Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Optimized Operation of a Micro-grid for Energy Resources ⋆

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Abstract: This paper presents the development of a dynamic optimization model to manage the generated energy in a micro-grid. Without loss of generality, the micro-grid consists of the following components: a wind energy system, an energy storage element, a load, and the interconnection with the utility grid. The optimization scheme considers the minimization of the associated cost due to the purchase of energy from the utility grid and simultaneously maximizing the profits associated with the sale of the generated energy to the utility grid. This energy generation and optimization schemes could represent the operation of a micro-grid, which considers different renewable resources and its energy management. The optimization model considers the dynamics of charge and discharge of the storage element, energy prices and forecasting for wind energy, buying and selling prices, and load energy demand, such that an efficient use of the generated energy in a micro-grid can be achieved.

1. INTRODUCTION

This contribution proposes an optimization scheme for an electrical micro-grid based on the forecast of time series, such as load energy demand, electrical energy prices and the forecast for generated wind energy in a micro-grid. The forecast task is realized by techniques for modelling time series, such as the Autoregressive integrated Moving Average (ARIMA) model [13, 10, 8, 3] and the Exponential Smoothing (ETS) model [4, 11, 18].

Since renewable energy is intermittent in nature, it motivates to developing an optimization algorithm to manage the energy in a micro-grid such that energy in the load is ensured by properly storing/extracting energy in/from an energy storage device and determining the amount of energy to be used from the utility grid. One of the main goals of the optimization scheme is to obtain the maximum profit by the sale of the generated or stored energy based on forecast information. The results of the optimization algorithm could be used as the reference values for power converters to manage the energy in a micro-grid.

The concept of micro-grid [14, 15, 6, 16] assumes a group of loads and micro-sources operating as a system that provides from energy to a local area. The micro-grid can be seen as a system designed to meet special needs, such as support local voltages, correct the voltage sags, etc.; moreover, the micro-grid could have the ability to respond in few period of time to meet the system requirements such as absorption or supply of harmonics, among others.

⋆ This work was supported by CONACyT, Mexico, under project 131678

The organization of this paper is described as follows. Section 2 describes the components of the micro-grid, whose operation is required to be optimized. In Section 3 it is presented the time series forecast models for load energy demand, buying and selling prices, and wind energy generation. Section 4 presents the optimization scheme for a micro-grid, which uses the results from the forecast process. Finally, Section 5 presents the conclusions.

2. COMPONENTS OF THE MICRO-GRID

This section describes the components of the micro-grid to be optimized in this paper, which includes wind energy generation, an energy storage device, a load and the connection with the utility grid to interchange energy. The addressed micro-grid has a power capacity of 1 kW, which is the average power to be consumed in a small house.

2.1 Wind Energy

It is possible to convert wind energy to electrical energy by generators [5]. The mechanical power of a wind turbine $P_{W_m,k}$ at time k is defined as

$$P_{W_m,\,k} = \begin{cases} 0, & v_k < v_1 \;\text{ or }\; v_3 \leq v_k \\ \dfrac{1}{2}\, C_p\, \rho\, A\, v_k^3, & v_1 \leq v_k < v_2 \\ P_{W_{max}}, & v_2 \leq v_k < v_3 \end{cases} \tag{1}$$

where C_p is the power coefficient which relates the amount of wind energy that is transferred to the electrical generator. In practice, this coefficient takes values between 0.25 and 0.45 [5]. The parameter ρ is the air density with approximate value of $1.225\,kg/m^3$, A is the swept area of the rotor in m^2 and v_k is the wind speed in m/s at

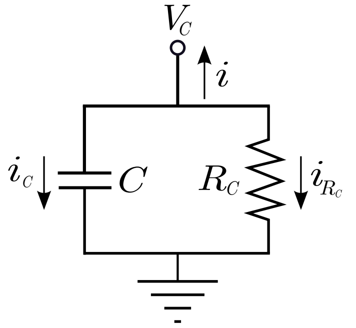

Fig. 1. Energy storage device.

time k; v_1 is minimum wind speed to generate energy, v_2 is the nominal wind speed for power generation and v_3 is the maximum wind speed allowed by the generator according to its construction and generation capacity. For an average wind speed v_{av} (m/s), it is common to consider $v_1 = 0.6v_{av}$, $v_2 = 1.5v_{av} \sim 1.75v_{av}$ and $v_3 = 3v_{av}$ [5].

