}^{(n)}$ and the current phasors $\bar{I}^{(n)} = \bar{i}^{(n)}\angle\bar{\xi}^{(n)}$ may thus be converted to the Cartesian coordinates as follows

$$\bar{U}_{\Re}^{(n)} = \bar{u}^{(n)}\cos\bar{\varphi}^{(n)}, \tag{30}$$

$$\bar{U}_{\Im}^{(n)} = \bar{u}^{(n)}\sin\bar{\varphi}^{(n)}, \tag{31}$$

$$\bar{I}_{\Re}^{(n)} = \bar{i}^{(n)}\cos\bar{\xi}^{(n)}, \tag{32}$$

$$\bar{I}_{\Im}^{(n)} = \bar{i}^{(n)}\sin\bar{\xi}^{(n)}, \tag{33}$$

$$\text{for} \quad n = 1, 2.$$

Substituting the measurements $\bar{U}^{(n)}$ and $\bar{I}^{(n)}$ to phasors $U^{(n)}$ and $I^{(n)}$, $n = 1, 2$, in equations (1) and (2), the measurement equations will be designed. As the measurements $\bar{I}^{(1)}$ and $\bar{I}^{(2)}$ are affected by random errors, it is appropriate to use the expected value of the current $\hat{I}$ given by

$$\hat{I} = \frac{1}{2}\left(\bar{I}^{(1)} + \bar{I}^{(2)} - \frac{Y}{2}\left(\bar{U}^{(1)} - \bar{U}^{(2)}\right)\right), \tag{34}$$

in (2) instead of one of the values calculated using the formulas (3) and (4).

So let the measurement equation at the time k be defined as

$$\mathbf{z}_k = \begin{bmatrix} \Delta\bar{U}_{\Re,k} \\ \Delta\bar{U}_{\Im,k} \\ \Delta\bar{I}_{\Re,k} \\ \Delta\bar{I}_{\Im,k} \end{bmatrix} = \mathbf{h}_k(\mathbf{p}_k) + \mathbf{e}_k, \tag{35}$$

where

$$\mathrm{h}_{1,k} = R_k\hat{I}_{\Re,k} - X_k\hat{I}_{\Im,k}, \tag{36}$$

$$\mathrm{h}_{2,k} = R_k\hat{I}_{\Im,k} + X_k\hat{I}_{\Re,k}, \tag{37}$$

$$\mathrm{h}_{3,k} = \left(\bar{U}_{\Re,k}^{(1)} + \bar{U}_{\Re,k}^{(2)}\right)\frac{G_k}{2} - \left(\bar{U}_{\Im,k}^{(1)} + \bar{U}_{\Im,k}^{(2)}\right)\frac{B_k}{2}, \tag{38}$$

$$\mathrm{h}_{4,k} = \left(\bar{U}_{\Im,k}^{(1)} + \bar{U}_{\Im,k}^{(2)}\right)\frac{G_k}{2} + \left(\bar{U}_{\Re,k}^{(1)} + \bar{U}_{\Re,k}^{(2)}\right)\frac{B_k}{2}, \tag{39}$$

where $\hat{I}_{\Re,k}$ and $\hat{I}_{\Im,k}$ are real and imaginary part of the current $\hat{I}_k$, which is computed according to (34) from the measurements obtained at the time k, $\mathbf{e}_k$ denotes a discrete time Gaussian white noise representing random error of measurement devices, so

$$\mathrm{E}\{\mathbf{e}_k\} = \mathbf{0}, \ \mathrm{cov}(\mathbf{e}_k) = \mathbf{\Sigma}_k. \tag{40}$$

The error covariance matrix $\mathbf{\Sigma}_k$ will be calculated using relations (27) – (29). The elements of $\mathbf{\Sigma}_k$ thus become

$$\mathbf{\Sigma}_k(1,1) = \sigma_\varphi^2\left((\bar{u}^{(1)})^2\sin^2\bar{\varphi}^{(1)} + (\bar{u}^{(2)})^2\sin^2\bar{\varphi}^{(2)}\right) + \sigma_u^2\left(\cos^2\bar{\varphi}^{(1)} + \cos^2\bar{\varphi}^{(2)}\right), \tag{41}$$

