

Taut Cable Control of a Tethered UAV^{*}

Marco M. Nicotra^{*} Roberto Naldi^{**} Emanuele Garone^{*}

^{*} *Université Libre de Bruxelles (ULB), Brussels, Belgium
(e-mail: [mnicotra/egarone]@ulb.ac.be).*

^{**} *Alma Mater Studiorum (Università di Bologna), Bologna, Italy
(e-mail: roberto.naldi@unibo.it)*

Abstract: This paper focuses on the design of a stabilizing control law for an aerial vehicle which is physically connected to a ground station by means of a tether cable. When the cable is taut, the resulting dynamic model is shown to be characterized by a new set of equilibria which untethered aircraft are unable to maintain in steady state. The control objective is to steer the UAV to a desired set-point while maintaining the cable taut at all times. This leads to a nonlinear control problem subject to constraints. A cascade control scheme is proposed and proven to asymptotically stabilize the overall system by means of ISS arguments. Constraint satisfaction is guaranteed using a modified thrust vector control coupled with a reference governor strategy. The effectiveness of the proposed control strategy is shown via numerical simulations.

Keywords: Unmanned Aerial Vehicles, Stability of Nonlinear Systems, Constrained Control.

1. INTRODUCTION

Recent advancements in the field of Unmanned Aerial Vehicles (UAVs) have led to the availability of inexpensive aerial robots with a growing range of applications. However, their full potential is often limited by key factors such as flight time, computing capabilities and safety regulations. To overcome these limitations, the UAV can be connected to a ground station by means of a tether cable that transmits energy, data and/or forces.

The cable can either hang loosely between the UAV and the ground station or remain taut. As discussed in Schmidt and Swik (1974), the loose cable configuration requires the stabilization of the tether cable oscillations. If the cable is taut, it can be used to improve flight stability in the presence of wind [Sandino et al. (2013)], guide landing procedures [Oh et al. (2006)], measure the position of the UAV [Lupashin and D’Andrea (2013)] or achieve full actuation [Naldi et al. (2012)]. In all these examples, the cable tension is controlled by an actuated winch and the UAV position is controlled by the UAV itself.

This paper investigates the dual case: the actuated winch controls the length of the cable whereas the UAV is charged with maintaining the cable taut. This control philosophy has the advantage of providing a guaranteed safety boundary imposed directly by the ground station winch. The proposed control law can also be applied to the case of a fixed-length cable and no winch. To the author’s best knowledge, this approach to the taut cable control of an UAV has not been previously addressed.

The paper is organized as follows. Section 2 presents a nonlinear model of the system where the cable tension is treated as a mechanical constraint instead of a control input. Section 3 defines the set of attainable equilibrium configurations. Section 4 describes the general control architecture in general which is then developed in Sections 5-7. Section 8 ends the paper with numerical simulations.

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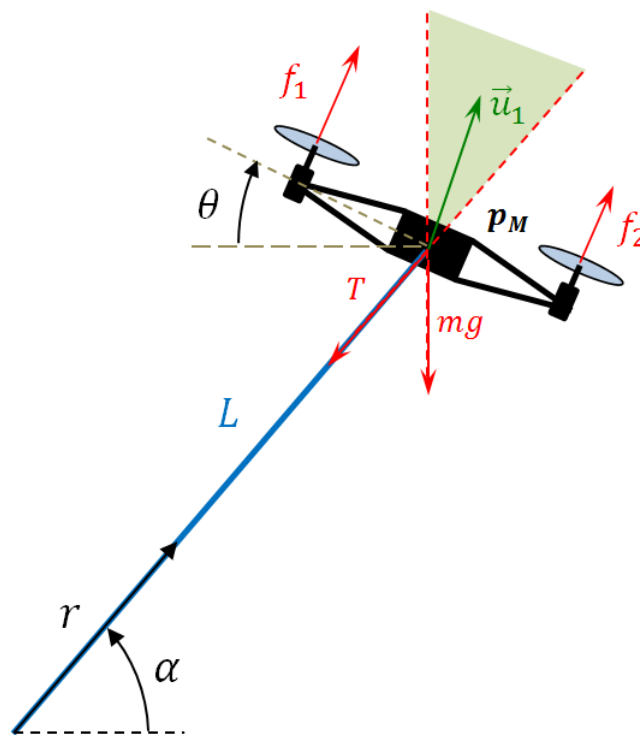


Fig. 1. 2D model of a tethered UAV with a taut cable

The main contributions of this paper include: the characterization of the set of attainable steady state attitudes of the described system, the adaptation of the vector thrust formalism [Hua et al. (2013)] to the case of a constrained UAV and the introduction of a Reference Governor strategy [see Kolmanovsky et al. (2014)] to ensure that the closed-loop system satisfies the constraints.

2. PROBLEM STATEMENT

Consider the planar model of an UAV of mass $m \in \mathbb{R}_{>0}$ and moment of inertia $\mathcal{J} \in \mathbb{R}_{>0}$ physically connected

to the origin of an inertial reference frame by means of a cable of length $L \in \mathbb{R}_{>0}$, as depicted in Figure 1. Let the polar coordinates $p_M = [r, \alpha]^T \in \mathbb{R}_{>0} \times [0, \pi]$ describe the position of the center of mass of the vehicle and $\theta \in [0, 2\pi]$ be the angle between the horizon and the UAV. Let $T \in \mathbb{R}_{\geq 0}$ be the tension acting on the cable and $f_1 \in \mathbb{R}_{>0}$, $f_2 \in \mathbb{R}_{>0}$ be the forces generated by the two propellers. To simplify the notation, define the thrust $u_1 := f_1 + f_2$ and the torque $u_2 := (f_1 - f_2)b$, where $b \in \mathbb{R}_{>0}$ is the lever-arm between the propellers and the center of mass. Furthermore, let $u_3 \in \mathbb{R}$ be the radial acceleration of the winch of radius $\rho \in \mathbb{R}_{>0}$.

