Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Design of a Robust Tracking PD Controller for a Class of Switched Linear Systems with External Disturbances

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Abstract: This paper concerns the design of a robust output feedback tracking controller for a class of Switched Linear Systems (SLS) subject to external disturbances. The proposed synthesis approach, based on a descriptor redundancy formulation, allows to avoid of the crossing terms appearance between the switched Proportional-Derivative (PD) controller's and the switched system's matrices. Using the multiple Lyapunov functional methods, a robust H_∞ output feedback tracking performance has been formulated in terms of set of Linear Matrix Inequality (LMI). The effectiveness of the proposed synthesis procedure has been illustrated by a numerical example

1. INTRODUCTION

Nowadays, several complex systems present hybrid behaviours. These ones are often modelled as Hybrid Systems (HS) by taking into account the interaction between the continuous and the discrete dynamics. Switched Systems (SS), as a class of HS, have attracted a lot of attention due to the widespread application in industrial, biological and economical dynamic processes (Balluchi *et al.,* 2000), (Babaali *et al.,* 2004) and (Belkhiat *et al.,* 2011). Generally, stability and stabilization problems are the main issues in the domain of SS. In this context, several studies have shown that the Lyapunov function techniques are effective to deal with these latter problems in the case of SS (Branicky, 1998), (Daafouz *et al.,* 2002) and (Zhang *et al.,* 2009). For more details about the basic problems in stability and stabilization for SS, the reader can refer to (Zhu *et al.,* 2012), and the references cited therein.

Recently, the output feedback tracking control of switched systems has received a considerable attention mainly with the fast development of switched system theory. In fact, the output feedback tracking control has a close relationship with the stability analysis and stabilization issues. The main objective of tracking control is to deal with the stabilization and the minimization of the error between the outputs of the system and of the desired reference model via designing a controller (Liu *et al.,* 2014). However, only a few works treating the output feedback tracking control for switched systems have been reported (Liu *et al.,* 2014), (Li *et al.,* 2009), (Lian *et al.,* 2013) and (Niu *et al.,* 2013). Then, an exponential L_1 output feedback tracking control for SLS with time-varying delays has been investigated in (Liu *et al.,* 2014). In the same context, the authors of the reference (Li *et al.,* 2009) have studied the problem of output feedback tracking control for a class of switched systems containing stabilizable and unstabilizable subsystems. Based on the average dwell time approach and the Lyapunov theory, the authors in (Lian *et al.,* 2013) propose a new controller design approach to satisfy the robust H_∞ output tracking control for a class of switched systems with time-varying delay under asynchronous switching. Nevertheless, in our knowledge, the output feedback tracking control problem of SLS has not been fully explored, which motivates the present study.

In this paper, a robust H_∞ output feedback tracking control for a class of SLS using switched PD controller is designed. The main contributions of the work consists in the proposition of new synthesis approach which takes account the advantage of descriptor redundancy formulation in order to avoid the appearance of the crossing terms between the controller's and the SLS system's matrices (Tanaka et *al.,* 2007) and (Jabri *et al.,* 2010). Therefore, the proposed approach leads to easier LMI conditions.

The present paper is organized as follows. The problem statement and some preliminaries are given in section 2. In Section 3, based on the Lyapunov function techniques, the robust H_∞ output feedback tracking control for a class of SLS using switched PD controller is developed. Then, sufficient conditions for the existence of a switched PD controller are formulated in terms of LMI. A numerical example illustrating the effectiveness of the proposed synthesis approach is provided in section 4. Section 5 summarized some conclusions and future works.

2. PROBLEM STATEMENT

We consider in this work a class of SLS composed of N linear continuous-time subsystems. Each linear subsystem is defined as follows:

$$\dot{x}(t) = A_q x(t) + B_q u(t) + B_{q_w} w(t) \tag{1}$$

$$y(t) = C_q x(t) \tag{2}$$

with $x(t) \in \Re^n$ is the state vector (unmeasurable), $u(t) \in \Re^m$ is the control input vector, $y(t) \in \Re^p$ is the measurement (output) vector and $w \in \Re^m$ is the L_2-norm bounded external disturbance. A_q, B_q, B_{q_w}, C_q are known matrices with appropriate dimensions, $q \in Q = 1,2,\ldots,N$ is the index indicating the active mode at instant t. q is known at any time.