Finally, the mechanical power $P_{W_m,k}$ is converter to electrical power $P_{W,k}$ through of a generator (with an efficiency η) as

$$P_{W,\,k} = \eta\, P_{W_m,\,k}. \tag{2}$$

2.2 Energy Storage

Without lost of generality, it is considered as energy storage device a supercapacitor as the described in Fig. 1. The continuous-time voltage dynamics of the charge/discharge of this device is given as

$$\dot{V}_c = -\frac{V_C}{R_C\, C} - \frac{1}{C}\, i$$

where V_C is the capacitor voltage, i is the current in the storage device, C is the capacitance and R_C is the internal resistance of the capacitor. Considering that the model of the supercapacitor is a linear one, an exact discretization [2] can be used to obtain a discrete-time dynamical model as

$$V_{C,k+1} = \alpha\, V_{C,k} + \beta\, i_k \tag{3}$$

where $\alpha = e^{-T_s/R_C\,C}$, $\beta = -R_C\left(1 - e^{-T_s/R_C\,C}\right)$ and T_s is the sampling time. Thus, the electrical power of this device at time k is given by $P_{S,k} = V_{C,k}\, i_k$. Additionally, a power constraint can be imposed to limit the energy exchange as

$$P_{S^-_{max}} \leq P_{S,\,k} \leq P_{S_{max}} \tag{4}$$

where $P_{S^-_{max}}$ is the maximum power stored in the supercapacitor and $P_{S^+_{max}}$ is the maximum power extracted form the storage device, or equivalently

$$V_{C_{min}} \leq V_{C,k} \leq V_{C_{max}} \qquad \text{and} \qquad i^-_{max} \leq i_k \leq i^+_{max}$$

where $V_{C_{min}}$ and $V_{C_{max}}$ are the minimum and maximum voltage value allowed for the capacitor, respectively, and i^-_{max} is the maximum amount of current injected in the storage device and i^+_{max} is the maximum amount of current extracted from the storage device.

2.3 Load in the Micro-grid

The energy consumption due to the load connected to the micro-grid can be represented in a general form as

$$P_{L_{min}} \leq P_{L,\,k} \leq P_{L_{max}} \tag{5}$$

where $P_{L,\,k}$ is the power of the load at time k, $P_{L_{min}}$ and $P_{L_{max}}$ are the minimum and maximum consumed power by the load, respectively.

2.4 Energy from/to the Utility Grid

The analysis and design procedure realized in this paper consider that a large amount of energy can be extracted by the micro-grid from the utility grid to provide energy to the load and to the storage device. Further, it is considered that a large amount of energy can be injected to the utility grid from the renewable resource and from the storage device in the micro-grid. Hence, a restriction of the power extracted or supplied to the utility grid is given as

$$P_{G^-_{max}} \leq P_{G,\,k} \leq P_{G^+_{max}} \tag{6}$$

where $P_{G,\,k}$ is the utility grid power at instant k, $P_{G^-_{max}}$ is the maximum power that can be generated in the micro-grid and inject the utility grid, while $P_{G^+_{max}}$ is the maximum power that can be extracted from the utility grid to provide of energy to the micro-grid.

3. TIME SERIES FORECASTING

This section gives a review for time series and forecasting techniques. Then, these tools are used to forecast time series, which will be required in the optimization scheme of the micro-grid.

For convenience purposes it is required to know or predict the behavior of a phenomena or a variable in order to plan or take decisions such that a desired objective can be accomplished. Hence, the analysis of time series can be used in order to forecast the future values for these variables, where the forecast process is based on past values of these variables. Thus, for a time series given as $Y_1\, Y_2$, ..., Y_t, it is desired to describe the behavior of that variable by means of a model, which is used to forecast future values.