Assumption 1. The cable is inextensible, massless and is attached to the center of mass of the UAV. \square

Under this assumption, the total kinetic \mathcal{K} and potential \mathcal{P} energy of the UAV are

$$\mathcal{K} = \frac{1}{2}mr\dot{\alpha}^2 + \frac{1}{2}mr^2\dot{\alpha}^2 + \frac{1}{2}\mathcal{J}\dot{\theta}^2 \quad \mathcal{P} = mgr \sin \alpha.$$

In the absence of dissipative forces, define the Lagrangian function $\mathcal{L} = \mathcal{K} - \mathcal{P}$. The dynamic model of the system can then be obtained via the Euler-Lagrange theorem, using

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = F_i \quad i = r, \alpha, \theta$$

$$F_r = u_1 \sin(\alpha + \theta) - T \quad F_\alpha = u_1 r \cos(\alpha + \theta) \quad F_\theta = u_2.$$

This leads to the following dynamic model

$$\begin{cases} m\ddot{r} = mr\dot{\alpha}^2 - mg \sin \alpha + \sin(\alpha + \theta) u_1 - T \\ mr^2\ddot{\alpha} = -2mr\dot{r}\dot{\alpha} - mgr \cos \alpha + r \cos(\alpha + \theta) u_1 \\ \mathcal{J}\ddot{\theta} = u_2. \end{cases} \quad (1)$$

Since the cable tension T is a reactive force that is lost if $r < L$, the following definition is given:

Definition 2. The cable is defined *taut* whenever p_M belongs to the manifold $r = L$. \square

For a weightless cable, the taut cable condition can be verified by enforcing the following constraint:

$$T(t) > 0 \quad \forall t \geq 0 \quad (2)$$

where $T(t) = T(r(t), \alpha(t), \theta(t))$ is

$$T = mr\dot{\alpha}^2 - mg \sin \alpha + \sin(\alpha + \theta) u_1 - m\ddot{r}. \quad (3)$$

At steady-state, condition (2) leads to the following.

Definition 3. The set of attainable configurations $\mathcal{S} \subset \mathbb{R}^3$ is defined as the set of equilibrium points $[\bar{r}, \bar{\alpha}, \bar{\theta}] \in \mathbb{R}^3$ such that $T(\bar{r}, \bar{\alpha}, \bar{\theta}) > 0$. \square

If the taut cable constraint (2) is satisfied at all times, the radial dynamics of the UAV can be reformulated as $\ddot{r} = \ddot{L}$ where $\ddot{L} = \rho u_3$. Therefore, the dynamic model of an UAV connected to a taut cable can also be expressed as

$$\begin{cases} \ddot{r} = \rho u_3 \\ \ddot{\alpha} = -\frac{1}{r}(2\dot{r}\dot{\alpha} + g \cos \alpha) + \frac{\cos(\alpha + \theta)}{mr} u_1 \\ \ddot{\theta} = \frac{1}{\mathcal{J}} u_2 \end{cases} \quad (4)$$

The objective of this paper is to stabilize the tethered UAV while simultaneously satisfying the taut cable constraint.

Control Objectives: Let system (4) be initialized in $[r(0), \alpha(0), \theta(0)] \in \mathcal{S}$ with limited velocities. Given a set-point $[\bar{r}, \bar{\alpha}, \bar{\theta}] \in \mathcal{S}$, design a control law such that

- (1) $\lim_{t \rightarrow \infty} [r(t), \alpha(t), \theta(t)] = [\bar{r}, \bar{\alpha}, \bar{\theta}]$
- (2) $T(r(t), \alpha(t), \theta(t)) > 0 \quad \forall t \geq 0$

3. ATTAINABLE EQUILIBRIUM POINTS

The goal of this section is to characterize the set \mathcal{S} of attainable configurations.

Proposition 4. Given system (4) subject to constraint (2), the set \mathcal{S} of attainable configurations is $\bar{r} > 0$, $\bar{\alpha} \in [0, \pi]$ and

$$\begin{cases} \bar{\theta} \in \left(0, \frac{\pi}{2} - \bar{\alpha}\right) \text{ if } \bar{\alpha} \in \left[0, \frac{\pi}{2}\right] \\ \bar{\theta} \in \left(\frac{\pi}{2} - \bar{\alpha}, 0\right) \text{ if } \bar{\alpha} \in \left(\frac{\pi}{2}, \pi\right]. \end{cases} \quad (5)$$

Moreover, the cable tension at equilibrium is

$$\bar{T} = \begin{cases} \text{any } \mathbb{R}_{>0} & \text{if } \bar{\alpha} = \frac{\pi}{2} \\ mg(\tan(\bar{\alpha} + \bar{\theta}) \cos \bar{\alpha} - \sin \bar{\alpha}) & \text{if } \bar{\alpha} \in [0, \pi] \setminus \left\{\frac{\pi}{2}\right\} \end{cases}$$

Proof. System (4) remains stationary for $\bar{u}_2 = 0$, $\bar{u}_3 = 0$ and \bar{u}_1 satisfying

$$\bar{u}_1 \cos(\bar{\alpha} + \bar{\theta}) = mg \cos \bar{\alpha}. \quad (6)$$

