Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Room for automatic control in combined sewer systems

Ronald R. P. van Nooijen * **Alla G. Kolechkina** **

* *Delft University of Technology, Stevinweg 1, 2628 CN, Delft, The Netherlands (e-mail:r.r.p.vannooyen@tudelft.nl).*
** *Aronwis, Den Hoorn Z.H., Netherlands (e-mail: a.g.kolechkina@aronwis.nl)*

Abstract: Combined sewer systems in low lying areas, such as those in the Western part of the Netherlands, have very specific characteristics. Due to a lack of available head gravity driven flow is possible only over short distances, so pumps are needed throughout the system. Lack of available gradient also means fairly large diameter pipes are needed to transport the maximum design flow, so storage volume is usually available. Systems tend to respond quickly and prevention of combined sewer overflows is strongly dependent on available pump capacity and storage volume. For a simplified system model a mathematical proof is given that for a system, designed by Dutch rules of thumb under spatially homogeneous load, local control is nearly as effective as central control when it concerns the prevention of CSO events.

Keywords: Urban systems; Local control; Bang-bang control; Control system design; Directed graphs; Environmental engineering.

1. INTRODUCTION

The design and operation of Dutch sewer systems, especially the combined sewer systems where both foul water and storm water are transported through the same system, is a source of interesting control problems. Occasional controlled spills into surface water, combined sewer overflows (CSO), are part of the normal operating procedure for these systems. While design of such a system is a complex matter, three simple rules given in NLingenieurs Sewer Systems Workgroup [2009] as an aid in establishing a first estimate of CSO frequency have very interesting implications from a control point of view. The rules are:

(1) all pumping stations must be able to transport a given fixed continuous precipitation intensity, the value given is $0.06\text{m}^3/\text{s}/\text{ha}$ where ha stands for hectare, $1\text{ha} = 10^4\text{m}^2$;
(2) all components must have a given fixed storage capacity per unit area (implicit in the reasoning on pages 95 and 97);
(3) all pumping stations must have a wet well of appropriate dimensions.

In this paper only a simplified model for a specific type of sewer system will be described, for a description of sewer systems in general and the theory of their control please consult Marinaki and Papageorgiou [2005], Ocampo-Martinez [2010]. Background information on a wide variety of systems can be found in the following case studies: Puig et al. [2009], Pleau et al. [2001, 2005], Ermolin [1999], Charron et al. [2001], Campisano et al. [2000], Ocampo-Martinez et al. [2005].

This paper makes rather strong assumptions about the system (no delays in transfers done by pumping stations, tree shaped networks) and the load on the system (homogeneous in space). These make it easier to obtain mathematical proofs. Numerical experiments with hydrodynamical models wil be needed to determine whether the conclusions carry over to more general cases. We do feel that the assumptions are not unreasonable for the particular systems under consideration here.

2. SOME REFERENCE QUANTITIES

The lecture notes [NLingenieurs Sewer Systems Workgroup, 2009, pp. 94–98], a freely available translation of ONRI-werkgroep riolering [2009], provides an short overview of the history of design rules for sewer systems in the Netherlands. From 1950s to the 1990s the concepts of storage per unit connected area and pump excess capacity (pump capacity in excess of that needed for dry weather flow) were important tools to get initial estimates of the behavior of a sewer system. In fact in 1985 a reference system was introduced to serve as representative for the systems in place at that time with 7mm/ha storage, 2mm/ha in a storage settling facility, and 0.7mm/h/ha excess pump capacity (where h stands for hour). Such a system was then to be combined with local data and used to calculate a foul water discharge. This was then to be used as an upper bound on the allowed foul water discharge to determine whether it was necessary to change the local system and to provide a target if this was the case. It was definitely not intended as a model implementation.