Definition 1. (Time Series). A time series, denoted by $Y_1, Y_2, \ldots, Y_t$, is a family of random variables, observations or measurements ordered according to an unit of time t [13].

The models for time series analysis are generally defined by three components: Trend T_t , seasonal S_t and a random component (a stochastic process) ε_t. These components are defined as:

Definition 2. (Trend). It is a function T_t describing the slow evolution and long-term average level of the series [8].

Definition 3. (Seasonal Component). The seasonal component S_t is the trend of the data of the time series, which presents a behavior that is repeated every L periods of time [12].

Definition 4. (Random Component). The random component ε_t are those movements which do not show a recognizable and periodical behavior, and they are considered random and assumed to be independent of each one, whit zero mean and variance σ^2 [17, 1].

In general, two models are used to describe a time series: the additive model and the multiplicative model. The additive model supposes that the value of the time series is composed of the addition of the three components as

$$Y_t = T_t + S_t + \varepsilon_t. \tag{7}$$

On the other hand, the multiplicative model considers that the value of the time series is composed of the multiplication of the three components as

$$Y_t = T_t S_t \varepsilon_t. \tag{8}$$

From the available information (past values of the series), the forecast analysis consists of estimating T_t and S_t as $\hat{T}_t$ and $\hat{S}_t$, respectively, then they are extracted from Y_t to obtain $\hat{\varepsilon}_t = Y_t - \hat{T}_t - \hat{S}_t$. Further, the resulting series $\hat{\varepsilon}_t$ is modeled and estimated. Finally, a time series model $\hat{Y}_t = \hat{T}_t + \hat{S}_t + \hat{\varepsilon}_t$ is obtained, and estimated future values of this series can be computed as $\hat{Y}_{T+h} = \hat{T}_{t+h} + \hat{S}_{T+h} + \hat{\varepsilon}_{T+h}$, where $h = 1, 2, \ldots, m$, with m the number of observations to be predicted.

The common forecast techniques to deal with time series are ARIMA and ETS, which are described as follows:

3.1 ARIMA

In late 60's, Box and Jenkins developed the methodology ARIMA for modeling time series. ARIMA is a statistical model that uses variations and statistical data regressions in order to forecast future values of the series. An ARIMA model is usually expressed as $ARIMA\,(p, d, q)$, where the parameters p, d and q are non-negative integers, which indicate the order of the model components such as autoregressive, integrated and moving average, respectively.

The ARIMA model can be generalized by considering the effect of seasonality. In this case, it is obtained a Seasonal ARIMA (SARIMA) model, which is denoted as $SARIMA\,(p, d, q)\,(P, D, Q)_s$ [10] and described as

$$\Phi\,(B^s)\,\phi\,(B)\,(1 - B^s)^D\,(1 - B)^d\,Y_t = \Theta\,(B^s)\,\theta\,(B)\,\varepsilon_t \tag{9}$$

where $\Phi\,(B^s)$) is the polynomial corresponding to the autoregressive part (AR) of order P, $\Theta\,(B^s)$ is the polynomial corresponding of the moving average part (MA) of order Q, $\phi\,(B)$ is the polynomial corresponding to the autoregressive (AR) of order p and $\theta\,(B)$ is the polynomial corresponding to the part of moving averages (MA) of order q, d is the number of differentiation so that the series is stationary, D is the number of seasonal differentiation so that the series is stationary, B is the shift operator in time $(B^j Y_t = BY_{t-j})$ and s is the seasonal frequency [7].

3.2 ETS

Exponential smoothing method was developed by Robert G. Brown [4] in 1956; however, a modelling framework which incorporates a model selection procedure was developed until recently by [11, 18]. Exponential smoothing is usually expressed as ETS, where the three letters refer to three components: error, trend and seasonality. The notation $ETS\,(\cdot, \cdot, \cdot)$ is used to represent the model of a time series, besides describes the type of the model components [9], for instance, $ETS\,(A, N, A)$ refers to a model with additive errors (denoted by a A in the model), no trend (denoted by N) and additive seasonality (denoted by A). ETS is a technique for time series prediction that exponentially weight the historical data, so that the most recent data may have greater weight with respect to (w.r.t.) previous data.