$$\mathbf{\Sigma}_k(2,2) = \sigma_\varphi^2\left((\bar{u}^{(1)})^2\cos^2\bar{\varphi}^{(1)} + (\bar{u}^{(2)})^2\cos^2\bar{\varphi}^{(2)}\right) + \sigma_u^2\left(\sin^2\bar{\varphi}^{(1)} + \sin^2\bar{\varphi}^{(2)}\right), \tag{42}$$

$$\mathbf{\Sigma}_k(1,2) = (\sigma_u^2 - (\bar{u}^{(1)})^2\sigma_\varphi^2)\sin\bar{\varphi}^{(1)}\cos\bar{\varphi}^{(1)} + (\sigma_u^2 - (\bar{u}^{(2)})^2\sigma_\varphi^2)\sin\bar{\varphi}^{(2)}\cos\bar{\varphi}^{(2)}, \tag{43}$$

$$\mathbf{\Sigma}_k(3,3) = \sigma_\xi^2\left((\bar{i}^{(1)})^2\sin^2\bar{\xi}^{(1)} + (\bar{i}^{(2)})^2\sin^2\bar{\xi}^{(2)}\right) + \sigma_i^2\left(\cos^2\bar{\xi}^{(1)} + \cos^2\bar{\xi}^{(2)}\right), \tag{44}$$

$$\mathbf{\Sigma}_k(4,4) = \sigma_\xi^2\left((\bar{i}^{(1)})^2\cos^2\bar{\xi}^{(1)} + (\bar{i}^{(2)})^2\cos^2\bar{\xi}^{(2)}\right) + \sigma_i^2\left(\sin^2\bar{\xi}^{(1)} + \sin^2\bar{\xi}^{(2)}\right), \tag{45}$$

$$\mathbf{\Sigma}_k(3,4) = (\sigma_i^2 - (\bar{i}^{(1)})^2\sigma_\xi^2)\sin\bar{\xi}^{(1)}\cos\bar{\xi}^{(1)} + (\sigma_i^2 - (\bar{i}^{(2)})^2\sigma_\xi^2)\sin\bar{\xi}^{(2)}\cos\bar{\xi}^{(2)}. \tag{46}$$

Obviously, the covariance matrix $\mathbf{\Sigma}_k$ depends on the present measurements and has to be calculated at each time k.

4.2 *Extended Kalman Filter*

Uncertainty in the difference of phasors is a crucial issue in parameter estimation due to the low signal to noise ratio. Therefore, it is necessary to use as much of data as possible. The EKF, Grewal and Andrews (2001), which takes into account the entire measurement history up to the time k over the traditional WLS approach, is given by the following two steps:

Measurement update step:

$$\mathbf{p}_{k|k} = \mathbf{p}_{k|k-1} + \mathbf{K}_{F,k}\left(\mathbf{z}_k - \mathbf{h}_k(\mathbf{p}_{k|k-1})\right), \tag{47}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^T\left[\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T + \mathbf{\Sigma}_k\right]^{-1}, \tag{48}$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k\mathbf{H}_k\mathbf{P}_{k|k-1}, \tag{49}$$

Time update step:

$$\mathbf{p}_{k+1|k} = \mathbf{f}(\mathbf{p}_{k|k}), \tag{50}$$

$$\mathbf{P}_{k+1|k} = \mathbf{F}_k\mathbf{P}_{k|k}\mathbf{F}_k^T + \mathbf{Q}, \tag{51}$$

where $\mathbf{p}_{k|k}$ is the parameter estimate based on measurement history up to the time k, $\mathbf{p}_{k+1|k}$ is the one-step ahead parameter prediction at the time $k+1$ based on measurement history up to time k,

$$\mathbf{H}_k = \left.\frac{\partial \mathbf{h}_k}{\partial \mathbf{p}_k}\right|_{\mathbf{p}_k=\mathbf{p}_{k|k-1}} = \begin{bmatrix} \left(\frac{\partial \mathrm{h}_{1,k}}{\partial \mathbf{p}_k}\right)^T \\ \left(\frac{\partial \mathrm{h}_{2,k}}{\partial \mathbf{p}_k}\right)^T \\ \left(\frac{\partial \mathrm{h}_{3,k}}{\partial \mathbf{p}_k}\right)^T \\ \left(\frac{\partial \mathrm{h}_{4,k}}{\partial \mathbf{p}_k}\right)^T \end{bmatrix},$$

$$\mathbf{F}_k = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{p}_k}\right|_{\mathbf{p}_k=\mathbf{p}_{k|k}} = \begin{bmatrix} \left(\frac{\partial \mathrm{f}_1}{\partial \mathbf{p}_k}\right)^T \\ \mathbf{0}_{4\times1} \ \mathbf{I}_{4\times4} \end{bmatrix},$$

$\mathbf{0}_{4\times1}$ is an 4-by-1 vector of zeros and $\mathbf{I}_{4\times4}$ is identity matrix of dimension 4.

