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Higher order sliding mode control based on adaptive first order sliding mode controller

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Abstract: This paper presents an adaptive higher order sliding mode controller based on first adaptive sliding mode control and an integral sliding variable. The idea is based on the definition of a nominal control which can stabilize a pure chain of integrators to zero in finite time. The novelty of the proposed approach is that uncertainties/perturbations effects are rejected by a first order sliding mode control tuned without any information about the bounds of the uncertainties. The efficiency of this controller is evaluated on an academic example with two kinds of nominal controls.

Keywords: higher order sliding mode, adaptive control, uncertain nonlinear systems.

1. INTRODUCTION

Sliding mode control (Utkin (1992)), (Zinober (1994)), (Edwards and Spurgeon (1998)) is considered as one of the most popular robust nonlinear control laws with respect to uncertainties and perturbations. The design strategy consists, first, in defining a manifold called "sliding surface" with desired properties of the closed-loop system dynamics; then, a discontinuous control is designed to enforce system trajectories to reach in a finite time the sliding surface and to generate the motion of the system on this manifold in spite of uncertainties and perturbations. However, the "sign" function which is used in the controller induces a high frequency control switching phenomenon, the so-called "chattering", which can causes dangerous damages to the actuators (Fridman (2001)), (Fridman (2003)).
Many solutions have been proposed these last decades in order to reduce this phenomenon. Among them, one can cite

- replacement of the "sign" function by a continuous one, as the saturation function (Slotine and Li (1991)). However, it enforces the system trajectories only to a vicinity of the sliding surface and it reduces the robustness with respect to uncertainties and perturbations;
- use of higher order sliding mode strategies (Levant (2003)). This class of controllers consists in driving the sliding variable and a finite number of its consecutive time derivatives to zero in a finite time. By this way, the discontinuous control acts on high order time derivative of the sliding variable which leads to chattering reduction.

An other solution for the reduction of the chattering phenomenon consists in combining higher order sliding mode control with dynamically adapted gain (Plestan et al. (2010)); in the case of the standard (here, "standard" means "with constant gain") sliding mode controllers, the tuning of the gains is made in the "worst" case by considering that uncertainties and perturbations equal their bounds. It means that the knowledge of these bounds is required for the control design.
The gain adaptation allows a gain adjustment with respect to a predefined criteria without any need of uncertainties and perturbations bounds. The idea is to increase the gain since higher order sliding mode is established; then, the gain is reduced and dynamically adapted when system trajectories are evolving on the sliding surface; the main interest of such strategies is that the gain is adapted to uncertainties/perturbations magnitude, the objective being to get the "just sufficient" gain in order to counteract the perturbations/uncertainties effects.
This approach has been applied to first order sliding mode control (Plestan et al. (2010)) and successfully used for the position control of an electropneumatic actuator. In (Bartolini et al. (2013)), (Taleb et al. (2013a)), (Shtessel et al. (2012)), adaptive second order sliding mode controllers have been proposed, which leads to adaptive versions of "twisting" and "supertwisting" controllers; these adaptive controllers have been also applied to the control of electropneumatic actuator.
In (Taleb et al. (2013b)), (Mondal and Mahanta (2013)), adaptive higher (*i.e.* larger than second) order sliding mode controllers have been proposed. A drawback of (Mondal and Mahanta (2013)) is inherent to the gain adaptation law: the gain increases until a higher order sliding mode is detected; then, the gain approximately stays constant even if the uncertainties and perturbations are reduced. The gain adaptation law is not totally efficient because it is not sufficiently reduced versus perturbations/uncertainties effects.
In (Taleb et al. (2013b)), the gain of discontinuous control increases and decreases according to an adaptation law: by this point-of-view, it is more efficient that (Mondal and Mahanta (2013)). However, the gain tuning is not optimized because it does not follow by a precise manner the variations of uncertainties and perturbations. The approach proposed in the sequel paper allows that the gain is

approximately tracking the variation of uncertainties and perturbations. By this way, the gain is "just sufficient" to counteract perturbations and uncertainties effects: it induces a reduced chattering phenomenon and, then, a better accuracy than previous results.
Section 2 states the problem whereas Section 3 presents the adaptive high order sliding mode controller. In order to show the applicability of this approach, the application to an academic example is presented in Section 4.