Furthermore, following from equation (3), the taut cable constraint is verified for \bar{u}_1 satisfying

$$\bar{u}_1 \sin(\bar{\alpha} + \bar{\theta}) - mg \sin \bar{\alpha} = \bar{T} \quad (7)$$

where $\bar{T} > 0$ is the cable tension at equilibrium. If $\alpha = \frac{\pi}{2}$, equations (6) and (7) become

$$\begin{cases} \bar{u}_1 \cos\left(\frac{\pi}{2} + \bar{\theta}\right) = 0 \\ \bar{u}_1 \sin\left(\frac{\pi}{2} + \bar{\theta}\right) = mg + \bar{T} \end{cases}$$

which can only be verified with $\bar{\theta} = 0$ and $\bar{u}_1 = mg + \bar{T}$. If $\alpha \neq \frac{\pi}{2}$, condition (6) is satisfied choosing

$$\bar{u}_1 = mg \frac{\cos \bar{\alpha}}{\cos(\bar{\alpha} + \bar{\theta})} \quad (8)$$

which exists if $\bar{\alpha} + \bar{\theta} \neq \frac{\pi}{2}$. The definition $u_1 := f_1 + f_2$ with $f_1, f_2 \in \mathbb{R}_{>0}$ implies $\bar{u}_1 > 0$. This is true only if

$$\text{sign}(\cos(\bar{\alpha} + \bar{\theta})) = \text{sign}(\cos \bar{\alpha}),$$

thus leading to the following bounds

$$\begin{cases} \bar{\theta} \in \left(-\frac{\pi}{2} - \bar{\alpha}, \frac{\pi}{2} - \bar{\alpha}\right) \text{ if } \bar{\alpha} \in \left[0, \frac{\pi}{2}\right] \\ \bar{\theta} \in \left(\frac{\pi}{2} - \bar{\alpha}, -\frac{\pi}{2} - \bar{\alpha}\right) \text{ if } \bar{\alpha} \in \left(\frac{\pi}{2}, \pi\right]. \end{cases} \quad (9)$$

As for condition (7), by substituting expression (8), the cable tension at equilibrium becomes

$$\bar{T} = mg(\tan(\bar{\alpha} + \bar{\theta}) \cos \bar{\alpha} - \sin \bar{\alpha})$$

which is strictly positive if $\tan(\bar{\alpha} + \bar{\theta}) \cos \bar{\alpha} > \sin \bar{\alpha}$. This inequality can be reformulated as

$$\begin{cases} \tan(\bar{\alpha} + \bar{\theta}) > \tan \bar{\alpha} \text{ if } \bar{\alpha} \in \left[0, \frac{\pi}{2}\right] \\ \tan(\bar{\alpha} + \bar{\theta}) < \tan \bar{\alpha} \text{ if } \bar{\alpha} \in \left(\frac{\pi}{2}, \pi\right] \end{cases}$$

Expression (5) is then obtained after a few trigonometric manipulations and taking into account (9). \blacksquare

Proposition 5 can be interpreted by the fact that the set of attainable equilibrium points are such that the thrust vector \vec{u}_1 belongs to the conic combination of vectors $-\vec{T}$ and $-\vec{g}$. This interpretation is shown in Figure 1.

4. CONTROL ARCHITECTURE

Having defined the set \mathcal{S} , the goal of this section is to describe the proposed control strategy. The first step is to adopt a hierarchical cascade approach [Marconi et al.

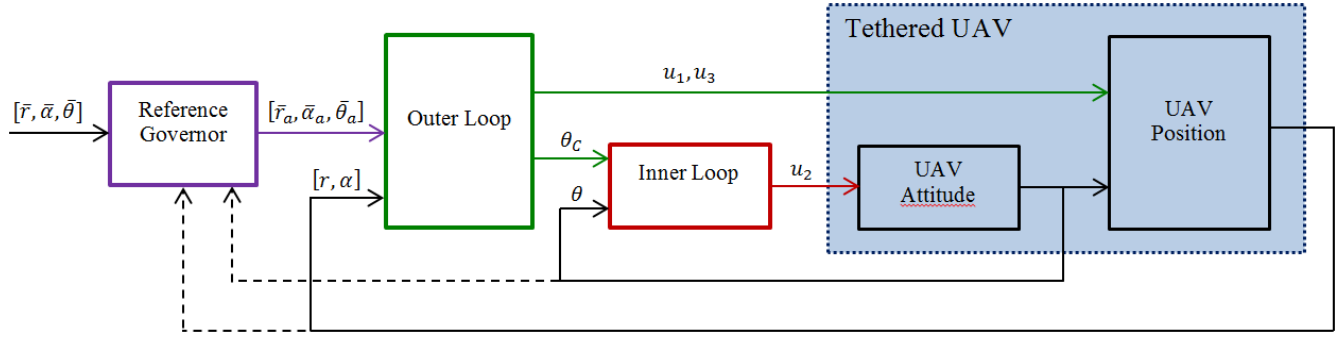


Fig. 2. Proposed control architecture.

(2011)]: the **inner loop** uses u_2 as a control input to obtain $\theta \approx \theta_C$; the **outer loop** then uses u_1, u_3 and θ_C as control inputs for the remaining system. Under the assumption that the inner loop is ideal, the outer loop is designed to asymptotically stabilize the UAV without violating the taut cable constraint. The assumption on the inner loop is then lifted. Stability of the inner/outer loop interconnection is proven using the small gain theorem. Constraint satisfaction is instead enforced by introducing a **reference governor** which, whenever necessary, provides intermediate way-points to the closed-loop so as to limit the error dynamics of the inner loop.

5. OUTER LOOP CONTROL

To design the outer loop control law, consider the auxiliary control variable θ_C as a virtual input for the subsystem

$$\ddot{r} = \rho u_3 \quad (10)$$

$$\ddot{\alpha} = -\frac{1}{r}(2\dot{r}\dot{\alpha} + g \cos \alpha) + \frac{\cos(\alpha + \theta_C)}{mr} u_1 \quad (11)$$

subject to constraint (2). Since the radial dynamics are independent from the rest of the system, they will be controlled separately using a nested saturation control so as to limit the unwinding acceleration of the cable.