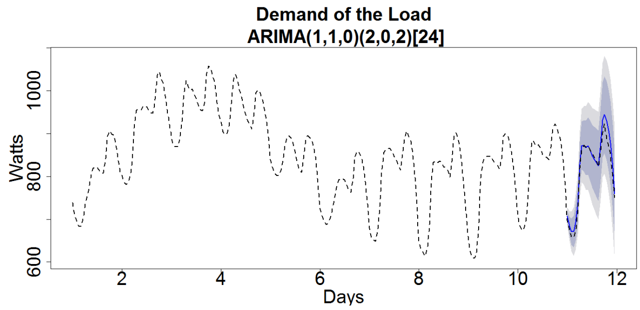

Fig. 2. Electrical demand forecasting (The dashed line shows the real series. The continuous-line shows the forecast of the series)

3.3 Forecasting for Demand, Costs and Wind Energy

In this paper, the series models are determined automatically by the software R, which is an open source software [R Development Core Team], for object-oriented programming and dedicated to financial and statistical computations. This software includes the package *forecast*, which is used to obtain the ARIMA and ETS models. Thus, it is obtained a model for the time series of demand, cost and wind energy. The models are used to perform a prediction of 24 values of each series, in intervals of 1 hour, i.e., it is realized a forecast of 24 hrs. This information will be used for the optimization algorithm to manage the micro-grid energy in order to decide the amount and direction of the stored energy and the energy extracted/injected from/to the utility grid.

Electrical Demand Forecasting of the Load The time series corresponding to the load demand was taken form the series of New York City, USA, during the period of January 2008 to February 2013, whose data were taken hourly [PJM Interconnection]. For our convenience, the demand values are scaled to meet the demand capacity specification of $1\,kW$, in accordance with the power of the proposed micro-grid. Using the software R, it is obtained automatically the models ARIMA and ETS by using historical data of last 10 days. The obtained ARIMA and ETS models, with their respective root-mean-square-error (RMSE), result in

$$ARIMA(1,1,0)(2,0,2)_{24} \;\; with \;\; RMSE = 9.21323$$
$$ETS(A, N, A) \;\; with \;\; RMSE = 11.7064.$$

The ARIMA model has a smaller RMSE error w.r.t. the ETS model; therefore, the ARIMA model is used for forecasting this series. The forecast result is shown in Fig. 2.

Forecasting for Energy Prices The series of prices is taken from [PJM Interconnection], where data refer to the purchase price of electric energy in the city of New York during the period of January 2008 to February 2013. Using the software R, the models ARIMA and ETS, and its respective errors, are given as

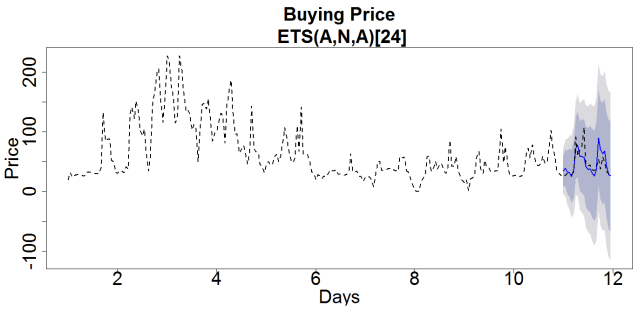

Fig. 3. Forecast of the purchase price of electrical energy (The dashed-line shows the real series. The continuous-line shows the forecast of the series)