2. PROBLEM FORMULATION

Consider the nonlinear uncertain system

$$\begin{aligned} \dot{x} &= f(x,t) + g(x,t)u \\ y &= \sigma(x,t) \end{aligned} \tag{1}$$

with $x \in I\!R^n$ the state vector, $u \in I\!R$ the control input, and $\sigma(x,t) \in I\!R$ a smooth output function (sliding variable). f and g are uncertain smooth vector fields and are differentiable. The uncertainties on $f(x,t)$ and $g(x,t)$ are due to parameter variations, unmodeled dynamics or external disturbances.

Assumption 1. The relative degree r of system (1) with respect to σ is constant and known, and the associated zero dynamics are stable. ■

The "ideal" r^{th}-order sliding mode is defined through the following definition

Definition 1. (Levant (2001)) Consider the nonlinear system (1) and the sliding variable $\sigma(x,t)$. Assume that the time derivative $\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}$ are continuous functions. The manifold defined as [1]

$$\Sigma^r = \left\{ x \mid \sigma(x,t) = \cdots = \sigma^{(r-1)}(x,t) = 0 \right\}$$

is called "r^{th}-order sliding mode set", which is non-empty and is locally an integral set in the Fillipov sens (Filippov (1988)). The motion on Σ^r is called "r^{th}-order sliding mode" with respect to the siding variable $\sigma(x,t)$. ■

By a practical point-of-view, this kind of sliding mode can not be established, due to, for example, sampling period or not considered dynamics. It is the reason why the notion of "real" higher order sliding mode is introduced.

Definition 2. (Levant (2001)) Consider the nonlinear system (1) and the sliding variable $\sigma(x,t)$. Assume that the time derivative $\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}$ are continuous functions. The manifold defined as (τ being the sampling period of the control law)

$$\Sigma^r_* = \left\{ x \mid |\sigma| \le \mu_0 \tau^r, \cdots, |\sigma^{(r-1)}| \le \mu_{r-1}\tau \right\}$$

with $\mu_i \ge 0$ (with $0 \le i \le r-1$), is called "real r^{th}-order sliding mode set", which is non-empty and is locally an integral set in the Fillipov sense (Filippov (1988)). The motion on Σ^r_* is called "real r^{th}-order sliding mode" with respect to the sliding variable σ. ■

[1] All over this paper, $\sigma(\cdot)^{(k)}$ ($k \in I\!N$) denotes the k^{th} time derivative of the function $\sigma(\cdot)$. This notation is also applied for every function.

The r^{th}-order sliding mode control approach allows the finite time stabilization at zero (or the finite time convergence to a vicinity of zero) of the sliding variable σ and its $(r-1)$ first time derivatives by defining a suitable discontinuous control function u. The r^{th} time derivative of σ reads as

$$\sigma^{(r)} = \psi(x,t) + \varphi(x,t)u \tag{2}$$

Assumption 2. Solutions of equation (2) with discontinuous right-hand side are defined in the Fillipov sense (Filippov (1988)). ■

Assumption 3. There exist positive constants C, φ_m and φ_M such as

$$|\psi(x,t)| < C \qquad 0 < \varphi_m < \varphi(x,t) < \varphi_M. \tag{3}$$

■

As shown in (Laghrouche et al. (2006)), an r^{th}-order sliding mode controller for system (1) *w.r.t* to sliding variable σ is equivalent to finite time stabilization of the following system

$$\begin{aligned} \dot{z}_i &= z_{i+1}, \quad i = 1 \ldots r-1 \\ \dot{z}_r &= \psi(x,t) + \varphi(x,t)u \end{aligned} \tag{4}$$

Consider now the following function, named "integral sliding variable", defined as (t_0 being the initial time)

$$s(z(t)) = z_r(t) - z_r(t_0) - \int_{t_0}^{t} v_{nom}(\tau)d\tau \tag{5}$$

with v_{nom} a control law allowing that the following system (chain of r integrators)

$$\begin{aligned} \dot{\xi}_i &= \xi_{i+1}, \quad i = 1 \ldots r-1 \\ \dot{\xi}_r &= v_{nom} + \gamma(t) \end{aligned} \tag{6}$$

is stabilized at zero **in a finite time**, with $\gamma(t)$ a small and bounded perturbation. Note that $s(z(t_0)) = 0$: then, the system is evolving on the *integral sliding mode set* defined as [2]

$$\Sigma^r_{\int} = \{ z \mid s(z(t)) = 0 \}$$

early from the initial time.

