Experimental evaluation of a DO-FPID controller with different filtering properties

M.Huba∗∗∗ I. Belai∗

∗ Institute of Automotive Mechatronics, Faculty of Electrical Engineering and IT, Slovak University of Technology, Ilkovičova 3, 812 19 Bratislava, Slovakia (e-mail: mikulas.huba,igor.belai@stuba.sk)

** FernUniversität in Hagen, FMI, PRT, Hagen, Germany

Abstract: The paper brings experimental results and evaluation to modular disturbance observer based filtered PD and PID controller design based on theoretical results of papers Huba (2013b,c). The approach is focused on achieving the possibly best filtering properties by keeping nearly constant dynamics of the setpoint responses. The developed controller applied to a positional DC motor control is evaluated for different values of the tuning parameter TD and different filter orders n by using time and shape related performance measures.

Keywords: noise attenuation, filter design, disturbance observer, PD control, position control

1. INTRODUCTION

Besides the traditional robustness-versus-speed trade off of different control approaches one is frequently faced with another trade off related to noise-versus-speed of transients. Is is important especially in situations with a dominant role of noise attenuation, e.g. in servo systems based on incremental position sensors. It is well known that a measurement, or quantization noise have a negative influence on the overall control performance. They contribute to an increased equipment wear, heat dissipation, production costs increase, reduced control precision, undesirable acoustic noise, etc. Thus, one may find many papers dealing with appropriate noise filtration (Åström and Hägglund, 2006; Hägglund, 2012; Larsson and Hägglund, 2012).

A higher attention to a filter design is paid in the disturbance observer based control (Ohishi et al., 1987; Schrijver and van Dijk, 2002; Radke and Gao, 2006; She et al., 2011). However, in all up to now known solutions, this holds just for the disturbance observer loop reconstructing an equivalent input disturbance that is then used for its compensation by a counteracting signal applied to the controller output. In all this publications, the filter design focusses on the possibly best closed loop robustness and does not cover filtration properties of the basic controller used to stabilize the plant. Despite the fact that the effects of noise exposure can be very similar to impact of a lower closed loop robustness.

For tasks with the dominant first-order dynamics this problem has been recently reformulated in papers Huba (2012a,b, 2013a). The corresponding experimental results achieved by an application to a DC motor speed control (Huba and Belai, 2014b) confirmed a huge performance improvement with respect to traditional two-degree-of-freedom (2DOF) PI control and gave a motivation to deal with this problem also in a more general setup.

An extension of the underpinning ideas to a modular design of a disturbance observer based filtered PD and PID control (FPD and DO-FPID) devoted to plants with a second order dominant dynamics has been treated in Huba (2013c,b). Whereas they have been based on analytical and simulation calculations, this paper reports experimental verification of the proposed approaches in their application to a DC motor positional control.

The paper is structured as follows. Section 2 deals briefly with a pole assignment control of second order plants. An expected performance and deviations from ideal shapes of transients at the plant input and output are discussed in Section 3. Section 4 summarizes basic features of the filtered PD control (FPD) and disturbance observer based filtered PID control (DO-FPID) and its optimal tuning for control loops extended by an unmodelled and filter dynamics. Section 5 describes application of the proposed DO-FPID control to a DC motor positional control. The achieved results are discussed in Section 6 and summarized in Section 7.

2. A SECOND ORDER PLANT AND ITS CONTROL

Firstly, let us is consider design of FPD and DO-FPID controllers for a dominant 2nd-order plant dynamics with the input disturbance \( d_1 \), with \( x = (y, \dot{y})' \), \( \dot{y} = dy/\text{dt} \) and y being the plant state, or output

\[
\dot{y} = K_s(u_1 + d_1) - a_1\dot{y} - a_0y
\]

Such a plant with parameters \( K_s, a_1 \) and \( a_0 \) may be described by a “pole-zero form” transfer function

\[
F(s) = \frac{Y(s)}{U_r(s)}\bigg|_{d=0} = \frac{K_s}{s^2 + a_1s + a_0}
\]

For stepwise constant setpoint values \( r \), under pole assignment control of the plant (2) one can require the
setpoint-to-output relation characterized by the closed loop poles \( \alpha_1, \alpha_2 \), or by the corresponding time constants \( T_{ri} = \frac{1}{\alpha_i} \).