To specify the desired trajectory, we consider the following reference model:

$$\dot{x}_r(t) = A_r x_r(t) + r(t) \tag{3}$$

$$y_r(t) = H_r x_r(t) \tag{4}$$

with $x_r(t) \in \Re^{n_r}$ and $y_r(t) \in \Re^p$ are the reference state vector and the reference output vector, respectively. $A_r \in \Re^{n_r \times n_r}$ is a specified asymptotically stable matrix and $r(t) \in \Re^{n_r}$ is L_2-norm bounded reference input. H_r is a known matrix with appropriate dimensions.

In order to ensure the robust H_∞ output feedback tracking performance of the overall switched linear systems, we give the following switched PD controllers (Huang *et al.,* 2004), (Ge *et al.,* 2003) and (Wang *et al.,* 2007):

$$u(t) = K_q^P e_r(t) + K_q^D \dot{y}(t) \tag{5}$$

where $e_r(t) = y_r(t) - y(t) \in \Re^p$ is the tracking error, K_q^P is the proportional gain and K_q^D is the derivative gain.

Notations. As usual, X^{-1} and X^T are the inverse and the transpose of matrix X, respectively. In a symmetric matrix, the star symbol $(*)$ indicates the transposed block in the symmetric position. Moreover, $I_.$ denotes an identity matrix with appropriate dimension.

The problem considered in this paper can be formulated as follows:

Problem 1. The objective consists in the design of the controller (5) such that the switched system (1)-(2) has a robust H_∞ output feedback tracking performance.

Definition 1. The switched linear systems (1)-(2) is said to have a robust H_∞ output feedback tracking performance, if the following conditions are satisfied:

- with zero disturbance input condition $w(t) \equiv 0$, the closed-loop switched system is stable.
- for all non zero $w(t) \in L_2[0\ \infty)$, under zero initial condition $x(t_0) \equiv 0$, it holds that:

$$\int_0^\infty e_r^T(t) e_r(t) dt \le \gamma^2 \int_0^\infty \left(w^T(t) w(t) + r^T(t) r(t) \right) dt$$

where γ is a positive constant.

As usual, the closed-loop dynamics consists on substituting the controller's equation (5) into the system's equation (1). This leads to:

$$\begin{aligned}\left(I_{n\times n} - B_q K_q^D C_q\right)\dot{x}(t) = \left(A_q - B_q K_q^P C_q\right) x(t) + \\ B_q K_q^P H_r x_r(t) + B_{q_w} w(t)\end{aligned} \tag{6}$$

Hence, the closed-loop dynamics (6) implicates several crossing terms between the gains controller K_q^P, K_q^D and the system's matrices ($B_q K_q^D C_q$, $B_q K_q^P C_q$ and $B_q K_q^P H_r$). These terms lead to very conservative conditions concerning the design of switched PD controller. In order to decouple the crossing terms appearing in equation (6) and to make easier LMI conditions, we use the descriptor redundancy approach (Tanaka et *al.,* 2007) and (Jabri *et al.,* 2010). Thus, we consider the following augmented state variable.

$$\tilde{x}^T(t) = \left[x^T(t)\ x_r^T(t)\ e_r^T(t)\ \dot{y}^T(t) \right],$$
$$\tilde{w}^T(t) = \left[r^T(t)\ w^T(t) \right]$$

Then, the equations of the switched system (1)-(2), the reference model (3)-(4) and the controller (5) are combined to obtain the following augmented system:

$$E\dot{\tilde{x}}(t) = \tilde{A}_q \tilde{x}(t) + \tilde{B}_q \tilde{u}(t) + \tilde{B}_{q_w} \tilde{w}(t) \tag{7}$$

$$e_r(t) = \tilde{C}_q \tilde{x}(t) \tag{8}$$

$$\tilde{u}(t) = \tilde{K}_q \tilde{x}(t) \tag{9}$$

where

$$E = \begin{bmatrix} I_{n\times n} & 0 & 0 & 0 \\ 0 & I_{n_r \times n_r} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ \tilde{B}_q = \begin{bmatrix} B_q \\ 0 \\ 0 \\ C_q B_q \end{bmatrix},$$

$$\tilde{A}_q = \begin{bmatrix} A_q & 0 & 0 & 0 \\ 0 & A_r & 0 & 0 \\ -C_q & H_r & -I_{p\times p} & 0 \\ C_q A_q & 0 & 0 & -I_p \end{bmatrix},\quad \tilde{B}_{q_w} = \begin{bmatrix} 0 & B_{q_w} \\ I_{n_r \times n_r} & 0 \\ 0 & 0 \\ 0 & C_q B_{q_w} \end{bmatrix},$$

$\tilde{C}_q = \begin{bmatrix} -C_q & H_r & 0 & 0 \end{bmatrix}$ and $\tilde{K}_q = \begin{bmatrix} 0 & 0 & K_q^P & K_q^D \end{bmatrix}$.