Assumption 4. The control v_{nom} is assumed to be bounded. There exists a positive constant v_M such as $|v_{nom}| \le v_M$. ■

3. ADAPTIVE CONTROLLER DESIGN

The ideas on which the structure of the controller is based are the following

- the control law has to force, via the integral sliding variable s, the system trajectories to evolve on
$$\Sigma^r_{\int *};$$
a (first order) real sliding mode behavior with respect to s is then established;

[2] By the same way as previously, a *"real" integral sliding mode set* can be introduced and defined as $\Sigma^r_{\int *} = \{ z \mid |s(z(t))| \le \mu\tau \}$.

- once this sliding mode established, the system dynamics has to be governed by the finite time convergence controller v_{nom}; by this way, an r^{th}-order sliding mode with respect to σ is established.

To summarize, the control strategy consists in defining an adaptive first order sliding mode controller ensuring the establishment of a first order real sliding mode with respect to s and inducing the convergence of $\dot{s}$ to a vicinity of zero. Then, this latter sliding mode induces the establishment, in a finite time, of a higher order sliding mode with respect to σ, which is the initial goal. The main advantage of this new approach is that the gain $K(t)$ is "automatically" and "dynamically" tuned to the "just sufficient" value able to counteract the perturbations and uncertainties.

The first time derivative of s reads as

$$\dot{s} = \sigma^{(r)} - v_{nom} = \psi(x,t) - v_{nom} + \varphi(x,t)u \tag{7}$$

which gives

$$\sigma^{(r)} = v_{nom} + \dot{s}. \tag{8}$$

Then, system (4) becomes

$$\begin{aligned} \dot{z}_i &= z_{i+1} \;\; i = 1 \ldots r-1 \\ \dot{z}_r &= v_{nom} + \dot{s} \end{aligned} \tag{9}$$

It is clear that, if $\dot{s} = 0$, then system (9) behaves like a pure chain of integrator. However, the objective $\dot{s} = 0$ is not reachable in a practical case. Then, a realistic objective consists in ensuring that, in a finite time, $\dot{s}$ is sufficiently small. The control law v_{nom} has to be sufficiently robust in order to stabilize, in a finite time, system (9) in spite of the term $\dot{s}$. The first time derivative of s can be written as

$$\dot{s} = \Gamma(x,t) + \varphi(x,t)u \tag{10}$$

with

$$\Gamma(x,t) = \psi(x,t) - v_{nom}. \tag{11}$$

The objective of the *adaptive first order sliding mode controller* is to establish a first order real sliding mode *w.r.t* s and to ensure the convergence of the first time derivative of sliding variable $\dot{s}$ to a vicinity of zero in a finite time t_F.

Adaptation law. The idea of the adaptation law is to increase K until that a real first order sliding mode *w.r.t* s is detected. Then, K is gradually reduced until the sliding mode is lost due to an insufficient control magnitude. When the real first order sliding mode is lost, the gain K is increased again by a finite value impulse in order to provide the convergence restoration. Then, the gain is decreased gradually until the real first order sliding mode is once more lost, and so on.

Sliding mode detector. Let $\tau > 0$ be the sampling period. It is well-known that a first order sliding mode provides an accuracy proportional to τ (see the definition of "real" sliding mode behavior). In order to have a criterion for the detection of the real first order sliding mode, consider a natural number N and some $\mu > 0$. Given the sampling time t_i ($i \in \mathbb{N}$), define the following criterion

$$\alpha(t) = \begin{cases} 1 \text{ if } \forall\, t_j \in [t_i, t_i + N\tau] : |s(t_j)| \leq \mu K(t_j)\tau \\ -1 \text{ if } \exists\, t_j \in [t_i, t_i + N\tau] : |s(t_j)| > \mu K(t_j)\tau \end{cases} \tag{12}$$

his criterion consists in verifying over the sliding window $[t_i, t_i + N\tau]$ if all values of s are less than a number proportional to the sampling time τ. In this case, the first sliding mode criterion is considered satisfied and $\alpha = 1$. If at least one value is out of the defined bounds over the sliding window, the criterion is violated and $\alpha = -1$.