Nevertheless it is interesting to examine the implications for a possible control system when these design rules are followed in a combined system that, like most Dutch systems, consists of small sub-networks of sewer pipes, where flow is gravity driven, that are linked to each other and to a Waste Water treatment Plant (WWTP) by pumps. To put this in context: mean annual rainfall within the country varies from 700 to 900 mm. An event that exceeds 100 mm in 24 hours is considered extreme. For a residential area in the Netherlands the peak foul water flow is approximately 0.2mm per hour and for industrial areas an upper bound of 0.72mm is used, see [NLingenieurs Sewer Systems Workgroup, 2009, pp. 74]. The formal Dutch reference for sewer design is Taakgroep Leidraad Riolering [2004]. The lecture notes NLingenieurs Sewer Systems Workgroup [2009] were used as a reference because they are in English and readily available on the Internet, they provide a condensed version of the information in Taakgroep Leidraad Riolering [2004]. Note that we concentrate on constant speed pumps. According to [NLingenieurs Sewer Systems Workgroup, 2009, pp. 89] the larger pumping stations in newer systems use variable speed pumps. For these stations a special local controller is used that minimizes level variation to realize longer pump run times and fewer switching moments. While it would be interesting to determine whether this has a positive or a negative effect on CSO volume and frequency, it is beyond the scope of this paper.

3. CONVENTIONAL CONTROL SCHEME

Our reference control scheme is local control, the pump is automatically switched on at full capacity when a given level h_{on} is exceeded in the wet well. It is switched off again once the level in the wet well drops below a second level $h_{\text{off}} < h_{\text{on}}$. The wet well and the levels are chosen to avoid an excessive amount of switching of the pumps.

4. THE MODEL SYSTEM

The model of the system is based on a directed tree (no parallel arcs, paths run from the leaves to the root) with n_{v} vertices, indexed by the elements of $I_{\text{v}} = \{1, 2, \ldots, n_{\text{v}}\}$, the root is assigned index n_{v} and there are $n_{\text{a}} = n_{\text{v}} - 1$ arcs. See also van Nooijen and Kolechkina [2013]. The arcs will be numbered as follows: the arc leaving node i gets index i. The graph is a directed tree so for all vertices there exists at exactly one (directed) path to the vertex with index n_{v}. The graph will be the basis for a network in which vertex n_{v} is a sink and all other vertices are time varying sources. We introduce the set of source indices $I_{\text{src}} = \{1, 2, \ldots, n_{\text{v}} - 1\}$, in our numbering this coincides with the set I_{arc} of arc indices. We will denote the incidence matrix of the tree by M. Consider the network in discrete time and assume transport along an arc to take one time step. The time step will be fixed, we denote it by Δt. Each source vertex i has a constant positive storage capacity $v_{\max,i}$ and a constant "connected surface" a_i. The vertex n_{v} has infinite storage and zero connected surface. The inflow due to precipitation into a source vertex i is given by $a_i \cdot p_i(t)$ where p_i is a function $p_i : \mathbb{R}_0^+ \to \mathbb{R}_0^+$. For $i \in I_{\text{src}}$ the dry weather flow (dwf, traditional name for foul water flow) into a vertex i is $a_i \cdot p_{\text{dwf},i}(t)$, where $p_{\text{dwf},i}$ is a function $p_{\text{dwf},i} : \mathbb{R}_0^+ \to \mathbb{R}_0^+$. Each arc j has a positive maximum discharge capacity $q_{\max,j}$.

If the system represents a sewer network where the WWTP is directly connected to multiple upstream districts and its maximum capacity q_{WWTP} is less than the sum of the upstream capacities, then this can be modeled by adding an additional node v_{WWTP} after $v_{n_{\text{v}}}$ with no precipitation inflow and zero storage, that links $v_{n_{\text{v}}}$ to v_{WWTP} by an arc with capacity q_{WWTP}. We will not consider this case in the rest of this paper.

The following notation will be used. For $i \in I_{\text{src}}$ a function $v_i(t)$ represents the volume stored at time t, this must be positive or zero. For $j \in I_{\text{arc}}$ a function $q_j(t)$ represents the flow along arc j at time t, this must be positive or zero. The usual constraints and conservation equations hold, for $i \in I_{\text{src}}$ and $t \in \mathbb{R}_0^+$

$$v_i(t) = v_i(0) + \int_{\tau=0}^{t} p_{\text{dwf},i}(\tau) \cdot a_i d\tau$$
$$+ \int_{\tau=0}^{t} \left(p_i(\tau) \cdot a_i - q_i(\tau) + \sum_{j \in N_{\text{s}} - \{i\}} M_{ij} \cdot q_j(\tau) \right) d\tau$$
$$v_i(0) = v_{0,i} \tag{1}$$