When considering plant (1) and solving the given problem for \( u_r \), one gets a PD controller

\[
u_r = K_P(r - y) - K_D \dot{y} + u_c; \quad u_c = a_0 r/K_s - d_i
\]

\[
K_P = \frac{\alpha_1 \alpha_2 - a_0}{K_s}; \quad K_D = -\frac{\alpha_1 + \alpha_2 + \alpha_1}{K_s}
\]

Thereby, the feedforward control \( u_c \) is necessary for keeping the output in a steady state at the required reference value \( r \) under influence of a constant disturbance \( d_i \).

A closed loop with the extended PD-controller (4) remains stable, when its poles remain negative, i.e.

\[
K_P K_s + a_0 > 0; \quad K_D K_s + a_1 > 0
\]

For stable and marginally stable plants \( (\alpha_1 \geq 0) \) this holds for any \( K_P K_s > 0 \), \( K_D K_s > 0 \) and stability will be satisfied for any \( 0 < T_{ri} < \infty \). For unstable plants with \( a_0 < 0 \) and \( T_{ri} = T_{ri} \) it must hold

\[
T_r < \sqrt{-1/a_0} = T_p
\]

i.e. the controller gain \( K_P \) cannot be arbitrarily decreased (the closed loop time constant \( T_r \) cannot be arbitrarily increased), just to a value fulfilling (6).

The paper deals with the problem that in tuning controllers for real plants, due to a non-modelled loop dynamics, intentionally used filters, as well as due to the always present measurement and quantization noise, a control designer has to look for some optimal controller gains \( K_P, K_D \), or the task may be expressed as looking for "optimal" closed loop poles \( \alpha_1, \alpha_2 \).

3. EXPECTED CONTROL PERFORMANCE

For the plant output changes, the optimal closed loop performance is frequently specified by nearly monotonic (MO) transients corresponding to setpoint changes (Huba, 2012c, 2013d,a). According to the plant dynamics inversion, the corresponding input has to yield "two-pulses" and will be denoted as a 2P input. At the plant output, ideal disturbance responses may be characterized by "one-pulse" shapes.

Measures for evaluating deviations of a signal \( u(t) \) from these ideal shapes may be proposed by modifying the Total Variance (TV) measure (Skogestad, 2003)

\[
TV = \int_0^\infty \frac{du}{dt} dt \approx \sum_i |u_{i+1} - u_i|
\]

For evaluating deviations from strictly MO plant output setpoint response \( y_s(t) \) with the initial value \( y_{s,0} \) and the final value \( y_{s,\infty} \) it is the TV0\((y_s)\) criterion

\[
TV_0(y_s) = \sum_i |y_{s,i+1} - y_{s,i}| - |y_{s,\infty} - y_{s,0}|
\]

(8)

\( TV_0(y_s) = 0 \) just for strictly MO response, else \( TV_0(y_s) > 0 \).

For disturbance responses \( y_d(t) \) with a 1P output shape having one extreme point \( y_{d,m} \neq (y_{d,0}, y_{d,\infty}) \) the \( TV_1 \) criterion may be defined as

\[
TV_1(y_d) = \sum_{d,i} |y_{d,i+1} - y_{d,i}| - |2y_{d,m} - y_{d,\infty} - y_{d,0}|
\]

(9)

Again, \( TV_1(y_d) = 0 \) just for strictly 1P response, else for output signals with superimposed higher harmonics \( TV_1(y_d) > 0 \).

Integral deviations from an ideal 2P input shape with two extreme points \( u_{m1}, u_{m2} \) may be characterized by

\[
TV_2(u) = \sum_i |u_{i+1} - u_i| - |2u_{m1} - 2u_{m2} + u_{\infty} - u_0|
\]

(10)

Again, for ideal 2P control functions \( u(t) \), \( TV_2(u) = 0 \).