5. CASE STUDY

Let us consider an overhead transmission line operated at $110kV$ with the maximum permitted current $700A$ and temperature $80°C$. The ACSR conductor with diameter $D = 26.5mm$ and length $l = 19km$ is considered. Further, line azimuth is $Z_1 = 288°$, total heat capacity is assumed $\tau = 1304.2J/(m \cdot °C)$. As the exact values of the solar absorptivity a and emissivity ϵ are unknown, $a = 0.7$ and $\epsilon = 0.5$ are set in accordance with suggestions in Bockarjova and Andersson (2007) and CIGRE (1992).The voltage and the current phasor measurements were obtained in the period of April 7 to September 9 with sampling period $\Delta t = 1min$.

The goal was to recursively identify the line parameters on basis of the obtained set of phasor measurements and the weather measurements, which are displayed in Fig. 2, in order to simulate the activity in real time. The proposed estimation algorithm was implemented in MATLAB.

Development of identification of the positive sequence parameters of the identified transmission line is shown in Fig. 3 and Fig. 4. It can be seen in Fig. 4 that the line was not overloaded within the observed period and that the conductor temperature was under the maximum permitted limit. Comparison of parameter estimates from the final time step with projected values of the line parameters is presented in table 2.

Finally, time development of the static line ampacity and its increase in accordance with the changes of the weather conditions are illustrated in Fig. 5.

Table 2. Designed values of the 110kV overhead transmission line parameters and their identified positive sequence component values.

Parameter	Projected value	Identified value
$R_{20}[\Omega]$	1.56	1.32
$X[\Omega]$	7.21	7.21
$G[\mu S]$	—	4.32
$B[\mu S]$	57.78	56.94

6. CONCLUSION

The paper was devoted to the monitoring the state of the transmission line in order to determine the load reserve of that line in accordance with actual weather conditions. The proposed procedure has adopted the EKF to estimate the line parameters in real time.

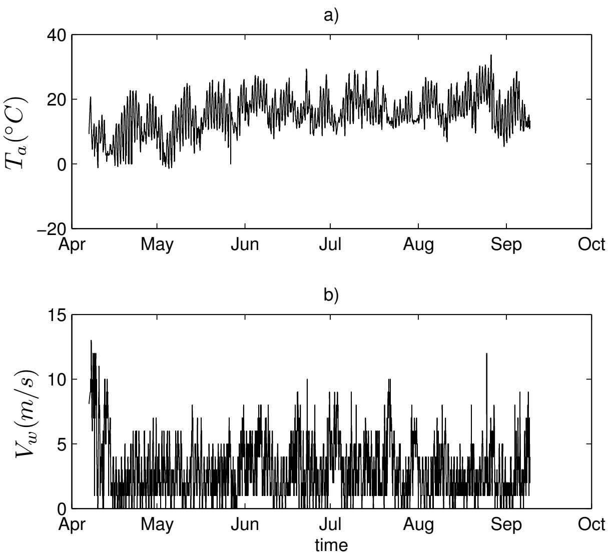

Fig. 2. Measured ambient temperature and wind speed at the line.