Proposition 5. Given the radial dynamics (10) and the control law

$$u_3 = -\frac{1}{\rho} \sigma_{\lambda_1}(k_{Dr}\dot{r} + \sigma_{\lambda_2}(k_{Pr}(r - \bar{r}))), \quad (12)$$

where $\sigma_{\lambda}(\dots)$ is a saturation function with saturation limit λ [see Liu et al. (1996)], then $[\bar{r}, 0]^T$ is a Globally Asymptotically Stable (GAS) equilibrium point for any $\lambda_1 > 0, \lambda_2 > \frac{\lambda_1}{k_{Dr}}, k_{Pr}, k_{Dr} > 0$. Moreover, $|\ddot{r}| \leq \lambda_1$.

Proof. Define $r_1 = r - \bar{r}$ and $r_2 = \dot{r}$. The controlled system is

$$\begin{cases} \dot{r}_1 = r_2 \\ \dot{r}_2 = -\sigma_{\lambda_1}(k_{Dr}\dot{r} + \sigma_{\lambda_2}(k_{Pr}(r - \bar{r}))) \end{cases}$$

The origin is GAS as proven by Marconi and Isidori (2000). Additionally, due to the saturation, $|\ddot{r}| \leq \lambda_1$. ■

Having stabilized the radial position of the UAV, it is necessary to define a control law that makes use of u_1 and θ_C to enforce $\lim_{t \rightarrow \infty} [\alpha(t), \theta(t)] = [\bar{\alpha}, \bar{\theta}]$ without violating the taut cable constraint $T(r, \alpha, \theta) > 0$. Although there are apparently three control objectives and two control inputs, the following lemma shows that the control objectives can be reduced to two independent conditions.

Lemma 6. Let system (1) be under the assumption that $\theta = \theta_C$. Given a desired configuration $[\bar{r}, \bar{\alpha}, \bar{\theta}] \in \mathcal{S}$, if

$$\begin{aligned} T &= \bar{T} + mr\dot{\alpha}^2, \\ \lim_{t \rightarrow \infty} [\alpha(t), r(t)] &= [\bar{\alpha}, \bar{r}] \end{aligned}$$

with

$$\bar{T} = mg(\cos \bar{\alpha} \tan(\bar{\alpha} + \bar{\theta}) - \sin \bar{\alpha}), \quad (13)$$

then $T(t) > 0 \forall t > 0$ and $\lim_{t \rightarrow \infty} \theta(t) = \bar{\theta}$.

Proof. Since reference $[\bar{r}, \bar{\alpha}, \bar{\theta}] \in \mathcal{S}$, it follows from Proposition 5 that $\bar{T} > 0$. Therefore $T = \bar{T} + mr\dot{\alpha}^2$ is strictly positive. Furthermore, substituting the expression of T and applying $\lim_{t \rightarrow \infty} [\alpha(t), r(t)] = [\bar{\alpha}, \bar{r}]$, it follows that

$$\begin{aligned} \lim_{t \rightarrow \infty} \begin{cases} m\ddot{r} = -mg \sin \alpha + \sin(\alpha + \theta_C) u_1 - \bar{T} \\ mr^2 \ddot{\alpha} = -2mrr\dot{\alpha} - mgr \cos \alpha + r \cos(\alpha + \theta_C) u_1 \end{cases} \\ = \begin{cases} 0 = -mg \sin \bar{\alpha} + \sin(\bar{\alpha} + \bar{\theta}) u_1 - \bar{T} \\ 0 = -mgr \cos \bar{\alpha} + \bar{r} \cos(\bar{\alpha} + \bar{\theta}) u_1 \end{cases} \end{aligned} \quad (14)$$

From the second equation of (14) it follows that

$$\lim_{t \rightarrow \infty} u_1(t) = mg \frac{\cos \bar{\alpha}}{\cos(\bar{\alpha} + \bar{\theta})}. \quad (15)$$

Substituting (13) and (15) into the first equation of (14),

$$\lim_{t \rightarrow \infty} mg \cos \bar{\alpha} (\tan(\bar{\alpha} + \bar{\theta}_C) - \tan(\bar{\alpha} + \bar{\theta})) = 0,$$

which, in terms, implies $\lim_{t \rightarrow \infty} \theta(t) = \lim_{t \rightarrow \infty} \theta_C(t) = \bar{\theta}$. ■

Thanks to Lemma 6, the three initial control objectives are reduced to two, making the control problem well-posed.

Proposition 7. Let system (10)-(11) be subject to the constraint (2) and controlled through

$$u_1 = \sqrt{u_T^2 + u_{\alpha}^2} \quad (16)$$

$$\theta_C = \text{atan2}(u_T, u_{\alpha}) - \alpha \quad (17)$$

with

$$u_T = \bar{T} + mg \sin \alpha + m\rho u_3 \quad (18)$$

$$u_{\alpha} = m(2\dot{r}\dot{\alpha} + g \cos \alpha) - mr(k_{P\alpha}(\alpha - \bar{\alpha}) + k_{D\alpha}\dot{\alpha}) \quad (19)$$

and where u_3 and \bar{T} are as in (12) and (13). Given $k_{P\alpha} > 0, k_{D\alpha} > 0$ and $\lambda_1 < \frac{\bar{T}}{m}$ the reference $[\bar{r}, \bar{\alpha}, \bar{\theta}]$ is an asymptotically stable equilibrium and $T(t) > 0 \forall t > 0$.