The speed of the transients at the plant output is usually quantified by the IAE (Integral of Absolute Error)

\[
IAE = \int_0^\infty |e(t)| dt
\]

(11)

4. DO-FPD AND DO-FPID TUNING

4.1 Extended loop dynamics

To deal with the implementation and filtration problem, the 2nd order plant approximation (1) will now be extended by filters \( F_n(s) \) used in all controller channels (Fig. 1) to

\[
S_n(s) = \frac{K_s e^{-T_{as}}}{(s^2 + a_1 s + a_0)(T_n s + 1)^n}; \quad F_n(s) = \frac{1}{(T_n s + 1)^n}
\]

(12)

In a limit for \( n \to \infty \) it was shown to be equivalent to a dead time \( T_D \)

\[
S_D(s) = \frac{K_s e^{-T_{ds}}}{s^2 + a_1 s + a_0}; \quad 0 < T_D << T_p = \sqrt{1/|a_0|}; \quad n = 1, 2, ...
\]

(13)

what allows to simplify the overall treatment. Critical and optimal tuning of such a configuration with \( a_0 = 0 \) have been analyzed in Huba (2013c,b, 2014).

Let us firstly consider a plant delay \( T_d \to 0 \). Then, for both \( F_n(s) \) and \( T_d \) located in the feedback one gets the input-to-output transfer functions

Fig. 1. FPD (with \( c_I = 0 \)) and DO-FPID control (\( c_I = 1 \)) using equal filtration of all channels of the state observer (SO) and of the disturbance observer (DO): \( \delta \)-quantization noise

\[
TV_1(y_d) = \sum_{d,i} |y_{d,i+1} - y_{d,i}| - |2y_{d,m} - y_{d,\infty} - y_{d,0}|
\]

Again, \( TV_1(y_d) = 0 \) just for strictly 1P response, else for output signals with superimposed higher harmonics \( TV_1(y_d) > 0 \).

Integral deviations from an ideal 2P input shape with two extreme points \( u_{m1}, u_{m2} \) may be characterized by

\[
TV_2(u) = \sum_i |u_{i+1} - u_i| - |2u_{m1} - 2u_{m2} + u_{\infty} - u_0|
\]

(10)

Again, for ideal 2P control functions \( u(t) \), \( TV_2(u) = 0 \).

The speed of the transients at the plant output is usually quantified by the IAE (Integral of Absolute Error)

\[
IAE = \int_0^\infty |e(t)| dt
\]

(11)
\[ F_{\text{fn}}(s) = \frac{Y(s)}{R(s)} = \frac{(K_P K_s + a_0) (T_n s + 1)^n}{(s^2 + a_1 s + a_0) (T_n s + 1)^n + K_s (K_P + K_D s)} \]

\[ F_{\text{in}}(s) = \frac{Y(s)}{D_i(s)} = \frac{(s^2 + a_1 s + a_0)(T_n s + 1)^n + K_s (K_P + K_D s)}{K_s (T_n s + 1)^n - 1} \]

It is important to note (Huba, 2013b) that both setpoint-to-output responses \(F_{\text{fn}}(s)\) and \(F_{\text{in}}(s)\) and thus also their corresponding characteristic polynomials are equal for the FPD and DO-FPID control.

### 4.2 Normalized loop parameters

By introducing parameters
\[ \begin{align*}
\overline{K}_{p_n} &= K_P K_s T^2_n; \quad \overline{K}_{D_n} = K_D K_s T_n \\
\overline{A}_{o_n} &= a_0 T^2_n; \quad \overline{A}_{oD_n} = a_1 T_n; \quad p = T_n s \\
\overline{K}_{PD} &= K_P K_s T_D^2; \quad \overline{K}_{DD} = K_D K_s T_D \\
\overline{A}_{oD} &= a_0 T^2_D; \quad \overline{A}_{oDD} = a_1 T_D; \quad p = T_D s
\end{align*} \]

they may be normalized to
\[\begin{align*}
A(p) &= (p^2 + A_1 p + A_0)(p + 1)^n + \overline{K}_{Dn} p + \overline{K}_{p n} \\
\dot{A}(p) &= (p^2 + A_1 d p + A_0 d) e^p + \overline{K}_{DDP} + \overline{K}_{PD}
\end{align*}\]