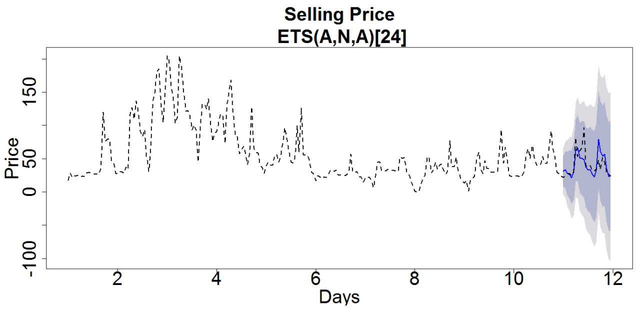

Fig. 4. Forecast of sale price of electrical energy (The dashed-line shows the real series. The continuous-line displays the forecast of the series)

$$ARIMA(4,1,2)(2,0,0)_{24} \;\; with \;\; RMSE = 20.94120$$
$$ETS(A,N,A) \;\; with \;\; RMSE = 19.42686.$$

The ETS model is selected based on its smaller RMSE w.r.t. the error of the ARIMA model. The result of the prediction is depicted in Fig. 3.

For the series corresponding to the sale prices of the injected energy toward the utility grid, it is considered that an energy selling price is 10% lower than the purchase price, that is, the selling price is the purchase prices series scaled by a factor of 0.9. Fig. 4 shows the forecast of selling price.

Forecasting for Wind Energy From data sampled each hour of the wind speed in the Ventosa, Oaxaca, Mexico, and using the software R, the models $ARIMA$ and ETS, and its respective errors, are obtained as

$$ARIMA(1,1,0)(2,0,1)_{24} \;\; with \;\; RMSE = 0.97254$$
$$ETS(A,A_d,A) \;\; with \;\; RMSE = 1.00982.$$

Therefore, the ARIMA model is selected due its lower RMSE w.r.t. the ETS model. The forecast of the wind speed is displayed in the upper graph on Fig. 5, whereas the lower graph shows the generated power by using (1)–(2), with $C_p = 0.40$, $A = 0,7854m^2$ and $\eta = 0.94$.

4. OPTIMIZATION OF THE MICRO-GRID

The proposed micro-grid to be optimized is shown in Fig. 6. The figure illustrates the energy flow between the components of the micro-grid. The amount and direction of the energy is determined by an optimization algorithm, which

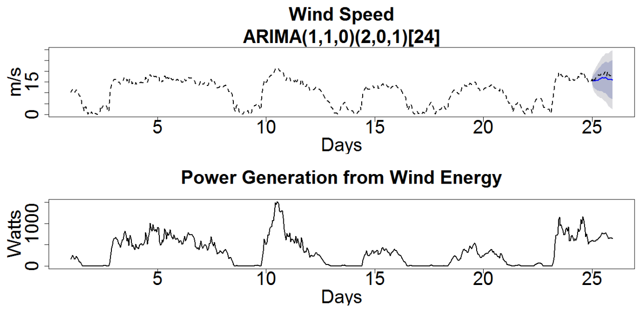

Fig. 5. Forecast of the wind speed and generated electrical power (The dashed-line shows the real series. The continuous-line shows the forecast of the series)

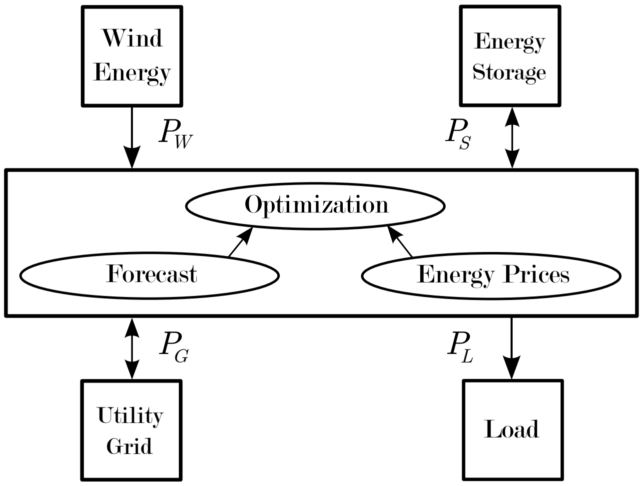

Fig. 6. Optimization scheme for the micro-grid.

considers the prediction of the wind energy availability, energy demand and the costs of electrical energy along a period of time T.

