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Approximating the Time Optimal Solution for Accessible Unknown Systems by Learning using a Virtual Output

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Abstract: Time optimal control of systems with bounded inputs is a numerically awkward problem as essentially a whole trajectory has to be designed on the basis of a single final point at unknown time. Analytical solutions are possible only for very few problems, in general numerical techniques will be needed, which, in view of the non convexity of the associated optimization problem, will typically converge to a local minimum. In case of an unknown but experimentally accessible nonlinear system, such a solution cannot be found numerically, but a learning algorithm has been proposed which typically converges to a solution not far from the optimal one. However, as in the case of numerical computations, no guarantee of global optimality can be given. In the numerical case, different randomization techniques can be used to ascertain the existence of better solutions, e.g. by choosing different initial conditions. This is more difficult in the case of an experimental method for unknown systems, but this paper proposes an approach based on virtual outputs (linear combination of measurable states) for the same goal which is shown to work in a classical problem of counterintuitive time optimal control - the fastest climbing of a plane.

Keywords: Time optimal control, aircraft control, iterative methods, learning control.

1. INTRODUCTION

Time optimal design has not received as much attention as the classical optimal control problem of the minimization of a cost function defined along a trajectory, as in the case of a standard LQR (Linear Quadratic Regulator) design. Roughly speaking, the challenge in time optimal control consists in designing a full trajectory based on one point unknown in time. In this paper with time optimality the minimal time to reach a certain time point is meant (cost function consisting only of an integral over time).

However, in view of its practical importance, different methods have been conceived and used for all kind of applications from biological (Moreno [1999]), chemical process (Schroeder and Mendes [1999]) to automotive challenges (Ortner et al. [2009]) or robotics (Bobrow et al. [1985]). The analytical solution is in general based on the gradient of the cost function, typically using the Euler Lagrange condition. For some problems, like linear systems with bounded inputs, Pontryagin's Maximum (or Minimum) Principle Pontryagin et al. [1962] can be used to derive a condition on the input. In general, however, the associated optimization problem will not be convex and an analytical solution is not possible, so that standard optimization methods (like multiple shooting Marler and Arora [2004], dynamic programming Bryson [1999], Stengel [1994]) can be used. This does not change the basic problem that a global optimum of a non convex function cannot be determined for sure in almost all cases, so that convergence of the numerical algorithms does not imply that the "real" time optimal value has been found. There are of course several methods which can be used to explore other solutions, hoping to get a better convergence, e.g. by variations of the initial conditions or variations of the input and change of the cost function (see e.g. Hintermüller et al. [2002] or Molga and Smutnicki [2005]).

All these methods are based on explicit models, but in many practical applications the control engineer is confronted with the need to estimate the minimum achievable transition time in order to allow a decision in terms of hardware design. In such a case, an explicit model is hardly available, and empirical methods are typically used, e.g. a parametrized family of possible solutions is established a priori and then experiments are performed on the prototype in order to determine the optimal weights. In the field of point-to-point control some model free versions are available as in Janssens et al. [2011], however such methods are restricted to specific classes, like linear time invariant systems. A time optimal result for such a system can be obtained by using Pontryagin and learning the switching point. Other point-to-point methods, like in Freeman and Tan [2012] require again a model of the system to compute the time optimal solution.

A new approach has been introduced in Trogmann and del Re [2012], which starts by requesting the system to track the ideal but almost always unfeasible response in terms of time optimality - a step change - and then relaxes it as far as necessary to allow the system to track it. This method has been found to converge to a sensible solution, and in practice this solution is frequently near to the time optimal one which can be calculated if a model and sufficient computational time is available - for instance it was shown even to recover the bang-bang behavior expected for nonlinear input affine systems (see Trogmann et al. [2013]). However, there is no guarantee that the solution is the global optimum, exactly as in the case of numerical, model based optimization. In addition a general statement of how near the obtained approximated solution lays to the "real" optimum cannot be given analytical and theoretical. Only an experimental value can be given and will be shown later in the paper.