### 4.3 Optimal loop tuning

Since, in the nominal case, the FPID and DO-FPID characteristic polynomials are equal, an optimal controller tuning based on the closed loop poles may be for both the FPD and DO-FPID control the same. The triple real dominant pole (TRDP) method represents one of the first methods used for an analytical controller tuning (see e.g. Oldenbourg and Sartorius (1944, 1951)). This method requires to fulfill identities \(A(p_0) = 0, \dot{A}(p_0) = 0\) and \(\dot{A}(p_0) = 0\). Although a corresponding solution exist for any \(a_0, a_1\), for the sake of simplicity it was shown just for the double integrator \((a_0 = a_1 = 0)\), when
\[ p_{oD} = \frac{-2 - \sqrt{2} n/(n + 1)}{(n + 2)}; \quad N = n(n + 1) \]

\[ \overline{K}_{oPn} = \frac{n + (5 n + 2) \sqrt{2 N - 7 N}}{(n + 2)^2 (N + \sqrt{2 N})} \]

\[ \overline{K}_{oDn} = \frac{\sqrt{2 N} - n}{N + \sqrt{2 N}} \]

For the limit case with dead time one gets
\[ \begin{align*}
p_{oD} &= -2 + \sqrt{2} \\
\overline{K}_{oPD} &= 2 e^{-2 + \sqrt{2}} (5 \sqrt{2} - 7) \\
\overline{K}_{oDD} &= 2 e^{-2 + \sqrt{2}} (\sqrt{2} - 1)
\end{align*} \]

As shown in Huba (2014), this tuning approximates well also the dynamics of considered positional motor control.

#### 4.4 IAE closed loop values

In the case of MO setpoint and 1P disturbance responses, when the control error does not change its sign, one may derive the IAE values corresponding to unit input steps of particular inputs by Laplace transform as
\[\begin{align*}
IAE_{s_n} &= \frac{a_1 + K_n K_{Dn} - K_p a_0 K_t N}{K_p T_n} \\
IAE_{o_n} &= \frac{K_p a_0}{K_p T_n} \\
IAE_{s_D} &= \frac{a_1 + K_n K_{DD} - K_p a_0 K_t D}{K_p T_D} \\
IAE_{o_D} &= \frac{T_D K_s}{K_{PD} K_s + a_0}
\end{align*}\]

#### 4.5 \(s_n\) - Equivalence of loop delays

Similarly as in tuning controllers for the 1st order plants (Huba, 2013a), in tuning the FPD and DO-FPID controllers one may get a closed loop performance nearly invariant against the filter order \(n\) by requiring a fixed dominant closed loop pole position in the complex \(s\) plane, which may be expressed by means of (18) and (19) as
\[ s_{oD} = \frac{p_{oD} T_D}{s_{oD} = p_{oD}/T_D} \]

what corresponds to a closed loop equivalence among the time constants \(T_n\) and \(T_D\)
\[ T_n = \frac{2(n + 1) \sqrt{2}}{2 - \sqrt{2}(n + 1)(n + 2)} \]

Thereby \(T_D\) may be used as a tuning parameter influencing primarily the speed of transients and the filter parameters (the order \(n\) and the time constant \(T_n\)) for modifying the noise attenuation.

#### 4.6 Control of loops with mixed delays

The above treated situations considered just the limit situations with either \(T_D = 0, T_n = 0\), or \(T_D \neq 0, T_n = 0\). Thereby, \(T_D\) has been considered as an equivalent limit case corresponding to \(T_n\) for \(n \to \infty\). In practice, one has usually to treat mixed situations, when a loop contains not only a filter dynamics \(F_{o}(s)\), but also a dead time \(T_D\) representing e.g. an nonmodelled plant dynamics. Analysis carried out in Huba (2013a) showed that for \(T_D << T_D\) the controller tuning may be simplified by using the values \(K_{oPD}, K_{oDD}\) (19), whereas the filter time constants are calculated according to (22) with \(T_D\) substituted for \(T_D\)
\[ T_D = T_D - T_d "]