Proof. Following from Proposition 5, the radial dynamics are GAS. As for the tangential dynamics (11) and the constraint (2), by using $u_T := \sin(\alpha + \theta_C) u_1$ and $u_{\alpha} := \cos(\alpha + \theta_C) u_1$, it follows that

$$\begin{cases} \ddot{\alpha} = -\frac{1}{r}(2\dot{r}\dot{\alpha} + g \cos \alpha) + \frac{1}{mr} u_{\alpha} \\ T = mr\dot{\alpha}^2 - mg \sin \alpha - m\rho u_3 + u_T. \end{cases}$$

Furthermore, by substituting (18) and (19) the system becomes

$$\begin{cases} \ddot{\alpha} = -k_{P\alpha}(\alpha - \bar{\alpha}) - k_{D\alpha}\dot{\alpha} \\ T = \bar{T} + mr\dot{\alpha}^2 \end{cases}$$

which satisfies the conditions given in Lemma 6 since $\alpha(t)$ is subject to a PD control law. As a result, the system asymptotically tends to $[\bar{r}, \bar{\alpha}, \bar{\theta}]$ without violating the taut cable condition. The control inputs u_1 and θ_C can be computed from u_T and u_α by solving

$$u_1 \begin{bmatrix} \sin(\alpha + \theta_C) \\ \cos(\alpha + \theta_C) \end{bmatrix} = \begin{bmatrix} u_T \\ u_\alpha \end{bmatrix}. \quad (20)$$

Note that θ_C is not defined if $u_T = u_\alpha = 0$. However, this condition is never verified since $u_3 \leq \frac{1}{\rho}\lambda_1 < \frac{1}{\rho}\frac{\bar{T}}{m}$ substituted in (18) implies $u_T \geq \bar{T} > 0$. ■

6. INNER LOOP CONTROL

So far, the design of the outer loop has been done under the assumption that $\theta_C = \theta$. The goal of this section is to study what happens in the presence of an attitude error $\tilde{\theta} = \theta - \theta_C$ and show how the stability of the interconnected loops can be achieved. As a first step, consider the dynamics of α (4) and the taut cable constraint (2) when $\theta = \theta_C + \tilde{\theta}$,

$$\begin{aligned} \ddot{\alpha} &= -\frac{1}{r}(2\dot{r}\dot{\alpha} + g \cos \alpha) + \frac{1}{mr} \cos(\alpha + \theta_C + \tilde{\theta}) u_1 \\ T &= mr\dot{\alpha}^2 - mg \sin \alpha + \sin(\alpha + \theta_C + \tilde{\theta}) u_1 - m\ddot{r}. \end{aligned} \quad (21)$$

After some trigonometric manipulations and taking into account (18)-(20), expression (21) becomes

$$\ddot{\alpha} = \kappa(\tilde{\alpha}, \dot{\alpha}) - \frac{1}{mr} u_\alpha (1 - \cos \tilde{\theta}) - \frac{1}{mr} u_T \sin \tilde{\theta} \quad (22)$$

$$T = mr\dot{\alpha}^2 + \bar{T} - u_T (1 - \cos \tilde{\theta}) + u_\alpha \sin \tilde{\theta} \quad (23)$$

where $\kappa(\tilde{\alpha}, \dot{\alpha}) = -k_{P\alpha}(\tilde{\alpha}) - k_{D\alpha}\dot{\alpha}$ and $\tilde{\alpha} = \alpha - \bar{\alpha}$. Equations (22)-(23) clearly show that $\tilde{\theta} \neq 0$ could destabilize the outer loop dynamics or lead to violations of the taut cable constraint. The two problems will be addressed separately: this section will use small-gain arguments to guarantee the stability of the inner/outer loop interconnection. The taut cable constraint will instead be enforced in the next section.

The inner loop is controlled with a simple PD. The following proposition discusses the properties of the inner loop subject to a time-varying reference $\theta_C(t)$.

Proposition 8. The system $\ddot{\theta} = \frac{1}{J}u_2$ subject to the control law

$$u_2 = -\mathcal{J} \left(k_{P\theta}\tilde{\theta} + k_{D\theta}\dot{\theta} \right) \quad (24)$$

with $k_{D\theta} = 2\zeta\sqrt{k_{P\theta}}$, $k_{P\theta} > 0$ and $\zeta \in (0, 1)$ is Input to State Stable (ISS) with respect to $\dot{\theta}_C$. Moreover, the asymptotic gain between $\dot{\theta}_C$ and $\tilde{\theta}$ can be made arbitrarily small given a sufficiently large $k_{P\theta}$.

Proof. Define $\theta_1 = \tilde{\theta}$ and $\theta_2 = \dot{\theta}$. The system may be rewritten as

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_{P\theta} & -2k_{D\theta} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \dot{\theta}_C \quad (25)$$

which is a GAS linear system. The remainder of the proof follows from Sontag (2004). ■

The following proposition guarantees the stability of the interconnected inner/outer loops.

Proposition 9. Given system (4) subject to the control laws (12), (17), (16) and (24) under the assumption that $\|\tilde{\theta}\|_\infty < \frac{\pi}{2}$, the set-point $[\bar{r}, \bar{\alpha}, \bar{\theta}] \in \mathcal{S}$ is Asymptotically Stable (AS) for a sufficiently large $k_{P\theta}$.