Therefore, the closed-loop system is given by:

$$E\dot{\tilde{x}}(t) = \left(\tilde{A}_q + \tilde{B}_q \tilde{K}_q\right)\tilde{x}(t) + \tilde{B}_{q_w} \tilde{w}(t) \tag{10}$$

Note that the system (7) is called switched descriptor system ($rank(E) < \dim(E)$). Using the augmented system, the problem 1 can be reformulated as follows:

Problem 2. The objective consists in the design of the controller (9) such that the system (7) has a robust H_∞ output feedback tracking performance.

At the end of this section, we introduce some definitions for the development of our results.

Definition 2. The switched descriptor system (7) is said to have a robust H_∞ output feedback tracking performance, if the following conditions are satisfied:

- with zero disturbance input condition $\tilde{w}(t) \equiv 0$, the closed-loop switched descriptor system (10) is admissible.
- for all non zero $\tilde{w}(t) \in L_2[0\ \infty)$, under zero initial condition $\tilde{x}(t_0) \equiv 0$, it holds that:

$$\int_0^\infty e_r^T(t) e_r(t) dt \le \gamma^2 \int_0^\infty \tilde{w}^T(t) \tilde{w}(t) dt \tag{11}$$

where γ is a positive constant.

Definition 3. The switched descriptor system (7) is said admissible if it is regular, impulse free and stable.

3. ROBUST PD CONTROLLER DESIGN

The main goal of this paper is to propose a sufficient LMI conditions in order to obtain the values of the gain matrices K_q^P and K_q^D such as the robust H_∞ output feedback tracking performance is satisfied. The main result is summarized in the following theorem.

Theorem 1. Given positive scalars κ, $\mu_{qq^+} \le 1$, for $q, q^+ \in Q$, $q \ne q^+$, if there exist matrices $X_q^{11} = X_q^{11T} > 0$, $X_q^{22} = X_q^{22T} > 0$, $X_q^{33} = X_q^{33T}$, $X_q^{44} = X_q^{44T}$, Y_q^p, Y_q^D such that the following LMIs hold:

$$\phi_q = \begin{bmatrix} \phi_q^{11} & 0 & \phi_q^{13} & \phi_q^{14} \\ (*) & \phi_q^{22} & \phi_q^{23} & 0 \\ (*) & (*) & \phi_q^{33} & \phi_q^{34} \\ (*) & (*) & (*) & \phi_q^{44} \end{bmatrix} < 0 \tag{12}$$

$$\Pi_{qq^+} = \begin{bmatrix} -\mu_{qq^+} X_q & X_q \\ (*) & -X_{q^+} \end{bmatrix} \le 0 \tag{13}$$

$$\Xi_q = \begin{bmatrix} \Xi_q^{11} & 0 & \phi_q^{13} & \Xi_q^{14} & -X_q^{11} C_q^T \\ (*) & \Xi_q^{22} & \phi_q^{23} & 0 & X_q^{22} H_r^T \\ (*) & (*) & \phi_q^{33} & \phi_q^{34} & 0 \\ (*) & (*) & (*) & \Xi_q^{44} & 0 \\ (*) & (*) & (*) & (*) & -I_{p\times p} \end{bmatrix} \le 0 \tag{14}$$

Then, the switched descriptor (7) is admissible and the robust H_∞ output feedback tracking performance is guaranteed with attenuation level $\sqrt{\kappa^{-1}}$. Moreover, the controller gains are constructed by $K_q^P = Y_q^P \left(X_q^{33}\right)^{-1}$ and $K_q^D = Y_q^D \left(X_q^{44}\right)^{-1}$.