5. ILLUSTRATIVE EXAMPLE

Since a DO-FPID control has been shown in Huba (2013c,b) to be much more noise sensitive than a FPD control, this comparative experiment deals just with the worse situation with a DO-FPID positional control of HSM 150 DC motor. The identified plant parameters are:

\[ J = 0.00012 \text{kgm}^2; \text{ moment of inertia} \]

\[ B = 0.00016 \text{ Nm.s.rad}^{-1}; \text{viscous friction} \]

\[ T_{GM} = 0.00025 \text{ s}; \text{torque generator time constant} \]

\[ \Delta \phi = 6.283/10000; \text{rad}; \text{position resolution} \]
When considering the plant model (1) one gets $K_s = 1/J = 8333.3$, $a_1 = B/J = 1.3333$ and $a_0 = 0$. All internal delays (including the torque generator time constant $T_{GM}$, the electrical time constant and the sampling period $T_s = 0.25 ms$) will be approximated by the dead time $T_d = T_{GM} + T_s = 0.5 ms$.

For a chosen $n$, the equivalent dead time allocated to filtration (23) is used instead of $T_D$ in calculating the filter time constant $T_{s,n}$ according to (22). The remaining FPID, or DO-FPID controller parameters $K_P$ and $K_D$ are set for $T_D$ according to (19).

Dependences of the simulated and really measured $IAE$, $TV_0(y_s)$, $TV_1(y_d)$ and $TV_2(u_s)$ values (for the setpoint and disturbance steps) on $n$ are shown in Figs 2-3. Examples of transient responses for two different $n$ are shown in Fig. 4.

The IAE curves in Figs 2-3 fully confirm that the equivalence of the loop delays (22) allows to tune the control loop for a different noise attenuation by changing $n$ without influencing significantly dynamics of the setpoint and disturbance responses. This holds despite the fact that the used equivalence of the loop delays has been derived for a double integrator, i.e. by neglecting the plant coefficient $a_1 \neq 0$ (Huba, 2014). Thereby, in terms of $TV_2(u_s)$, the noise impact may be significantly reduced (nearly 10 times), but not so strongly as indicated by the simulation results. By shortening $T_s$ this improvement factor increases. Discrepancy between the simulated and really measured setpoint responses could be explained by an additional noise source from the moment generator (PWM, commutator effect). The much larger differences appearing for the disturbance...
responses are obviously caused by a noise produced by the second motor used as a load torque generator. These differences increase by weakening the primary motor control. This noise impact contributes also to increased IAE values of real responses. For a more detailed study, one should identify these up to now non-modelled noise sources and thus to increase a matching between the simulation and real time control.

Regarding the choice of an optimal filter order \( n \) the measured results do not offer a simple interpretation. In each case it is obvious that, when working with a minimal filter order \( n = 2 \) required for a proper inversion of the plant dynamics, one gets a relatively high noise impact. This may be significantly reduced by \( n > 2 \). To get a deeper insight into the loop properties, a loop sensitivity analysis Åström and Hägglund (2006) might be helpful. For a noise attenuation evaluated at the control output, from the "gang of four", the noise-to-control frequency gains are important. For \( T_d = T_D \), when \( F_d(s) = \exp(-T_d s) \approx 1/(1 + T_d s) \), one gets

\[
C(s) = K_F + K_D s; F_m(s) = K_a/(s^2 + a_1 m s) \\
S_F(s) = F_a(s)/(1 - F_u(s) F_d(s)) \\
L(s) = C(s) S_F(s) F_d(s) F(s) \left(1 + \frac{1}{C(s) F_m(s)}\right) \\
F_{\text{sum}}(s) = \frac{U_r(s) \delta(s)}{\delta(s)} = \frac{C(s) S_F(s)}{1 + L(s)} \frac{1}{1 + \frac{1}{C(s) F_m(s)}}
\]

Fig. 5 shows that for the minimal DO order \( n = 2 \) the high frequency measurement noise amplification is the highest and for \( \omega \to \infty \) it converges to \( |F_{\text{sum}}(\infty)| > 0 \). For this conclusion seems to be in a contrast to the analysis in Sariyıldız and Ohnishi (2013), to get a deeper insight into this problem, a more detailed sensitivity loop analysis, or...
Fig. 6. DO-FPID: Implementation scheme

an analysis by the performance portrait method should be
carried out and confronted with these experimental results.
Recently, results of this paper have been augmented by a
comparison with several traditional PID control structures
(Huba and Bélaï, 2014a) and extended to a case of con-
strained control (Huba and Bélaï, 2014c).