We define the following special sets that exist for all directed trees with paths directed towards the root. The set of upstream adjacent vertices (parents) for vertex i, $I_{\text{par}}(i) = \{j \in I_{\text{src}} \mid M_{ji} < 0\}$, the set of leaves, $I_{\text{leaf}} = \{i \in N_{\text{s}} \mid I_{\text{par}}(i) = \emptyset\}$ and the set of vertices linked by a path to i (its ancestors)

$$I_{\text{anc}}(i) = I_{\text{par}}(i) \cup \left(\bigcup_{m \in I\text{par}(i)} I_{\text{anc}}(m) \right) \tag{2}$$

in our case $I_{\text{src}} = I_{\text{anc}}(n_{\text{v}})$.

5. DEFINITIONS AND CONJECTURES

We provide a definition of a homogeneous sewer system design and a homogeneous load. In both definitions accuracy refers to the desired accuracy for the volume of the simulated CSO's.

Definition 1. A *homogeneous sewer system design* is a design whose dynamic behavior can be modeled to sufficient accuracy by a model system of the form described in the previous section and for which there exist three positive real numbers ρ, r and $\tau_{\min}$ such that

$$q_{\max,i} = \left(a_i + \sum_{j \in I_{\text{anc}}} a_j \right) \rho \tag{3}$$

and

$$v_{\max,i} = a_i r + q_{\max,i} \tau_{\min} \tag{4}$$

Equation 3 corresponds to design rule 1 cited in the introduction with ρ as the transport capacity per hectare. Equation 4 incorporates design rules 2 and 3 given there, r is the storage capacity per hectare and $\tau_{\min}$ corresponds to the hysteresis time for an on/off pump controller which in turn specifies a reasonable size for the wet well.

Definition 2. A *homogeneous load* for a homogeneous sewer system design is a combination of precipitation and dry weather flow such that there is a function p' such that for all source nodes $p'(t) = p_i(t) + p_{\text{dwf},i}(t)$ to sufficient accuracy.

We now formulate our conjectures.

Conjecture 3. For a homogeneous sewer system design and homogeneous precipitation central control will not perform better than local control.

Conjecture 4. For a homogeneous sewer system design and homogeneous precipitation use of a precipitation forecast will not improve control performance.

6. CHECK OF THE CONJECTURES FOR A SINGLE LEAF

For leaves three general Lemmas can be proven.

Lemma 5. For a leaf i with pump capacity $q_{\max,i} = \rho a_i$ and storage capacity $v_{\max,i} = r a_i + q_{\max,i}\Delta t_{\min}$ starting at $v_i(0) = 0$ any non-negative locally integrable input signal $p'(t) = a_i(p_i(t) + p_{\mathrm{dwf},i}(t))$ for which there exist times $t_0 < t_1$ such that

$$P(t_0, t_1) = \int_{t=t_0}^{t_1} p'(t)\,dt > \rho(t_1 - t_0 + \Delta t_{\min}) + r \tag{5}$$

will lead to

$$\frac{v(t_1) - v_{\max,i}}{a_i} > 0 \tag{6}$$

Proof. This follows immediately from $v_i(t_0) \geq 0$ and the upper bound on the pumping capacity $q_i(t) \leq q_{\max,i}$.

As a reference controller we will use a rule that combines delay-free level control when below a certain minimum storage and use of maximum pump capacity with a small delay above the minimum storage. If we let the delay go to zero this results in bang-bang control.

Definition 6. For a given delay time δ and vertex i with local stored volume $v_i(t)$, area a_i and outgoing pump capacity $q_{\max,i}$ we define the following control signal:

$$q_i(t) = \begin{cases} q_{\mathrm{lim},i}(t) & : t \leq \delta \\ q_{\mathrm{lim},i}(t) & : t > \delta, v_i(t-\delta) < q_{\max,i}\delta \\ q_{\max,i} & : t > \delta, v_i(t-\delta) \geq q_{\max,i}\delta \end{cases} \tag{7}$$

with

$$q_{\mathrm{lim},i}(t) = \min\left(a_i p'(t) + \sum_{j \in I_{\mathrm{par}}} q_j(t), q_{\max,i}\right) \tag{8}$$

where we assumed time starts at 0.