Adaptation law algorithm. The adaptation law of the gain K has to depend on the value of the criterion α. Thus, the gain must decrease when $\alpha = 1$ whereas, when $\alpha = -1$, the gain will increase. Introduce the constants K_m and q satisfying

$$K_m > 0, \quad q > 1 \tag{13}$$

and let the adaptation law be

$$\dot{K} = \begin{cases} -\alpha\lambda K & \text{if } K > K_m \\ \lambda & \text{if } K \leq K_m \end{cases} \tag{14}$$

where λ is a positive adaptation parameter. Thus, K is never less than K_m, which is taken arbitrarily small. In addition, in order to increase the convergence time, a gain increment is implemented at each sampling instant t_i if the first order sliding mode criterion is violated, *i.e.* when the criterion passes from true to false state, which gives

$$K(t_i) = \begin{cases} qK(t_{i-1}) & \text{if } \alpha(t_{i-1}) = 1 \text{ and } \alpha(\mathrm{t_i}) = -1 \\ K(t_i) & \text{if } \alpha(t_{i-1}) \neq 1 \text{ or } \alpha(\mathrm{t_i}) \neq -1 \end{cases} \tag{15}$$

The parameter λ should be chosen according to system response time. However, it can be non critical since after system convergence, the sliding mode is established, when lost, by the gain increment (15).

Theorem 1. (Bartolini et al. (2013)) Consider the nonlinear system (10), and suppose that the control u reads as $u = -K(t)\,\mathrm{sign}(s)$. Consider also the adaptation law (14)-(15) with λ, μ and N sufficiently large. For a given q, there exists $q^* > q$ such that, with sufficiently small τ, the variable gain $K(t)$ features local maxima such that

$$K(t) < \max\left[q^*\frac{|\Gamma|}{\varphi}, K_m\right].$$

Furthermore, the accuracy $|s| \leq \eta\tau K(t)$ is obtained in finite time. ■

Note that it has been proved in (Bartolini et al. (2013)) that the gain $K(t)$ with the adaptation law (14)-(15) approximately tracks $|\Gamma|/\varphi$. It means that the gain is then "just sufficient" versus uncertainties and perturbations magnitudes. The main result is then given by the following theorem.

Theorem 2. Consider system (1) with Assumptions 1,2, 3 and 4 fulfilled, and the sliding variable σ. Let the control u be defined as

$$u = -s - K(t)\,\mathrm{sign}(s) \tag{16}$$

for which the gain $K(t)$ is adapted according to adaptation law (14)-(15), and consider the integral sliding variable s (5) with v_{nom} a controller sufficiently robust to stabilize in a finite time the uncertain system (9). Then, an r^{th}-order sliding mode versus σ is established in a finite time. ■

Proof. From (7)-(16), one has

$$\dot{s} = \psi(.) - v_{nom} - \varphi s - \varphi K\,\text{sign(s)} \quad (17)$$

Given the adaptation law (14)-(15), it has been proved in (Bartolini et al. (2013)) that the term $-K\,\text{sign(s)}$ follows the uncertain term $-\Gamma/\varphi = -(\psi - v_{nom})/\varphi$, which means that, in a finite time, one has

$$-K\,\text{sign}(s) \simeq -\frac{\psi - v_{nom}}{\varphi}. \quad (18)$$

Then, one gets

$$\dot{s} \simeq -\varphi s \quad (19)$$

which means that s and $\dot{s}$ reach a vicinity of the origin in finite time. From (4) and (18), one has

$$\begin{aligned} \dot{z}_r &= \psi + \varphi u \\ &= \psi - \varphi s - K(t)\varphi \cdot \text{sign}(s) \\ &\simeq v_{nom} - \varphi s \end{aligned} \quad (20)$$