Where
$\phi_q^{11} = X_q^{11} A_q^T + A_q X_q^{11}$, $\phi_q^{13} = -X_q^{11} C_q^T + B_q Y_q^P$,
$\phi_q^{14} = X_q^{11} A_q^T C_q^T + B_q Y_q^D$, $\Xi_q^{22} = \phi_q^{22} + \kappa I_{n_r \times n_r}$, $\phi_q^{23} = X_q^{22} H_r^T$,
$\phi_q^{22} = X_q^{22} A_r^T + A_r X_q^{22}$, $\Xi_q^{11} = \phi_q^{11} + \kappa B_{q_w} B_{q_w}^T$,
$\phi_q^{34} = Y_q^{PT} B_q^T C_q^T$, $X_q = diag\left(X_q^{11} \quad X_q^{22} \quad X_q^{33} \quad X_q^{44}\right)$,
$\phi_q^{33} = -X_q^{33} I_{p\times p} - I_{p\times p} X_q^{33}$, $\Xi_q^{14} = \phi_q^{14} + \kappa B_{q_w} B_{q_w}^T C_q^T$,
$\Xi_q^{44} = \phi_q^{44} + \kappa C_q B_{q_w} B_{q_w}^T C_q^T$ and
$\phi_q^{44} = -X_q^{44} I_{p\times p} - I_{p\times p} X_q^{44} + Y_q^{DT} B_q^T C_q^T + C_q B_q Y_q^D$.

Proof. Without loss of generality, we assume that the descriptor system (7) is regular and impulse free (Sajja *et al.*, 2013). According to the definition 2, the proof is composed of two steps.

- Step 1:

With zero disturbance input condition $\tilde{w}(t) \equiv 0$, the objective is to give a sufficient conditions to ensure that the closed-loop switched descriptor system (10) is stable, Then it is admissible. Therefore, we consider the following multiple Lyapunov-like functional candidate:

$$V_q(\tilde{x}(t)) = \tilde{x}^T(t) E^T P_q \tilde{x}(t)$$

with $E^T P_q = P_q^T E > 0$ and $q \in Q = 1, 2, \ldots, N$. Hence, P_q is considered diagonal matrix:

$P_q = P_q^T = diag\left(P_q^{11} \quad P_q^{22} \quad P_q^{33} \quad P_q^{44}\right)$ with $P_q^{ii} = P_q^{iiT} > 0$ for $i = \{1, 2\}$ and $P_q^{ii} = P_q^{iiT}$ for $i = \{3, 4\}$.

The closed-loop switched descriptor is stable if the conditions (15) and (16) are satisfied:

$$\dot{V}_q(\tilde{x}(t)) < 0 \tag{15}$$

and for $q = 1, \ldots, N$, $q^+ = 1, \ldots, N$ and $q \ne q^+$

$$V_{q^+}(\tilde{x}(t)) \le \mu_{qq^+} V_q(\tilde{x}(t)) \tag{16}$$

where the decreasing rate $\mu_{qq^+} \le 1$ is positive scalar describing the Lyapunov-like evolution at the switching time $t_{q \to q^+}$.

We develop now the condition (15).

$$\begin{aligned} \dot{V}_q(\tilde{x}(t)) &= \dot{\tilde{x}}^T(t) E^T P_q \tilde{x}(t) + \tilde{x}^T(t) P_q E \dot{\tilde{x}}(t) < 0 \\ &= \tilde{x}^T(t) \left[\left(\tilde{A}_q + \tilde{B}_q \tilde{K}_q\right)^T P_q + P_q \left(\tilde{A}_q + \tilde{B}_q \tilde{K}_q\right) \right] \tilde{x}(t) < 0 \end{aligned} \tag{17}$$

The condition (17) is verified if

$$\left(\tilde{A}_q + \tilde{B}_q \tilde{K}_q\right)^T P_q + P_q \left(\tilde{A}_q + \tilde{B}_q \tilde{K}_q\right) < 0$$

Multiplying by P_q^{-1} and doing the following change of variable $X_q = P_q^{-1}$, we obtain:

$$X_q\left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right)^T + \left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right)X_q < 0 \quad (18)$$

where $X_q = X_q^T = diag\left(X_q^{11} \quad X_q^{22} \quad X_q^{33} \quad X_q^{44}\right)$, with $X_q^{ii} = X_q^{iiT} > 0$ for $i = \{1,2\}$ and $X_q^{ii} = X_q^{iiT}$ for $i = \{3,4\}$.

We substitute $\tilde{A}_q$, $\tilde{B}_q$, $\tilde{K}_q$ in (18). After considering the following change of variable $Y_q^P = K_q^P X_q^{33}$, $Y_q^D = K_q^D X_q^{44}$, the LMI (12) is provided.