Against this background, it would be interesting to explore possible "better" optima not immediately resulting from the straightforward application of the method. As the method relies on an internal ILC (Iterative Learning Control) loop to enforce repeatability of the experiments, some techniques are not easily implementable. In this paper, we suggest using appropriate variations of the target function, to obtain a learning quantity which can be used for the trajectory update algorithm presented in Trogmann and del Re [2012]. As a virtual output in this paper a linear combination of measurable states of the system is meant, so that the output does not exist really, however it is possible to compute it out if measurements. In addition it will be shown that it works again for cases, for which it would fail in standard configuration of the method. The rationale underlying of this idea is a special property of time-optimal control, for which only the initial and final point are specified, but the intermediate values are free. In other terms, any target function which ensures the desired transition will do.

The paper is structured as follows. First in Section 2 we recall a known example of counter-intuitive time-optimal solution from Bryson and Denham [1962], then in Section 3 we discuss the classical time optimal solution and show the non convexity of the problem, then in Section 5 we use our method on the problem and on variations of it and in Section 6 finally show that changing the target function does lead to a better solution in terms of time optimality. As last part a conclusion is given in Section 7.

2. MOTIVATING EXAMPLE

A particularity of the used aircraft (which has been presented in Bryson and Denham [1962]) is the necessary interruption in the ascent to gain additional speed. The result of this maneuver is the time optimal solution to reach the desired final point (flight level: 30 and 300$\frac{m}{s}$ speed). By using the standard approach to approximate the time optimal solution, this would fail, as the method does not recognize a necessary descent after a certain point has been reached. So to cope with this phenomena a change of the learning quantity will be necessary. Without knowing the system it can not be assumed that the system is strict increasing the output to obtain the time optimal solution. For this it will be necessary to find a strict increasing quantity to reach the time optimal solution.

To be able to compare the result of the learning method and the standard optimization methods a model of the aircraft is used. For simplicity a 3 degree of freedom model of the aircraft is build up. In such a model the number of forces acting on the aircraft are reduced as it can be seen in Figure 1 to lift, drag, weight and thrust. The aircraft can be described basically by two differential equation for aircraft speed and pitch respectively,

$$\begin{aligned} \dot{v} &= \frac{F(h,M)}{m}\cos\alpha - \frac{D(h,M,\alpha)}{m} - g\sin\gamma \\ \dot{\gamma} &= \frac{L(h,M,\alpha)}{mv} + \frac{F(h,M)}{mv}\sin\alpha - \frac{g}{v}\cos\gamma \end{aligned} \tag{1}$$

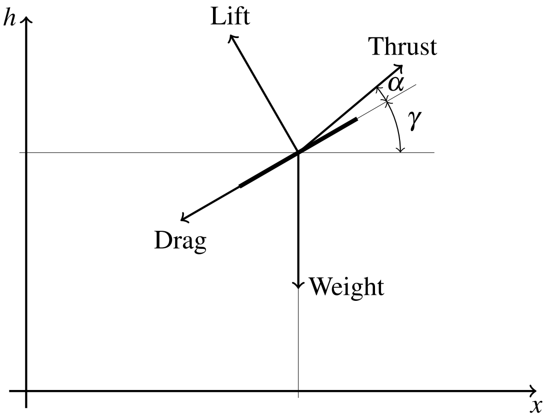

Fig. 1. Acting Forces on the air plane, α angle of attack, γ pitch of the aircraft

with $F(h,M)$ the thrust as tabular function, Figure 2, M the Mach number depending on the height, $D(h,M,\alpha)$ the drag value calculated out of (3), g the gravity depending on the height, α the angle of attack as input and $L(h,M,\alpha)$ the lift value calculated in (5). For simplicity the time dependency of the variables in (1) has been omitted. Out of these two essential