Given that s is evolving in the vicinity of the origin, the quantity φs can be viewed as a small bounded perturbation: as previously denoted, the control law v_{nom} has to be designed in order to stabilize the following system

$$\begin{aligned} \dot{z}_i &= z_{i+1}, \quad i = 1 \dots r-1 \\ \dot{z}_r &= v_{nom} - \varphi s \end{aligned} \quad (21)$$

where φs can be considered as a bounded perturbation. In fact, given that a first order real sliding mode is established with respect to s, one has

$$|s| \leq \mu K \tau \quad (22)$$

Given that

$$K < \max(q^*|\Gamma|/\varphi, K_m),\ 0 < \varphi_m < \varphi < \varphi_M$$

and

$$|\Gamma| = |\psi - v_{nom}| < C + v_M,$$

one gets

$$|\varphi s| < \max(\mu q^* \tau (C + v_M)\varphi_M/\varphi_m, \mu\varphi_M K_m \tau). \quad (23)$$

By denoting $\gamma_M = \max(\mu q^* \tau (C+v_M)\varphi_M/\varphi_m, \mu\varphi_M K_m \tau)$, the control v_{nom} has to be tuned such the following system

$$\begin{aligned} \dot{z}_i &= z_{i+1}, \quad i = 1 \dots r-1 \\ \dot{z}_r &= v_{nom} + \gamma_M \end{aligned} \quad (24)$$

is stablized in zero in finite time. Thus, an r^{th} order sliding mode is established in finite time for system (1) *w.r.t* sliding variable σ. ∎

Remark 1. The quantity γ_M is, in many practical cases, small given that it is proportional to the sampling period τ. It means that the effort required from v_{nom} to ensure robust behavior of the closed-loop system is relatively limited. ∎

4. SIMULATION

Consider the following nonlinear uncertain system

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= 10 + 2\sin(0.1\,\text{t}) + (2 + \cos(\text{t}))\text{u} \end{aligned} \quad (25)$$

with the sliding variable σ defined as $\sigma = z_1$. It means that

$$\sigma^{(3)} = \psi(t) + \varphi(t)\,u \quad (26)$$

with $\psi(t) = 10 + 2\sin(0.1\,\text{t})$ and $\varphi(t) = 2 + \cos(\text{t})$. Thus, the relative degree of system (25) *w.r.t* to σ is equal to 3. Moreover, functions ψ and φ are bounded and

$$|\psi(t)| \leq 12 \text{ and } 0 < 1 \leq \varphi(\text{t}) \leq 3. \quad (27)$$

The control objective is to establish a 3^{rd} order sliding mode *w.r.t* $\sigma = z_1$ in a finite time. The integral sliding variable s is tuned as

$$s = z_3(t) - z_3(0) - \int_0^t v_{nom}(\tau)\,d\tau \quad (28)$$

with v_{nom} defined as [3] (Levant (2003))

$$v_{nom} = -5\frac{z_3 + 2\left(|z_2| + |z_1|^{2/3}\right)^{-1/2}\left(z_2 + |z_1|^{2/3}\text{sign}z_1\right)}{|z_3| + 2\left(|z_2| + |z_1|^{2/3}\right)^{1/2}}$$

The control u is defined by (16) with an adapted gain $K(t)$ according to (14)-(15)-(13). The adaptation parameters are chosen as follow (the tuning has been made in order to get good compromise between accuracy and smooth response)

$$\tau = 2\,10^{-5}\,sec;\ \mu = 50;\ q = 1.2;\ K_m = 0.05;\ \lambda = 20$$

Moreover, any information about functions ψ and φ was used to tune the control.