Proof. Consider the interconnection of the outer loop dynamics (10), (22) and the inner loop dynamics $\tilde{\theta}(t)$

$$\begin{cases} \dot{r}_1 = r_2 \\ \dot{r}_2 = -\sigma_{\lambda_1}(k_{D_r}r_2 + \sigma_{\lambda_2}(k_{P_r}r_1)) \\ \dot{\alpha}_1 = \alpha_2 \\ \dot{\alpha}_2 = -k_{P_\alpha}\alpha_1 - k_{D_\alpha}\alpha_2 + \Gamma_1(\alpha, r, y_{In}) \\ y_{Out} = \dot{\theta}_C \end{cases} \quad (26)$$

$$\begin{cases} \dot{\theta}_1 = \theta_2 + y_{Out} \\ \dot{\theta}_2 = -k_{P_\theta}\theta_1 - k_{D_\theta}\theta_2 \\ y_{In} = \theta_1 \end{cases} \quad (27)$$

where $r_1 = r - \bar{r}$, $r_2 = \dot{r}$, $\alpha_1 = \alpha - \bar{\alpha}$, $\alpha_2 = \dot{\alpha}$

$$\dot{\theta}_C = \frac{d}{dt} \text{atan2}(u_T, u_\alpha) - \alpha_2 = \frac{u_\alpha \dot{u}_T - u_T \dot{u}_\alpha}{u_T^2 + u_\alpha^2} - \alpha_2$$

$$\Gamma_1(\alpha, r, y_{In}) = -\frac{1}{mr} (u_\alpha (1 - \cos y_{In}) + u_T \sin y_{In}).$$

Following from Proposition 5, $r(t)$ asymptotically tends to \bar{r} regardless of all the other states. As for $\alpha(t)$, by substituting (19) with $\dot{r} \rightarrow 0$ and $\|y_{In}\|_\infty < \frac{\pi}{2}$, its dynamics becomes

$$\dot{\alpha}_2 = -\tilde{k}_{P_\alpha}\alpha_1 - \tilde{k}_{D_\alpha}\alpha_2 - \frac{u_T}{mr} \sin y_{In} - rg(1 - \cos y_{In})$$

where $\tilde{k}_{P_\alpha} = k_{P_\alpha} \cos \|y_{In}\|_\infty$ and $\tilde{k}_{D_\alpha} = k_{D_\alpha} \cos \|y_{In}\|_\infty$. Since u_T and r are bounded, the system $[\alpha_1, \alpha_2]^T$ is clearly ISS with respect to $y_{In} < \frac{\pi}{2}$. Moreover, since y_{Out} is a limited function of $\alpha(t)$ and $r(t)$, subsystem (26) has a finite asymptotic gain γ between y_{In} and y_{Out} . As for subsystem (27), Proposition 8 shows that it is ISS with respect to y_{Out} with an arbitrarily small gain χ given a sufficiently large $k_{P\theta}$. Since $y_{In} = \theta_1$, the asymptotic gain between y_{Out} and y_{In} is equal to χ . Therefore, it is possible to choose a sufficiently large $k_{P\theta}$ such that $\gamma\chi < 1$. The asymptotic stability of the origin follows from the application of the small gain theorem Khalil (1996). ■

Although Proposition 9 proves that the inner loop can be designed so that system (4) asymptotically tends to the set-point $[\bar{r}, \bar{\alpha}, \bar{\theta}]$, the obtained results are regional and give no guarantee that the cable will remain taut at all times. Both problems will be addressed in the following section by generating a suitable sequence of way-points so as to limit the attitude error and recover the properties of the outer loop.

7. REFERENCE GOVERNOR

The Reference Governor (RG) is a reference management scheme that, whenever necessary, substitutes the desired reference $[\bar{r}, \bar{\alpha}, \bar{\theta}]$ with a succession of applied way-points $[\bar{r}_a, \bar{\alpha}_a, \bar{\theta}_a]_k$ so as to prevent the violation of constraints. A survey on the subject is presented by Kolmanovsky et al. (2014). This paper will focus on the RG strategy for nonlinear systems proposed by Bemporad (1998). The idea is as follows.

First of all, the set of attainable set-points is reduced so as to obtain a safety margin T_{min} .

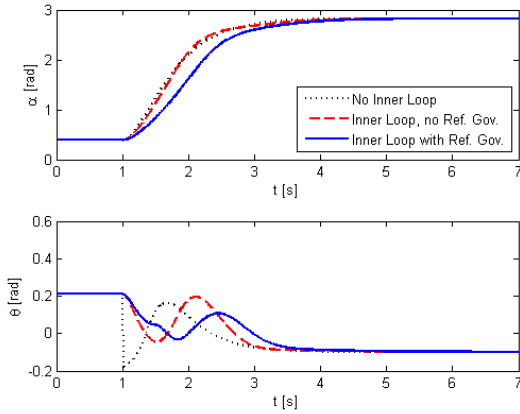


Fig. 3. Tether angle and UAV attitude during Task 1.

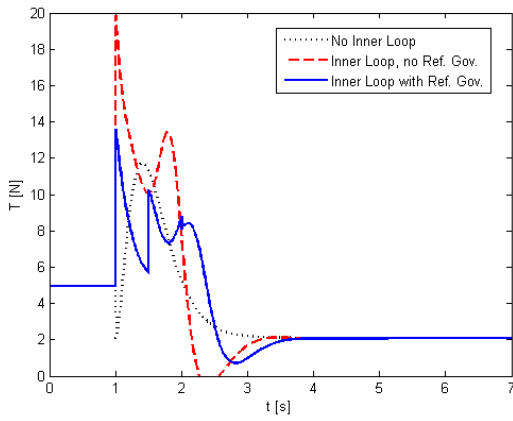


Fig. 4. Cable tension $T(t)$ during Task 1.

Definition 10. The set of attainable configurations $\mathcal{S}_{T_{\min}}$ is defined as the set of equilibrium points $[\bar{r}, \bar{\alpha}, \bar{\theta}] \in \mathbb{R}^3$ such that $T(\bar{r}, \bar{\alpha}, \bar{\theta}) \geq T_{\min} > 0$.