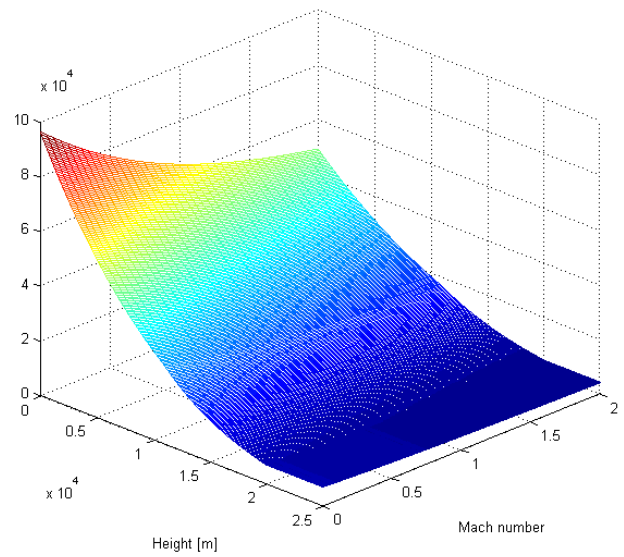

Fig. 2. Maximal thrust of the engine depending on height and Mach number

states of the aircraft 3 important additional states the height h, the position over ground x and the mass are calculated,

$$\begin{aligned} \dot{h} &= v\sin\gamma \\ \dot{x} &= v\cos\gamma \\ \dot{m} &= \dot{m}(h,M) \end{aligned} \tag{2}$$

with $\dot{m}$ as a function of height and velocity, which is available as a tabular function. To obtain the drag, the drag coefficient is needed, in general it is a tabular function valid only for a specific air plane, however it is possible to calculate it depending on the angle of attack (α) and the Mach number (M).

$$D(h,M,\alpha) = \left(C_{D0} + \eta\frac{dC_L}{d\alpha}\alpha^2\right)\frac{\rho v^2 A_{Drag}}{2} \tag{3}$$

with C_{D0}, η and $\frac{dC_L}{d\alpha}$ as tabular functions Figure 3

$$\frac{dC_L}{d\alpha} = \begin{cases} 3.5 & M < 0.8 \\ 3.5 + \sin^2\left(\frac{M-0.8}{0.4}\pi\right) & 0.8 \le M \le 1.02 \\ 4.475 - 2.2\sqrt{1 - \left(\frac{M_{max}-M}{0.98}\right)^2} & 1.02 \le M \end{cases}$$

$$\eta = \begin{cases} 0.55 & M < 0.7 \\ 0.55 + 0.15\left(\frac{M-0.7}{0.2}\right)^6 & 0.7 \le M \le 0.9 \\ 0.7 + 0.25\sqrt{1 - \left(\frac{M_{max}-M}{1.1}\right)^2} & 0.9 \le M \end{cases}$$

$$C_{D0} = \begin{cases} 0.013 & M < 0.8 \\ 0.013 + 0.027\sin^2\left(\frac{M-0.8}{0.8}\pi\right) & 0.8 \le M \le 1.25 \\ 0.039 - 0.004\sqrt{1 - \left(\frac{M_{max}-M}{0.75}\right)^2} & 1.25 \le M \end{cases} \tag{4}$$

The lift coefficient is calculated in a similar way by using the

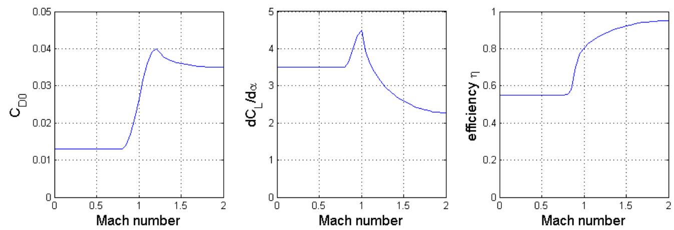

Fig. 3. Single coefficients to calculate the drag and lift coefficient and further the corresponding force.

same tabular function for $\frac{dC_L}{d\alpha}$

$$L(h,M,\alpha) = \frac{dC_L}{d\alpha}\alpha\frac{\rho v^2 A_{Lift}}{2}. \tag{5}$$

As already mentioned air pressure, density and gravity depend on the actual height of the aircraft, more details can be found in Roskam [1995].