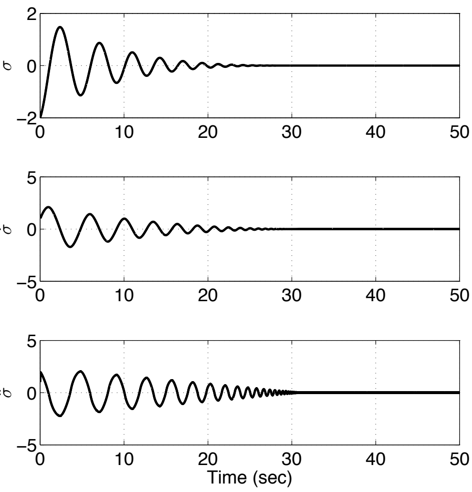

Fig. 1. Sliding variable σ and its time derivatives - σ (**top**), $\dot{\sigma}$ (**middle**) and $\ddot{\sigma}$ (**bottom**) versus time (sec).

Figures 1 and 2 respectively show the convergence of σ, $\dot{\sigma}$, $\ddot{\sigma}$ from a first hand-side, and s, $\dot{s}$ from the other hand-side. Figure 3 displays the evolution of control u, gain $K(t)$ and $(\psi - v_{nom})/\varphi$ versus time. It is clear that the gain $K(t)$ approximately tracks the value of $|\Gamma|/\varphi = |\psi - v_{nom}|/\varphi$,

[3] This choice of control strategy for v_{nom} is arbitrary. The required features are the finite time convergence and the robustness.

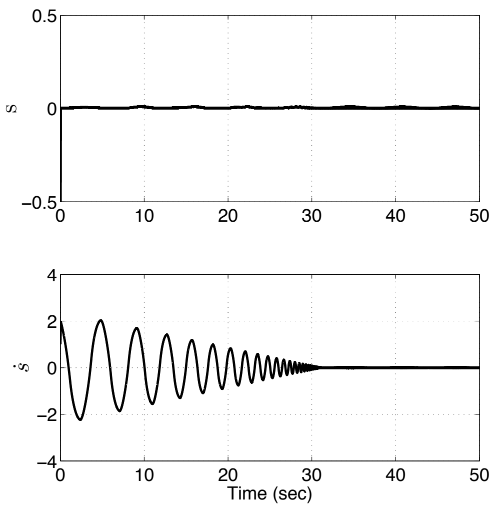

Fig. 2. Integral sliding variable s and its time derivative - s (**top**) and $\dot{s}$ (**bottom**) versus time (sec).

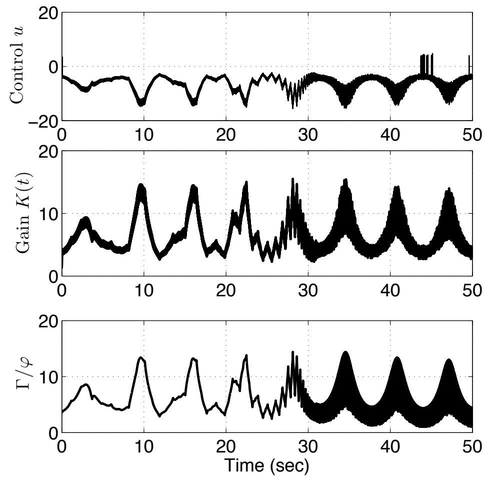

Fig. 3. **Top.** Control u versus time (sec). **Middle.** Gain $K(t)$ versus time (sec). **Bottom.** Ratio $|\Gamma|/\varphi$ versus time (sec).

which is a good situation with respect to gain adaptation ("just sufficient" gain). Moreover, the chattering in control u is very small camparing with standard 3^{rd} order sliding controller.
Simulations have been made with a sampling time $\tau = 4.10^{-5}$ sec to show the establishment of a 3^{rd} order real sliding mode. The results of simulation can be summarized in Table 1 with R_* defined as

$$R_{|\sigma|} = \frac{|\sigma|_{\tau=4\,10^{-5}\,s}}{|\sigma|_{\tau=2\,10^{-5}\,s}},\; R_{|\dot{\sigma}|} = \frac{|\dot{\sigma}|_{\tau=4\,10^{-5}\,s}}{|\dot{\sigma}|_{\tau=2\,10^{-5}\,s}},\quad R_{|\ddot{\sigma}|} = \frac{|\ddot{\sigma}|_{\tau=4\,10^{-5}\,s}}{|\ddot{\sigma}|_{\tau=2\,10^{-5}\,s}} \tag{29}$$

	$\tau = 2\,10^{-5}\,sec$	$\tau = 4\,10^{-5}\,sec$	R_*
$\lvert\sigma\rvert <$	$2\,10^{-6}$	$1.55\,10^{-5}$	~ 8
$\lvert\dot{\sigma}\rvert <$	$2.5\,10^{-4}$	10^{-3}	4
$\lvert\ddot{\sigma}\rvert <$	0.05	0.1	2

Table 1. Accuracy obtained for σ, $\dot{\sigma}$ and $\ddot{\sigma}$ versus sampling period τ.