The main objective of the optimization scheme is to determine the optimal amount of energy to be transferred from/to the storage element and the utility grid, such that it is minimized the cost associated to the energy consumption from the utility grid and simultaneously it is maximized the profit due to the sale of electrical energy stored and generated in the utility grid.

4.1 Optimization Model

The objective function to be minimized in the optimization process corresponds to a function composed of 24 future values (i.e., $T = 24$) for the variables, where the values are taken in periods of 1 hour. The model considers the predicted values from Section 3 for each respective time series. The proposed model is given as

$$\min \sum_{k=0}^{T-1} \lambda^k \left[C_{B,\,k} P_{L,\,k} P_{G,\,k} - C_{S,\,k} P_{W,\,k} P_{S,\,k} \right] \qquad (10)$$

subject to:

$$P_{G,\,k} + P_{S,\,k} = P_{L,\,k} - P_{W,\,k}$$
$$V_{C,k+1} = \alpha\, V_{C,k} + \beta\, i_k$$
$$(i_{k+1} - i_k)^2 \le \gamma^2$$
$$V_{C_{min}} \le V_{C,k} \le V_{C_{max}}$$
$$i^-_{max} \le i_k \le i^+_{max}$$
$$P_{G^-_{max}} \le P_{G,\,k} \le P_{G^+_{max}}$$
$$V_{C,0} = 0.70\, V_{C_{nom}}$$
$$V_{C,T} = 0.85\, V_{C_{nom}}$$

where $0 < \lambda \le 1$ is a discount factor, $C_{B,\,k}$ is the cost per kWh associated to the buying of electrical energy from the utility grid, $C_{S,\,k}$ is the cost per kWh associated to the selling of electrical energy toward the utility grid, $V_{C,min} = 0.7V_{C_{nom}}$ and $V_{C,max} = V_{C_{nom}}$, $V_{C_{nom}}$ is the nominal voltage of the storage device, γ is a positive constant which limits the current flow. The decision variables are $P_{G,k}$ and $P_{S,k}$. In order to determine $P_{G,k}$ in (10), two issues are taken into account, the energy buying cost and the load demand, which allows to obtain a lower value for (10), and a similar consideration is realized for determining $P_{S,\,k}$.

The value for $T = 24$ is selected by considering that this time interval is enough to take in advance the decision at time $k = 1$ for charging or discharging the supercapacitor in accordance with energy prices, energy demand and energy availability. Hence, the optimization is realized at each time k by considering 24 hours in advance, which results in an optimization scheme that needs to be updated each hour.

4.2 Optimization Results

The values used for the optimization algorithm are: $T = 24$ hrs, $\lambda = 0.995$, the sampling time $T_s = 1\,hr$, $\gamma = 4$, $R_C = 10 \times 10^6$ ohms, $C = 100F$, $V_{C_{nom}} = 12\,V$, $i^-_{max} = -10\,A$ and $i^+_{max} = 10\,A$.

Fig. 7 shows the forecast-based optimization results, where in the top of the figure it is shown the power flow of the micro-grid components and in the bottom figure the respective prices for the energy. The results are described as follows:

- For the time $(1, 6.15)$ hrs: the generated energy is greater than the consumed by the load, and besides the energy is cheap, then energy is extracted from the utility grid and stored in the supercapacitor.
- For the time $(6.15, 10.5)$ hrs: the energy consumption by the load is greater than the generated by the wind system, and the energy is expensive, thus the energy is taken from the supercapacitor and only a small amount of energy is taken from the utility grid.
- For the time $(10.5, 17.38)$ hrs: the generated energy is greater than the consumed by the load, and the energy is cheap, hence there is energy to be sold to the utility grid and to charge the supercapacitor.
- Finally, for the time $(17.38, 23.4)$ hrs: the load energy consumption is greater than the generated, and besides the energy is expensive, therefore the stored energy is used by the load and additionally, it is required to buy energy from the utility grid in order to fully provide from energy to the load.