Now, let us focus on the stability condition (16). Their aim is to ensure the global behavior of the like-Lyapunov function at the switching time $t_{q\to q^+}$. We assume that, we have not state jump at switching time.

According to the condition (16), we can write:

$P_{q^+} \le \mu_{qq^+}P_q$, for $q = 1,\ldots,N$, $q^+ = 1,\ldots,N$ and $q \ne q^+$

which implicates

$$X_{q^+}^{-1} \le \mu_{qq^+}X_q^{-1}$$

Multiplying by X_q, we obtain:

$$X_q X_{q^+}^{-1} X_q - \mu_{qq^+} X_q X_q^{-1} X_q \le 0$$

Applying Schur's complement, the LMI (13) is provided.

- Step 2:

In this step, we consider the external disturbances $\tilde{w}(t) \in L_2[0\ \infty)$, under zero initial condition $\tilde{x}(t_0) \equiv 0$.

From the stability condition (11), we can develop

$$\int_0^\infty \left(e_r^T(t)e_r(t) - \gamma^2\tilde{w}^T(t)\tilde{w}(t)\right)dt \le 0 \quad (19)$$

Let $V_q(\tilde{x}(t)) = \tilde{x}(t)E^T P_q \tilde{x}(t) > 0$, with $E^T P_q = P_q^T E > 0$, be a Lyapunov-like function candidate. Hence, the inequality (19) can be written as:

$$J = \int_0^\infty \left(e_r^T(t)e_r(t) - \gamma^2\tilde{w}^T(t)\tilde{w}(t) + \frac{dV_q(\tilde{x}(t))}{dt}\right)dt$$
$$-V_q(\tilde{x}(t)) \le 0$$

$J \le 0$ if

$$e_r^T(t)e_r(t) - \gamma^2\tilde{w}^T(t)\tilde{w}(t) + \dot{V}_q(\tilde{x}(t)) \le 0 \quad (20)$$

The latter condition (20) can be reformulated such as:

$$\begin{bmatrix}\tilde{x}(t)\\ \tilde{w}(t)\end{bmatrix}^T \begin{bmatrix}\Lambda_q & P_q\tilde{B}_{q_w}\\ (*) & -\gamma^2 I_{2p\times 2p}\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ \tilde{w}(t)\end{bmatrix} \le 0 \quad (21)$$

with $\Lambda_q = \left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right)^T P_q + P_q\left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right) + \tilde{C}_q^T\tilde{C}_q$.

Applying the inverse of Schur's complement, we can write (21) as follows:

$$\left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right)^T P_q + P_q\left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right) + \tilde{C}_q^T\tilde{C}_q$$
$$+\kappa P_q\tilde{B}_{q_w}\tilde{B}_{q_w}^T P_q \le 0 \quad (22)$$

with $\kappa = \left(\gamma^2\right)^{-1}$.

Multiplying by P_q^{-1} and considering the following change of variable $X_q = P_q^{-1}$, we obtain.

$$X_q\left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right)^T + \left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right)X_q$$
$$+X_q\tilde{C}_q^T\tilde{C}_q X_q + \kappa\tilde{B}_{q_w}\tilde{B}_{q_w}^T \le 0 \quad (23)$$

Using Schur's complement, the inequality (23) can be written as follow.

$$\begin{bmatrix}\Theta_q^{11} & X_q\tilde{C}_q^T\\ (*) & -I_{p\times p}\end{bmatrix} \le 0 \quad (24)$$

with $\Theta_q^{11} = X_q\left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right)^T + \left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right)X_q + \kappa\tilde{B}_{q_w}\tilde{B}_{q_w}^T$.

We substitute $\tilde{A}_q$, $\tilde{B}_q$, $\tilde{K}_q$, $\tilde{B}_{q_w}$ in the inequality (24). Using the following change of variable $Y_q^P = K_q^P X_q^{33}$, $Y_q^D = K_q^D X_q^{44}$, the LMI (14) is provided. ■

In order to simplify the conditions given in theorem 1, let consider the inequality (23), with

$$\lambda = X_q\left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right)^T + \left(\tilde{A}_q + \tilde{B}_q\tilde{K}_q\right)X_q$$

and $\beta = X_q\tilde{C}_q^T\tilde{C}_q X_q + \kappa\tilde{B}_{q_w}\tilde{B}_{q_w}^T$ such that $\beta > 0$. Then the inequality (18) ($\lambda < 0$) is verified when the condition (23) ($\lambda + \beta \le 0$ with $\beta > 0$) is satisfied. Hence, the theorem 1 can be resumed in the following corollary.