Given the desired set-point $[\bar{r}, \bar{\alpha}, \bar{\theta}] \in \mathcal{S}_{T_{\min}}$ and assuming that the currently applied reference $[\bar{r}_a, \bar{\alpha}_a, \bar{\theta}_a]_k \in \mathcal{S}_{T_{\min}}$ does not lead to constraint violation if maintained constant, the RG computes at regular time intervals a new way-point

$$[\bar{r}_a, \bar{\alpha}_a, \bar{T}_a]_{k+1} = (1 - c) [\bar{r}_a, \bar{\alpha}_a, \bar{T}_a]_k + c [\bar{r}, \bar{\alpha}, \bar{T}].$$

The parameter $c \in [0, 1]$ is maximized under the condition that if the new way-point is maintained constant, the system will not violate the constraints. The optimization of c is performed via online simulations over a sufficiently long receding horizon.

Since the constraints are convex, the only thing left to prove is that the RG will never get stuck in an intermediate way-point. Proposition 11 will show that if the system state is in proximity of the current way-point, it is always possible to change the way-point in such a way that the closed-loop system will not violate the constraints.

Proposition 11. Let system (4) be subject to the control laws (16), (24), (12) and (17). Given the initial coordinates $[r(0), \alpha(0), \theta(0)] \in \mathcal{S}$ and the initial velocities $|\dot{r}(0)| \leq \dot{r}_{\max}$, $|\dot{\alpha}(0)| \leq \dot{\alpha}_{\max}$, $|\dot{\theta}(0)| \leq \dot{\theta}_{\max}$, there exists a region of way-points $[\bar{r}_a, \bar{\alpha}_a, \bar{\theta}_a] \in \mathcal{S}_{T_{\min}}$ such that $T(t) > 0 \forall t \geq 0$ and $\|\tilde{\theta}\|_{\infty} < \frac{\pi}{2}$.

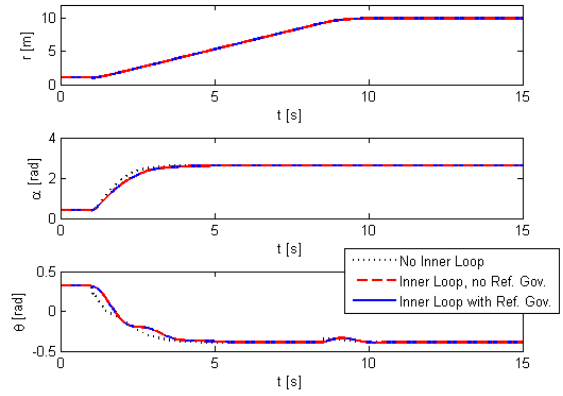


Fig. 5. System evolution during Task 2.

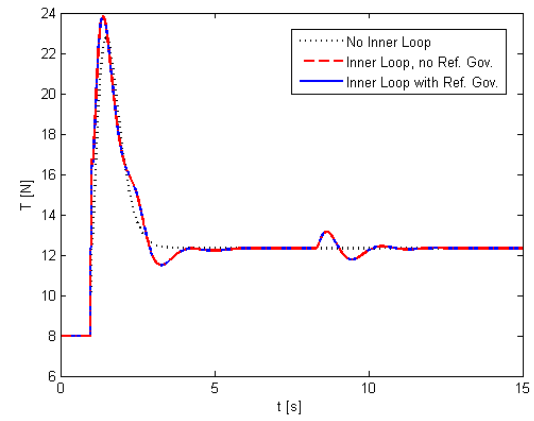


Fig. 6. Cable tension during Task 2.

Proof. See Appendix.

By proceeding from one way-point to the next, the governed system is able to move freely between any two references that belong to the set $\mathcal{S}_{T_{\min}}$.

8. SIMULATIONS

Consider a planar UAV of mass $m = 2 [kg]$ and moment of inertia $\mathcal{J} = 0.015 [kg m^2]$ attached to a winch of radius $\rho = 0.1 [m]$. The system is subject to the control law (16), (24), (12) and (17) with $k_{Pr} = k_{P\alpha} = 6$, $k_{P\theta} = 30$ and $\zeta = 0.9$. To show the effectiveness of the proposed control strategy, two different tasks are shown. In the first task, the control law must steer the UAV from its current configuration $r(0) = 1 [m]$, $\alpha(0) = \frac{\pi}{8}$ and $\theta(0) = \frac{\pi}{15}$ to the desired reference $\bar{r} = 1 [m]$, $\bar{\alpha} = \frac{9\pi}{10}$ and $\bar{\theta} = -\frac{\pi}{32}$. Figures 3 and 4 depict the cable tension $T(t)$ and the angles $\alpha(t)$ and $\theta(t)$, respectively. Three different control schemes are compared: **No Inner Loop**, **Inner Loop without RG**, **Inner Loop with RG**. The reference was updated every 0.5 seconds. The simulations clearly show that if the inner loop is not ideal, the RG is needed to enforce the taut cable constraint. In the second task, the control law is charged with steering the UAV from the initial condition $r(0) = 1 [m]$, $\alpha(0) = \frac{\pi}{8}$ and $\theta(0) = \frac{\pi}{10}$ to the set-point $\bar{r} = 10 [m]$, $\bar{\alpha} = \frac{5}{6}\pi$ and $\bar{\theta} = -\frac{\pi}{8}$. In this case, the RG is not required and, as shown in Figures 5 and 6, the system asymptotically tends to the desired set-point without violating the taut cable constraint.