In the normal case the pilot would control besides the angle of attack, the thrust of the aircraft. For the comparison the propulsion will not be controlled by the method, it will be set to the maximal value for the actual flight position and aircraft velocity.

As this model is highly complex as the original system it would be necessary to see how the standard time optimal control looks like.

3. STANDARD TIME OPTIMAL SOLUTION

Is it possible to obtain a solution using the standard methods for time optimal control. To answer this question and for the following observations a system in the form

$$\begin{aligned} \dot{x} &= f(x,u,t) \\ y &= g(x,t) \end{aligned} \tag{6}$$

is used.

In general, an optimization problem can be stated in the form

$$J = \int_0^T \Phi(x,u,t)\,dt + \Gamma(x(T),T) \tag{7}$$

consisting of a time depend part $\Phi(x,u,t)$ and a terminal cost part $\Gamma(x(t),T)$ and additional constraints like

$$\begin{aligned} y(T) &= g(x(T),T) = y^* \\ Ax(t) &\le b \end{aligned} \tag{8}$$

where $g(\cdot)$ is the output function, y^* the required end point and - in this example - linear conditions for the states x are used specified by A and b. The optimization itself consists of a minimization of the cost function (7) by respecting the constraints (8) and will be written as

$$\begin{aligned} &\min_u \quad J \\ &s.t. y(T) = y^* \\ &Ax(t) \le b \end{aligned} \tag{9}$$

with the already presented constrained variables.

Analytical methods like the Hamiltonian can be used when the system is known and a description is available. With the Hamiltonian in the form

$$H(x,u,t,\lambda_0,\lambda) = \lambda_0\Phi(x,u,t) + \lambda^T f(x,u,t) \tag{10}$$

with λ the states of the auxiliary system, an optimal solution can be obtained. It is well known that from the first variation of the cost function J a condition on optimality is derived

$$\frac{\partial H(x,u,t,\lambda)}{\partial u} = 0. \tag{11}$$

and on convexity which is given if

$$\frac{\partial^2 H(x,u,t,\lambda)}{\partial u^2} > 0 \tag{12}$$

is valid $\forall x^*$ and $t \in [t_0, t_f]$.

For linear and input affine nonlinear systems it is not possible to calculate the optimal input using condition (11). In the case of time-invariant linear systems $\dot{x} = Fx + Bu$ (11) is constant

$$\frac{\partial H(x,u,t,\lambda)}{\partial u} = \lambda^T B \tag{13}$$

and convexity is not given as the second partial deviation is zero. A similar result is obtained for input affine time-invariant nonlinear systems $\dot{x} = f(x) + g(x)u$, in this case (11) gets to

$$\frac{\partial H(x,u,t,\lambda)}{\partial u} = \lambda^T g(x) \tag{14}$$

and the second partial derivative is zero. The solution for this two cases is obtained by a constrained input and Pontryagin's Minimal Principle.

In the case of general time-invariant nonlinear systems $\dot{x} = f(x,u)$ it is necessary to calculate the first and second partial derivative of the Hamiltonian. The condition gets to

$$\frac{\partial H(x,u,t,\lambda)}{\partial u} = \lambda^T \frac{\partial f(x,u)}{\partial u} \tag{15}$$

For the aircraft in Section 2 it can be shown that the optimal control input can not be computed out of (15). Indeed we obtain

$$\begin{aligned} \frac{\partial H(x,u,t,\lambda)}{\partial \alpha} &= \lambda_1\left(-\frac{F(h,M)}{m}\sin\alpha - \frac{1}{m}2\alpha\eta\frac{dC_L}{d\alpha}\frac{\rho v^2 A_{Drag}}{2}\right) \\ &+ \lambda_2\left(\frac{dC_L}{d\alpha}\frac{\rho v^2 A_{Lift}}{2mv} + \frac{F(h,M)}{mv}\cos\alpha\right). \end{aligned}$$

As the result of this equation should be zero for all $t \in [0,T]$ independently of λ_1 and λ_2, it is not possible to obtain an α to ensure that both parts are zero.