REFERENCES

Control. ISA, Research Triangle Park, NC.
In Proc. 16th Mediterranean Conference on Control and
Automation, 326–331. IEEE, Ajaccio, France.
IFAC Conf. Advances in PID Control PID’12, 2, Part
1.
Han, J. (2009). From PID to Active Disturbance Rejection
Control. Industrial Electronics, IEEE Transactions on,
56(3), 900 –906.
Huba, M. (2012a). Modular disturbance observer based
constrained PI controller design. In Int. Conf. Advances
in Motion Control. IEEE, Sarajevo, BH.
Huba, M. (2012b). Open and flexible P-controller de-
sign. In Int. Conf. Advances in Motion Control. IEEE,
Sarajevo, BH.
of the IPDT plant. IFAC Conf. Advances in PID Control
PID’12.
Disturbance Observer Based Filtered PI Control. Jour-
nal of Process Control, 23, 10, 1379–1400.
Huba, M. (2013b). Modular PID-controller design with
different filtering properties. In 39th Annual Conference
of the IEEE Industrial Electronics Society (IECON).
IEEE, Vienna, Austria.
Huba, M. (2013c). Open flexible PD-controller design for
different filtering properties. In 39th Annual Conference
of the IEEE Industrial Electronics Society (IECON).
IEEE, Vienna, Austria.
limits and optimal PI control for the IPDT plant. Journal
of Process Control, 23, 4, 500–515.
Huba, M. (2014). Tuning of a Filtered Pole Assignment
Controller for an Integral Plant. In 15th Int. Carpathian
Control Conference - ICCC. Velké Karlovice, Czech
Republic.
Approaches to a Positional Servo Control. In 15th
Int. Carpathian Control Conference - ICCC. Velké
Karlovice, Czech Republic.
Huba, M. and Bélaï, I. (2014b). Noise attenuation mo-
tivated controller design. Part I: Speed control. In
Speedam Symposium 2014, Capri, Italy.
Huba, M. and Bélaï, I. (2014c). Noise attenuation mo-
tivated controller design. Part II: Position control.
In Speedam Symposium 2014, Capri, Italy.
PID Position Controller Including Zero-Phase Error
Tracking Performance for Direct Drive Rotation Motor.
JPE, 9(1), 74–84.
Robust PID and Predictive PI Controllers with Con-
strained Control Signal Noise Sensitivity. IFAC Conf.
Advances in PID Control PID’12, 2, Part 1, 175–180.
Ohishi, K., Nakao, M., Ohnishi, K., and Miyachi, K.
(1987). Microprocessor-controlled dc motor for load-
sensitive position servo system. IEEE Trans. Indus-
trial Electronics., IE-34(1), 44 –49.
selbsttätiger Regelungen. 2nd Ed. R. Oldenbourg-Verlag,
München.
disturbance observers for practitioners. In American
Sariyildiz, E. and Ohnishi, K. (2013). Performance and
Robustness Trade-off in Disturbance Observer Design.
In 39th Annual Conference of the IEEE Industrial Elec-
tronics Society (IECON), 3679–3684. IEEE, Vienna,
Austria.
Schrijver, E. and van Dijk, J. (2002). Disturbance ob-
servers for rigid mechanical systems: Equivalence, sta-

dibility, and design. ASME, J. Dyn. Sys., Meas., Control,
124 (4), 539–548.
She, J.H., Xin, X., and Pan, Y. (2011). Equivalent-
input-disturbance approach: analysis and application to
disturbance rejection in dual-stage feed drive control
system. Mechatronics, IEEE/ASME Transactions on,
16(2), 330 –340.
reduction and PID controller tuning. J. Process Control,
13, 291–309.
Żabiński, T. and Trybus, L. (2010). Tuning P-PI and PI-
PI controllers for electrical servos. Bulletin of the Polish