9. CONCLUSIONS

This paper provides a novel approach for the study of tethered UAVs in the taut cable configuration. The cable tension is modeled as a reaction force caused by a mechanical constraint. The system dynamics are then obtained under the hypothesis that the taut cable condition is verified at all times. The attainable equilibrium points are discussed and interpreted geometrically. An inner/outer loop control strategy is developed with the dual objective of controlling the UAV and guaranteeing the taut cable condition. The outer loop is designed to automatically satisfy the constraints given under the assumption of an ideal inner loop. The inner loop error dynamics are then accounted for using a reference governor to avoid constraint violation. Future work will aim at the extension to the three-dimensional case as well as the investigation of a more advanced reference governor strategy to improve the system response.

APPENDIX

The proof of Proposition 11 requires the following Lemma:
Lemma 12. Let system (4) be controlled with (16), (24), (12) and (17). For each applied set-point $[\bar{r}_a, \bar{\alpha}_a, \bar{\theta}_a] \in \mathcal{S}_{T \min}$, there exists a maximum attitude error $0 < \Theta < \frac{\pi}{2}$ that ensures the control objectives, i.e.

$$\|\tilde{\theta}\|_{\infty} < \Theta \Rightarrow \begin{cases} \lim_{t \rightarrow \infty} \alpha(t) = \bar{\alpha} \\ \dot{T}(t) > 0 \forall t > 0 \end{cases}.$$

Furthermore, Θ increases for decreasing norms of the velocities $|\dot{r}(0)|, |\dot{\alpha}(0)|, |\dot{\theta}(0)|$ and offsets $|\bar{r}_a - r(0)|, |\bar{\alpha}_a - \alpha(0)|, |\bar{\theta}_a - \theta(0)|$.

Proof. Considering expression (23), it is possible to guarantee a positive cable tension if $\|\tilde{\theta}\|_{\infty} < \Theta$ where $\Theta = \min_{i=1,2} (|\Theta_i|, \frac{\pi}{2})$ and Θ_1, Θ_2 are the solutions of

$\|u_T\|_{\infty} (1 - \cos \Theta_i) + \|u_{\alpha}\|_{\infty} \sin \Theta_i = \bar{T}_a$. Since $\hat{T} \geq T_{\min} > 0$, then $\Theta > 0$ is a decreasing function of $\|u_T\|_{\infty}$ and $\|u_{\alpha}\|_{\infty}$. Due to expressions (18)-(19), $\|u_T\|_{\infty}$ and $\|u_{\alpha}\|_{\infty}$ are bounded by $\|\bar{r}_a - r\|_{\infty}, \|\dot{r}\|_{\infty}, \|\bar{\alpha}_a - \alpha\|_{\infty}$ and $\|\dot{\alpha}\|_{\infty}$. Following from Proposition 9, $[\bar{r}_a, \bar{\alpha}_a, \bar{\theta}_a]$ is AS, therefore the infinity norms are bounded by the initial conditions. As a result, given a decreasing norm of the initial conditions, the acceptable error Θ increases. ■

Proof of Proposition 11: Following from Proposition 9, if $\|\tilde{\theta}\|_{\infty} \leq \frac{\pi}{2}$, the interconnection between the inner and outer loop verifies the small gain theorem. Under these conditions, the following expression holds true

$$\|\tilde{\theta}\|_{\infty} = \|y_{\text{In}}\|_{\infty} \leq \frac{1}{1 - \gamma\chi} (\beta_{\text{In}} + \chi\beta_{\text{Out}})$$

where β_{In} is the free output response of the inner loop initialized in $[\tilde{\theta}(0), \dot{\theta}(0)]$ whereas β_{Out} is the free output response of the outer loop initialized in $[r(0) - \bar{r}_a, \dot{r}(0), \alpha(0) - \bar{\alpha}_a, \dot{\alpha}(0)]$. Since $\|\tilde{\theta}\|_{\infty}$ depends on the initial conditions of the system, the objective is to show that there exists $[\bar{r}_a, \bar{\alpha}_a, \bar{\theta}_a]$ such that $\|\tilde{\theta}\|_{\infty} < \Theta$ where $0 < \Theta < \frac{\pi}{2}$ is the acceptable error discussed in

Lemma 12. Since the free system responses of (26)-(27) are linear, there exists a finite forward invariant set of initial conditions $\Omega_{\text{In}} = \{\tilde{\theta}, \dot{\theta}\}$ such that

$$[\tilde{\theta}(0), \dot{\theta}(0)] \in \Omega_{\text{In}} \Rightarrow \beta_{\text{In}} (\tilde{\theta}(0), \dot{\theta}(0)) < (1 - \gamma\chi) \Theta.$$

Likewise, there exists a finite forward invariant set $\Omega_{\text{Out}} = \{\tilde{r}, \dot{r}, \tilde{\alpha}, \dot{\alpha}\}$ such that

$$[r(0) - \bar{r}_a, \dot{r}(0), \alpha(0) - \bar{\alpha}_a, \dot{\alpha}(0)] \in \Omega_{\text{Out}}$$

$$\Downarrow$$

$$\beta_{\text{In}} (r(0) - \bar{r}_a, \dot{r}(0), \alpha(0) - \bar{\alpha}_a, \dot{\alpha}(0)) \leq \frac{1 - \gamma\chi}{\chi} \Theta - \frac{1}{\chi} \beta_{\text{In}}.$$

By imposing $\dot{\theta}_{\max} \in \Omega_{\text{In}}, \dot{r}_{\max} \in \Omega_{\text{Out}}$ and $\dot{\alpha}_{\max} \in \Omega_{\text{Out}}$, it is possible to choose a way-point $[\bar{r}_a, \bar{\alpha}_a, \bar{\theta}_a] \in \mathcal{S}_{T \min}$ such that Ω_{In} and Ω_{Out} contain $[r(0), \alpha(0), \theta(0)]$. ■

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