Lemma 7. For a leaf i with pump capacity $q_{\max,i} = \rho a_i$, storage capacity $v_{\max,i} = r a_i + q_{\max,i}\Delta t_{\min}$ with $v_i(0) = 0$, a (small) $\delta > 0$ such that $\delta < \Delta t_{\min}$ and a non-negative locally integrable input signal $p'(t) = a_i(p_i(t) + p_{\mathrm{dwf},i}(t))$ such that for all times $0 \leq t_0 < t_1$

$$P(t_0, t_1) = \int_{t=t_0}^{t_1} p'(t)\,dt < \rho(t_1 - t_0 + \Delta t_{\min} - \delta) + r \tag{9}$$

the control action given by Eq. 7 will result in $0 \leq v_i(t) \leq v_{\max,i}$ for al finite t.

Proof. Thanks to $\delta > 0$ the control rule given by Eq. 7 is well defined on $[0, \delta]$ and by extension on $[k\delta, k\delta + \delta]$. Now suppose that

$$T_1 = \{\tau \geq 0 \,:\, v_i(\tau) \geq v_{\max,i}\} \tag{10}$$

is non-empty and therefore $t_1 = \inf T_1$ exists. As $v_i(0) = 0$

$$T_0 = \{0 \leq \tau \leq t_1 \,:\, v_i(t) < q_{\max,i}\delta\} \tag{11}$$

is non-empty and $\sup T_0$ exists. For $t_0 < \tau \leq t_1$ it follows that $v_i(\tau) > 0$ so for $t_0 + \delta < \tau \leq t_1$ the value of $q_i(t)$ is $q_{\max,i}$. This implies that for $t_0 + \delta < t \leq t_1$

$$\begin{aligned} v_{\max,i} \leq v_i(t_1) &= v_i(t_0) + \int_{\tau=t_0}^{t_1} p'(t) \cdot a_i - q_i(\tau)\,d\tau \qquad (12) \\ &= \int_{\tau=t_0}^{t_1} p'(t) \cdot a_i d\tau - q_{\max,i}(t_1 - t_0 - \delta) \\ &= a_i\left(\int_{\tau=t_0}^{t_i} p'(t)\,d\tau - \rho(t_1 - t_0 - \delta)\right) \end{aligned}$$

which is equivalent to

$$r + \rho \leq \int_{\tau=t_0}^{t_i} p'(t)\,d\tau - \rho(t_1 - t_0 - \delta) \tag{13}$$

and this contradicts the assumption on $P(t_0, t_1)$.

Remark 8. The $\delta > 0$ in the above Lemma is there to simplify reasoning about the existence of a solution v.

Definition 9. For vertex i with local stored volume $v_i(t)$, area a_i and outgoing pump capacity $q_{\max,i}$ we define a control signal corresponding to on/off control with a processing delay δ and switch levels $0 < r_{\mathrm{off}} < r_{\mathrm{on}}$ as follows

$$q_i(t) = \begin{cases} 0 & 0 \leq t \leq \delta \\ 0 & : \tau_{\mathrm{on}}(t-\delta) \leq \tau_{\mathrm{off}}(t-\delta) \\ q_{\max,i} & : \tau_{\mathrm{on}}(t-\delta) > \tau_{\mathrm{off}}(t-\delta) \end{cases} \tag{14}$$

with

$$\tau_{\mathrm{off}}(t) = \begin{cases} 0 & t < 0 \\ \sup(\{0\} \cup T_{\mathrm{off}}(t)) & t \geq 0 \end{cases} \tag{15}$$

$$\tau_{\mathrm{on}}(t) = \begin{cases} 0 & t < 0 \\ \sup(\{0\} \cup T_{\mathrm{on}}(t)) & t \geq 0 \end{cases} \tag{16}$$

$$T_{\mathrm{off}}(t) = \{0 \leq \tau \leq t \mid v(t) \leq a_i r_{\mathrm{off}}\} \tag{17}$$

$$T_{\mathrm{on}}(t) = \{0 \leq \tau \leq t \mid v(t) \geq a_i r_{\mathrm{on}}\} \tag{18}$$

and where we assumed time starts at 0.