In order to illustrate Remark 1, simulations have been made with v_{nom} defined as

$$v_{nom} = -z_1 - 3\,z_2 - 3\,z_3. \tag{30}$$

Note that, in this case, it is not possible to ensure the establishment of a real 3^{rd} order sliding mode given that, in this case, the finite time convergence to a vicinity of the origin is not ensured. The objective here is to show that, even if the control law v_{nom} is not a robust one, the closed-loop system behavior is very efficient. In fact, the main part of the robustness is provided by the discontinuous control law u, and not by v_{nom}.
Figure 4 displays the variables σ, $\dot{\sigma}$ and $\ddot{\sigma}$ exponentially converging to a vicinity of the origin. Figure 5 displays control u, gain $K(t)$ and $|\Gamma|/\varphi$. It is clear that the controller is robust versus perturbations and uncertainties, and the gain $K(t)$ is tracking, as previously, the uncertain function Γ/φ.

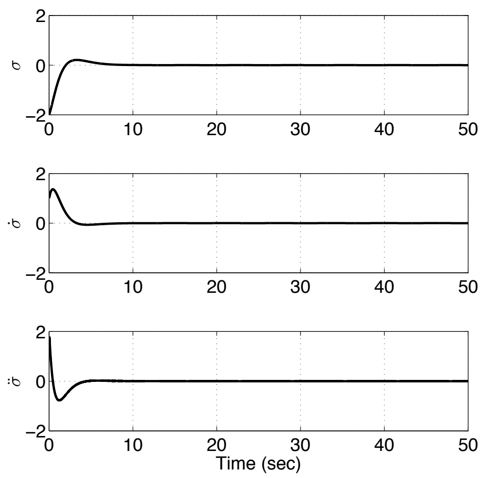

Fig. 4. Sliding variable σ and its time derivatives - σ (**top**), $\dot{\sigma}$ (**middle**) and $\ddot{\sigma}$ (**bottom**) versus time (sec).

5. CONCLUSION

The paper has proposed a new strategy for adaptive higher order sliding mode control. This controller is based on integral sliding mode concept and is using an adaptive first order sliding mode control law in order to stabilize, in a finite time, the integral sliding variable. Once this stabilization achieved, a "nominal" control law is used in order to ensure finite time convergence of successive time derivatives of the sliding variable to a vicinity of 0, *i.e.* a higher order sliding mode with respect to the sliding variable is established. Simulations on an academic example have shown the efficiency of the approach. Future works concern the application of this class of controllers

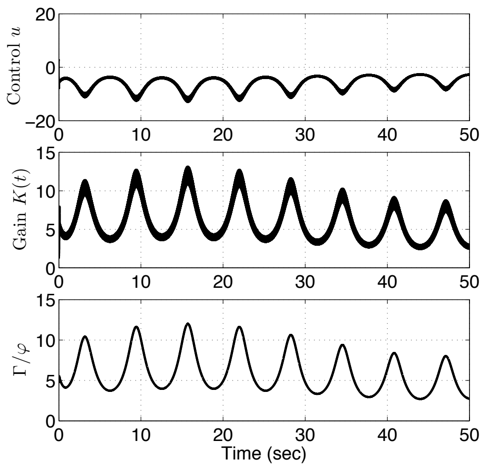

Fig. 5. **Top.** Control u versus time (sec). **Middle.** Gain $K(t)$ versus time (sec). **Bottom.** Ratio $|\Gamma|/\varphi$ versus time (sec).

on experimental systems (for example, application to electropneumatic set-up (Taleb et al. (2013a)),(Girin et al. (2009))).

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