Corollary 1. Given positive κ, $\mu_{qq^+} \ge 1$, for $q, q^+ \in Q$, $q \ne q^+$, if there exist matrices $X_q^{11} = X_q^{11T} > 0$, $X_q^{22} = X_q^{22T} > 0$, $X_q^{33} = X_q^{33T}$, $X_q^{44} = X_q^{44T}$, Y_q^p, Y_q^D such that the following LMI hold:

$$\Pi_{qq^+} = \begin{bmatrix}-\mu_{qq^+}X_q & X_q\\ (*) & -X_{q^+}\end{bmatrix} \le 0 \quad (25)$$

$$\Xi_q = \begin{bmatrix}\Xi_q^{11} & 0 & \phi_q^{13} & \Xi_q^{14} & -X_q^{11}C_q^T\\ (*) & \Xi_q^{22} & \phi_q^{23} & 0 & X_q^{22}H_r^T\\ (*) & (*) & \phi_q^{33} & \phi_q^{34} & 0\\ (*) & (*) & (*) & \Xi_q^{44} & 0\\ (*) & (*) & (*) & (*) & -I_{p\times p}\end{bmatrix} \le 0 \quad (26)$$

Then, the switched descriptor (7) is admissible and the robust H_∞ output feedback tracking performance is guaranteed with attenuation level $\sqrt{\kappa^{-1}}$. Moreover, the controller gains are constructed by $K_q^P = Y_q^P \left(X_q^{33}\right)^{-1}$ and $K_q^D = Y_q^D \left(X_q^{44}\right)^{-1}$.

where

$\phi_q^{11} = X_q^{11} A_q^T + A_q X_q^{11}$, $\phi_q^{13} = -X_q^{11} C_q^T + B_q Y_q^P$,
$\phi_q^{14} = X_q^{11} A_q^T C_q^T + B_q Y_q^D$, $\Xi_q^{22} = \phi_q^{22} + \kappa I_{n_r \times n_r}$, $\phi_q^{23} = X_q^{22} H_r^T$,
$\phi_q^{22} = X_q^{22} A_r^T + A_r X_q^{22}$,
$\Xi_q^{11} = \phi_q^{11} + \kappa B_{q_w} B_{q_w}^T$, $\phi_q^{34} = Y_q^{PT} B_q^T C_q^T$,
$X_q = diag\left(X_q^{11} \quad X_q^{22} \quad X_q^{33} \quad X_q^{44}\right)$,
$\phi_q^{33} = -X_q^{33} I_{p\times p} - I_{p\times p} X_q^{33}$, $\Xi_q^{14} = \phi_q^{14} + \kappa B_{q_w} B_{q_w}^T C_q^T$,

$\Xi_q^{44} = \phi_q^{44} + \kappa C_q B_{q_w} B_{q_w}^T C_q^T$ and
$\phi_q^{44} = -X_q^{44} I_{p\times p} - I_{p\times p} X_q^{44} + Y_q^{DT} B_q^T C_q^T + C_q B_q Y_q^D$.

4. SIMULATION AND RESULTS

In this section, a numerical example is provided to illustrate the effectiveness of the proposed approach. We consider a switched system S with two modes and a reference system S_r.

Switched system S :

Mode 1:

$$A_1 = \begin{bmatrix} -2.6 & 1 \\ 0.5 & -1.1 \end{bmatrix},\ B_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix},\ B_{1_w} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},\ C_1 = \begin{bmatrix} 4 & 2 \end{bmatrix}.$$

Mode 2:

$$A_2 = \begin{bmatrix} -1 & 0.25 \\ 0.39 & -1.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, B_{2_w} = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix},\ C_2 = \begin{bmatrix} 2 & 3 \end{bmatrix}.$$

Reference system S_r :

$$A_r = \begin{bmatrix} -2105.8 & 268.7 \\ 2105.2 & -2154.2 \end{bmatrix},\ H_r = \begin{bmatrix} 1 & -0.3 \end{bmatrix}.$$

Reminding that the initial condition is assumed equal to zero ($x(t_0) \equiv 0$).

A PD controller, composed of a set of two controllers, is synthesized based on the matrix inequalities (25)-(26) of corollary 1 via the Matlab LMI toolbox. Hence, for the attenuation level $\kappa = 0.01$ and the decreasing rates $\mu_{12} = \mu_{21} = 0.15$, we obtain the PD controller parameters as follows:

$K_1^P = 25.3$, $K_1^D = -0.001$, $K_2^P = 121$, $K_2^D = 0.01$.