The convexity in this case depends mainly on the sign of the auxiliary system λ

$$\begin{aligned} \frac{\partial^2 H(x,u,t,\lambda)}{\partial \alpha^2} &= \lambda_1\left(-\frac{F(h,M)}{m}\cos\alpha - \frac{1}{m}2\eta\frac{dC_L}{d\alpha}\frac{\rho v^2 A_{Drag}}{2}\right) \\ &+ \lambda_2\left(-\frac{F(h,M)}{mv}\sin\alpha\right) \end{aligned}$$

under the assumption of arbitrary values of λ convexity is not given if

- λ_1 and λ_2 are positive
- $\lambda_1 = 0$ and $\alpha\lambda_2 > 0$

for single time points $t \in [0,T]$.

Time optimal control is just a special kind of optimal control defined in the form

$$\begin{aligned} \Phi(x,u,t) &= 1 \\ \Gamma(x,T) &= c\,(x(T) - x^*) \end{aligned} \tag{16}$$

with c as a weighting of the final deviation.

Of course, all preceding remarks on non convexity apply here as well. In this form the time is regarded explicitly and the minimum of the cost function will return the time optimal solution for the given system. The difficulty for this type of optimal control is to find the connection between the input u (which is used to minimized the cost function) and the time T. One solution would be if it is possible to compute

$$\frac{dT}{du} \tag{17}$$

so it would be possible to determine how a change of the input would affect the end time to reach the desired point. Another possibility is using the Hamiltonian which provides for input affine system a simple solution by using Pontryagin's Maximal Principle.

4. LEARNING APPROACH

The method Trogmann and del Re [2012] consists of two ILC loops, an inner loop to control the unknown system by knowing only some essential parts of the system and an outer loop acting in some sense as a serial ILC. For the ILC there are two time axis and so it is necessary to introduce the following notation $z_i(k)$ represents the variable $z(t)$ at time $k\tau$ (where τ is the sampling time) at the i-th iteration.

A key difference to standard methods is that the learning method does not use a cost function depending on time. Basically, if the inner ILC receives as reference feasible trajectory, it forces the input to behave like the optimal one. In the case of an unknown system, a first attempt consists in using the fastest possible reference, the step, which, however is almost always unfeasible. An unfeasible point can be detected by using the actual and an initial output error e_1 e.g. using a barrier function like

$$\begin{aligned} f_{change}(k) &= \frac{\delta}{\|e_l(k) - e_{l+1}(k)\|} \\ \bar{f}_{change} &= \min\left\{\max\left\{f_{change}, 0\right\}, 1\right\} \end{aligned} \tag{18}$$

where $\bar{f}_{change}$ denotes the limited changing factor of each time point k, l the iteration index of the update loop and δ is a factor for the steepness when the barrier (e_l) is hit. Depending on the value of $\bar{f}_{change}(k)$ the new reference value is computed as

$$ref_{m+1}(k) = \left(1 - \frac{\bar{f}_{change}(k)}{2}\right) ref_m(k) + \frac{\bar{f}_{change}(k)}{2} y_m(k) \tag{19}$$

with y_m the output of the m^{th} iteration.

A possible drawback for this methods is that for a remaining initial error, transients during the learning can occur. In Longman [2000] the phenomena is explained in more detail the reason for this is that in the case of unknown systems it can not be ensured by the design of the ILC that all errors converge to zero. Especial in the case of unfeasible trajectories and trajectory points. To avoid this, it is essential that if

$$\|e_{i+1}\|_2 < \|e_i\|_2 \tag{20}$$

with i as iteration index of the inner ILC, is violated the inner loop has to stop and a trajectory update is needed.

The critical point in the design of the algorithm is the selection of the right learning quantity. An intuitive selection would be using the real target quantity. However, in the example shown in Section 2, the method does converge to the desired final point, however the resulting solution is not time optimal and a final time of $800s$ is returned (see Figure 4).