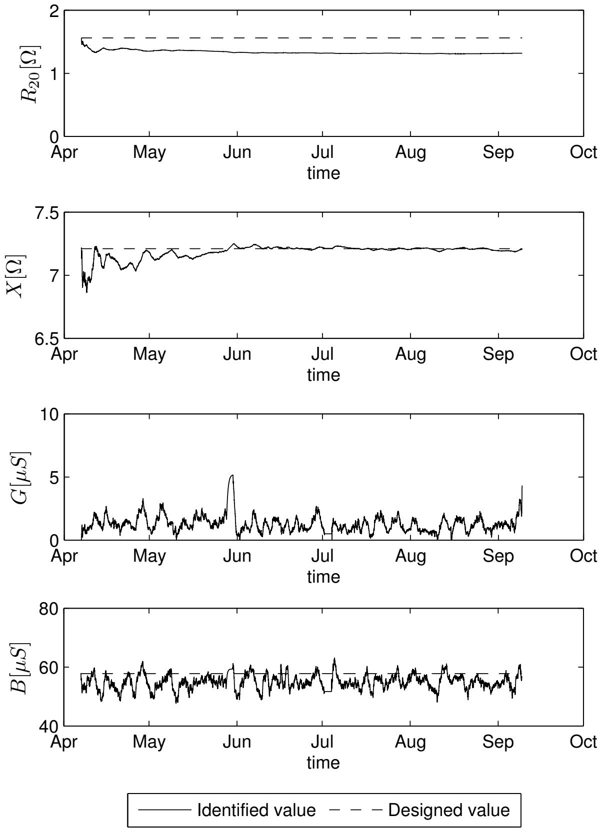

Fig. 3. Development of identified parameters of the transmission line with respect to the increasing number of phasor measurements from the line.

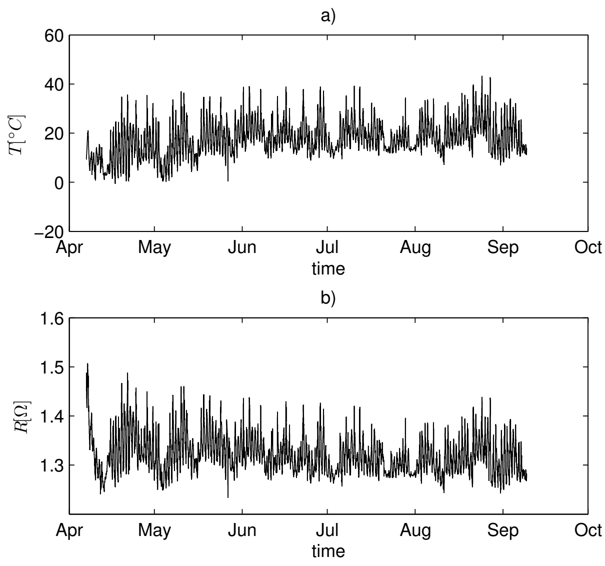

Fig. 4. Development of the identified temperature and actual resistance of the transmission line.

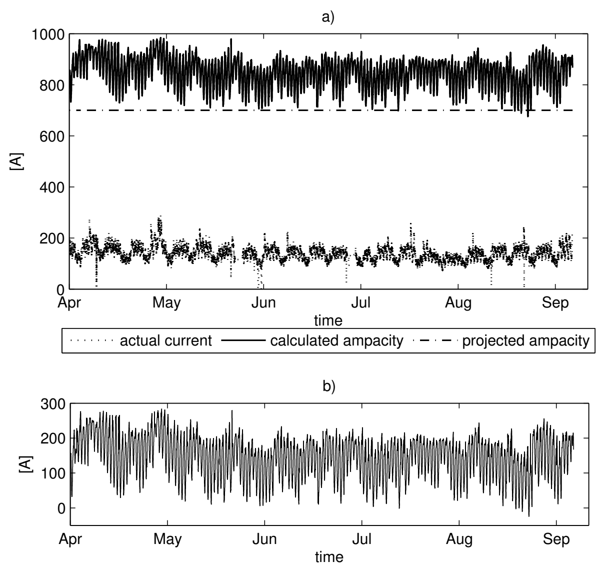

Fig. 5. a) Time development of the static line ampacity in accordance with the changes of the weather conditions; b) Difference between the calculated static line ampacity and the projected ampacity.

The proposed transmission line model incorporates the influence of ambient environment, such as ambient temperature, wind speed and direction and solar heating, on the line parameters. The main advantage of the used model is the estimation of reference resistance and actual conductor temperature instead of direct estimation of the actual positive sequence resistance. It makes possible to identify the line parameters with taking into account the whole data history and to reduce the impact of the random measurement errors. Consequently, it improves resultant estimates of the overhead transmission line parameters. The proposed approach can be used for on-line monitoring of the static ampacity of the transmission line. In conclusion it should be noted that any other nonlinear estimator can be used for the line parameter estimation instead of the aforementioned EKF.

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