In order to illustrate the effectiveness of the proposed approach, simulation curves are presented in Figs. 1-3. Fig. 1 shows the switching signal evolution of the switched system S, where the dwell time of each subsystem is considered respectively $T_1 = 3s$ for the first subsystem and $T_2 = 2s$, for the second.

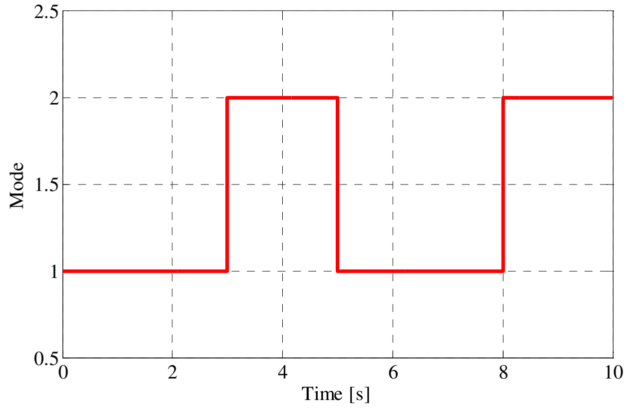

Fig. 1. Switching signal evolution of the switched system.

For simulations, the reference signal $r(t)$ is considered as:

$$r(t) = \begin{cases} 500 \times \sin(0.04 \times t) & t \le 5s \\ 0 & t > 5s \end{cases}$$

and the external disturbances signal $w(t)$ as a white noise sequence.

The state evolutions of the closed-loop switched system S with external disturbances as well as the tracking performances are given in figs 2 and 3, respectively.

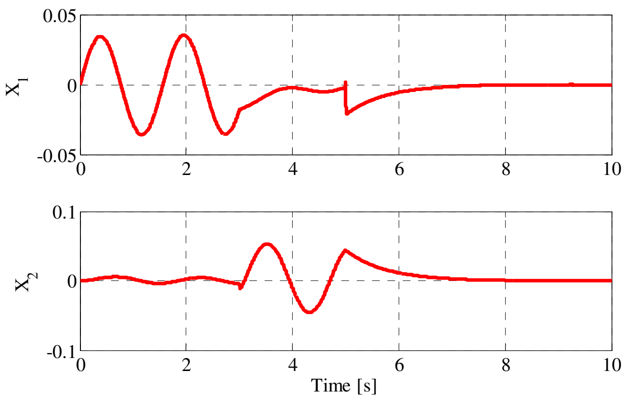

Fig. 2. State evolutions of the closed-loop switched system.

As expected, the output $y(t)$ of the switched system S can track the desired signal $y_r(t)$ after a finite time interval.

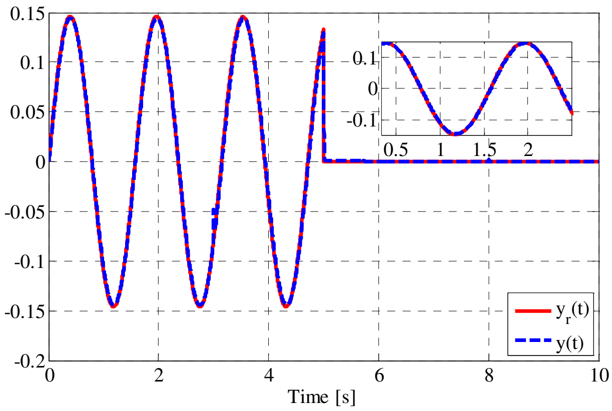

Fig. 3. Trajectory of the switched system's and the reference model's outputs.

5. CONCLUSION

In this paper, a robust H_∞ output feedback tracking control has been considered for a class of switched system with external disturbances. Thanks to the descriptor redundancy formulation of the closed-loop dynamics, crossing terms between the controller's and the switched system's matrices have been avoided. Beside, multiple Lyapunov functional method has been employed. These lead to easier LMI conditions for stability analysis and controller design. Finally, the efficiency of the proposed approach has been illustrated by a numerical example. Moreover, in this work, it is assumed that the SLS modes are known at any time. Further relaxation of this assumption and extension of the proposed approach to more general hybrid systems will be the focus of future work.

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