5. ADAPTATION OF THE LEARNING METHOD

The counterintuitive time optimal solution of the basic problem is well known, as it can be found in Bryson and Denham [1962]. The solution for the time optimal challenge for the aircraft ascent is to find the right combination of pitch and velocity over the time interval $[0,t_0]$ as these are the main components influencing the height (2). To be able to gain height a pitch > 0 is needed and by taking a closer look to (1) it can be seen that a positive value for the angle of attack (α) increases the pitch and at the same time decreases the velocity. However as the pitch gets greater in the case of the velocity the mass of the aircraft plays an important role and depending on the actual thrust (which declines with the altitude and the velocity) a further increase is only possible if

$$\gamma < \sin^{-1}\left(\frac{F\,(h,M)}{mg}\cos\alpha - \frac{D\,(h,M,\alpha)}{mg}\right) \tag{21}$$

is valid. Due to the learning of the inner loop

$$\begin{aligned} \alpha_{i+1}(k) = \alpha_i(k) &+ p\left(h_{ref}(k-1) - h_{i+1}(k-1)\right) \\ &+ d\left(\dot{h}_{ref}(k-1) - \dot{h}_{i+1}(k-1)\right) \end{aligned} \tag{22}$$

with p and d as learning gains, the angle of attack is increased and the pitch of the aircraft rises.

To still be able to learn a time optimal solution for an unknown system we examine the use of a different learning quantity. In the case of the standard learning method a single quantity with a start value $y(0)$ and the desired end value $y(t_0)$ where t_0 denotes the minimal time is used to learn the time optimal trajectory. In this paper we propose the use of a function $w(k)$ instead of single learning quantity e.g. h_{ref} with the same properties of a start value $w(0) \triangleq y(0)$ and an end value $w(t_0) \triangleq y(t_0)$. This is a more general form of the learning as the output of the system is already a possible function $y(t) = g(x,t)$. In the case of the presented aircraft as function

$$w(k) = ah_{ref}(k) + bv_{ref}(k) \tag{23}$$

a linear combination of the height and the velocity with the weightings a and b. The weightings are used to compensate the difference of the numerical value of the used system states (the height is two power of ten higher than the velocity).

The numerical computation using the proposed function in (23) by setting $a = 1$ and $b = -20$ in combination with the learning function (22) ($p = 1e-6\frac{\pi}{180}$ and $d = 5e-7\frac{\pi}{180}$) returns the learned optimal trajectory shown in Section 6. For this setup the desired final point of flight height $20.000m$ will be achieved in a time of $319s$ (see Figure 4). With this experiment it is shown that the change of the learning quantity it is possible to achieve the desired goal by learning for systems which does not belong to the class of input affine systems.

6. COMPARISON TO STANDARD METHOD

The standard optimization with a time optimal cost function and additional with a terminal cost has been implemented on the same machine as the learning method once with standard learning quantity (height) and the adapted learning quantity. To perform the computation a standard PC with an Intel© Core™ i5 – 2400 with 3.1 GHz, 8 GB of RAM, Windows 7 64-bit and *MATLAB/Simulink* is used. In the case of the standard optimization the available *MATLAB*- functions have been used. The same has been done for the learning method.

6.1 Standard vs. adapted learning

In a first attempt the standard learning method using the height as quantity (desired end value is a specific height) was used. As it turned out that only the end time is reached and the time remains still high (around 800 seconds), the adapted learning method has been used. In Figure 4 the first obvious disparity

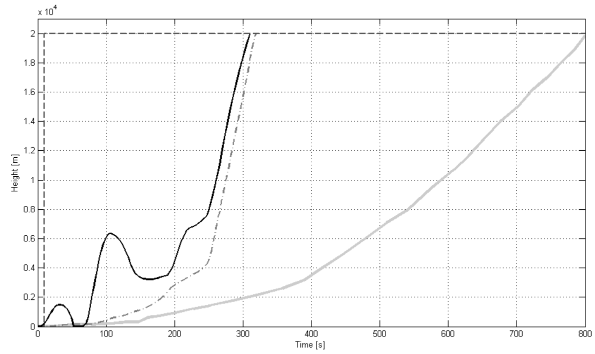

Fig. 4. Position comparison: black solid line time optimal solution, gray solid line the standard approach with the height as reference and dash dotted gray with the new virtual output.

is the final time until the aircraft reaches the desired height. Another clear difference between the two learning methods can be seen by comparing the velocity Figure 5. The different velocity behavior is the main reason why the adapted learning method is able to achieve an approximation of the time optimal result.