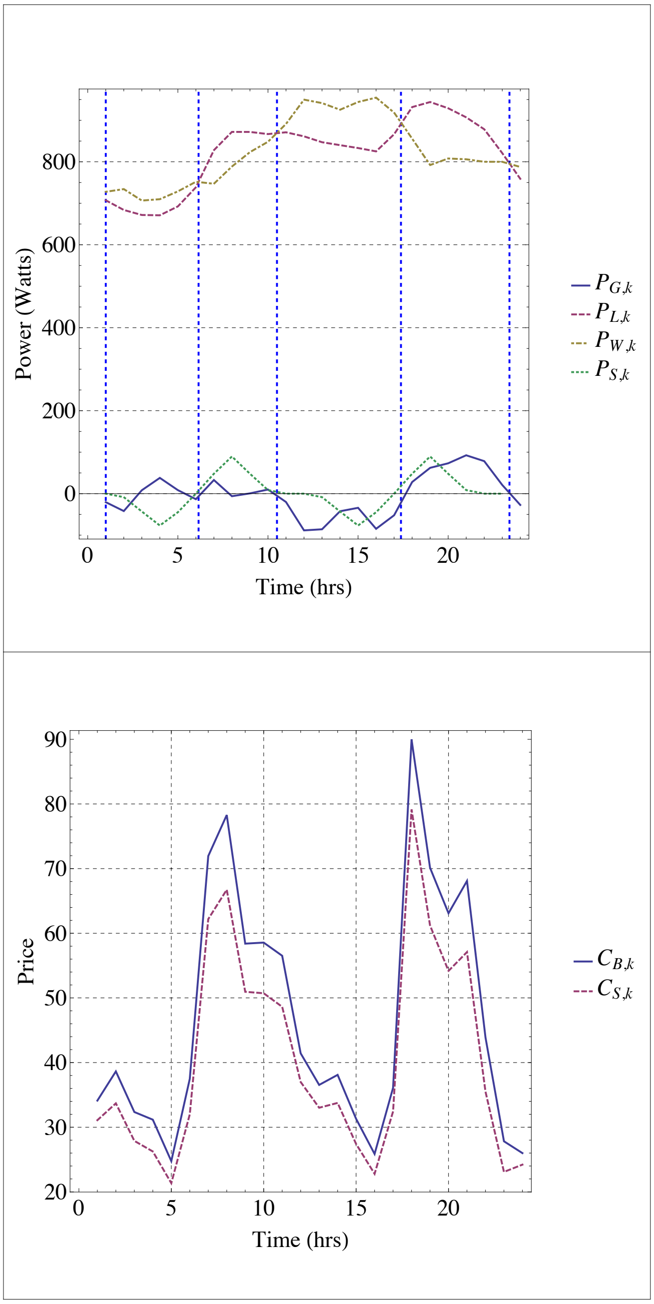

Fig. 7. Optimization results.

As a result of the optimization algorithm, the charge-discharge of the supercapacitor is depicted in Fig. 8.

5. CONCLUSIONS

This paper proposes an optimization algorithm to manage in an optimized way the energy in a micro-grid. The values of the amount and direction of the energy to be transfered in the storage device and in the utility grid are determined as a result of the optimization process based on an interval of 24 hrs, which considers the predicted values for energy availability, energy prices and load energy demand. A posteriori, the optimization scheme will be used to manage the energy reference values for power converters, which are the elements that interchange the energy in a micro-grid,

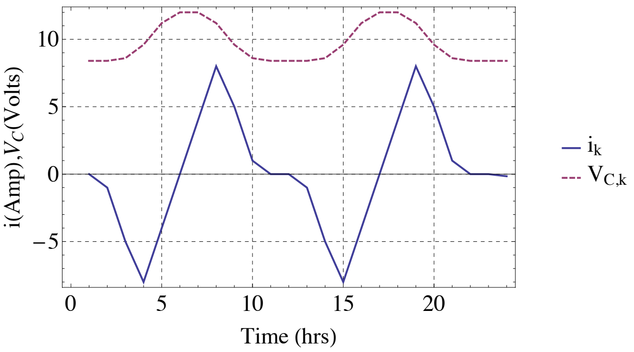

Fig. 8. Voltage and current in the supercapacitor.

and where the energy losses due the converters need to be considered.

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