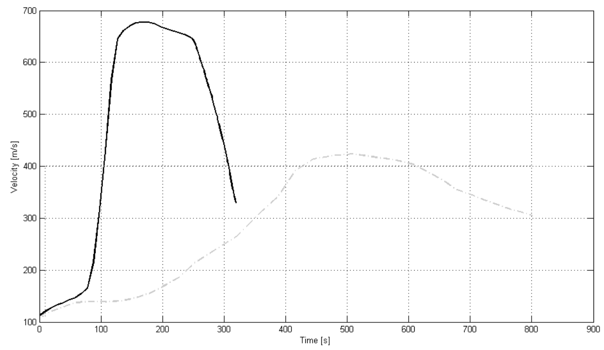

Fig. 5. Velocity comparison: solid black line approximated time optimal (with virtual output), dash dotted line the standard approach with the height as learning.

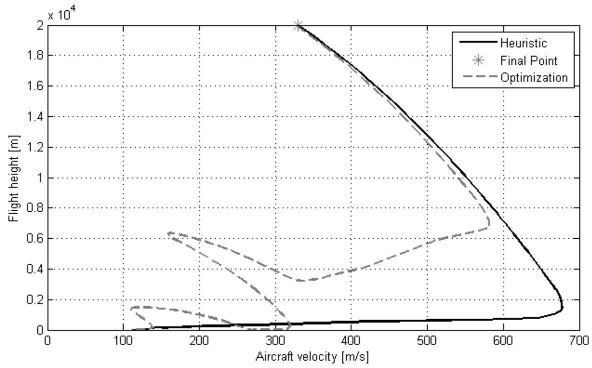

Fig. 6. Height over velocity diagram for the steepest ascent problem. Optimization in green, learned solution in blue

6.2 Optimization vs. Learning

In Figure 6 the optimization and the adapted learning method use the maximal possible energy of the system.

By taking a look at the height over time for both cases some distinctive points can be seen Figure 4. Around 200 seconds the two methods are very close to each other, this suggests that this point can be achieved with different control inputs up to this time point. An additional fact is that the sudden ascent in the case of the optimization is responsible for a faster ascent of 9 seconds or 3% faster result compared to the learned value (see Table 1). It seems that the learning has found another solution (as the optimal result is not convex) and is not able to track such a step increase at this point.

6.3 Time until height reached

Table 1. Times until final point

	time [sec]	time [%]
optimization (only time)	310	100
optimization (time + terminal)	311	100, 3
learned solution (height + velocity)	319	103
learned solution (only height)	> 800	> 258

The difference between the two learning methods can be explained by the velocity difference and the used learning quantity. In the case of the more interesting comparison between optimization and learning the convexity of the system plays an important role. Both results are close to each other and obtained by different runs of the height.

Nevertheless, the learning methods do not use any information of the system a priori (information is gained due to iteration) a difference in the range of 3% can be neglected as it is an approximation of the time optimal result. Probably by tuning the ILC accordingly to a priori available information the result will be closer to the time optimal one. A similar problem is the first ascent and decline of the standard method, which depends mainly of the search direction of the optimization method and can have different shapes by starting the optimization again.

7. CONCLUSION AND OUTLOOK

In this paper it has been shown that by changing the learning quantity, the time optimal solution can be obtained for highly complex nonlinear systems. For this a linear combination of states instead of a single quantity is applied to the learning process. It turned out that the selection of the learning quantity is essential for the success of obtaining the time optimal solution. The difficulty furthermore lays in the possibility to recognize the use of a misleading quantity as the process returns in any case a solution. By using the “wrong” learning quantity no approximation of the time optimal solution will be possible. So the process of selecting the learning quantity has to consist of different linear combination of the states and compare the results, to determine the “right” quantity.

A further work will treat the selection of the final quantity to reduce the influence of the user and to lower error-proneness of the method.

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