Lemma 10. Take a leaf i with pump capacity $q_{\max,i} = \rho a_i$, storage capacity $v_{\max,i} = r a_i + q_{\max,i}\Delta t_{\min}$. Suppose there are $r_{\mathrm{off}}, r_{\mathrm{on}}, \delta$ such that $0 < 2\delta < a_i(r_{\mathrm{on}} - r_{\mathrm{off}})/q_{\max,i}$, $\delta < a_i r_{\mathrm{off}}/q_{\max,i}$, $\delta < \Delta\tau$. For any a non-negative locally integrable input signal $p'(t) = a_i(p_i(t) + p_{\mathrm{dwf},i}(t))$ such that for all times $0 \leq t_0 < t_1$

$$P(t_0, t_1) = \int_{t=t_0}^{t_1} p'(t)\,dt < \rho(t_1 - t_0 + \Delta t_{\min} - \delta) + r - r_{\mathrm{on}} \tag{19}$$

and any initial condition such that $v_i(0) \leq a_i r_{\mathrm{off}}$ the control action defined by Eq. 14 results in $0 \leq v_i(t) \leq v_{\max,i}$ for al finite t.

Proof. Note that $\tau_{\mathrm{off}}(t) = \tau_{\mathrm{on}}(t) > 0$ would imply that $T_{\mathrm{off}}(t) \neq \emptyset$ and $T_{\mathrm{on}}(t) \neq \emptyset$ and therefore that for all $\epsilon > 0$ there are

$$t_0 \in T_{\mathrm{off}}(t) \tag{20}$$

$$t_1 \in T_{\mathrm{on}}(t) \tag{21}$$

such that $|t_1 - t_0| < \epsilon$ and

$$\int_{t=t_0}^{t_1} p'(t)\,dt \geq r_{\text{on}} - r_{\text{off}} \tag{22}$$

but p' is a locally integrable function so this leads to a contradiction. We can therefore assume that $\tau_{\text{off}}(t) = \tau_{\text{on}}(t)$ implies $\tau_{\text{off}}(t) = \tau_{\text{on}}(t) = 0$. Thanks to the exception for $0 \leq t \leq \delta$ the control rule is well defined for $0 \leq t \leq \delta$ where $q_i(t) = 0$. Therefore the solution is well defined for $0 \leq t \leq \delta$ and is given by

$$\begin{aligned} v_i(t) &= v_i(0) + a_i \int_{\tau=0}^{t} p'(\tau)\,d\tau \qquad (23)\\ &< v_i(0) + a_i\left(\rho\left(t_1 - t_0 + \Delta t_{\min} - \delta\right) + r - r_{\text{off}}\right)\\ &< a_i\left(\rho \Delta t_{\min} + r\right) = v_{\max,i} \end{aligned}$$

For $\delta \leq t \leq 2\delta$ we have either $T_{\text{on}}(\delta) = \emptyset$ and therefore $v_i(\delta) < v_{\text{off}}$ so

$$\begin{aligned} v_i(t) &= v_i(\delta) + a_i \int_{\tau=\delta}^{t} p'(\tau)\,d\tau \qquad (24)\\ &< a_i\rho\left(\delta + \Delta t_{\min} - \delta\right) + a_i r = v_{\max,i} \end{aligned}$$

or $0 < \inf T_{\text{on}}(\delta) = \tau \leq \delta$ and because of the restrictions on δ we have $\inf T_{\text{off}}(\delta) > \tau + 2\delta > 2\delta$ so

$$\begin{aligned} v_i(t) &= v_{\text{off}} + a_i \int_{\tau=\tau}^{t} p'(\tau)\,d\tau - (t-\tau)\,q_{\max,i} \qquad (25)\\ &< a_i\rho\left(t - \tau + \Delta t_{\min} - \delta\right) + a_i r - (t-\tau)\,q_{\max,i}\\ &= v_{\max,i} \end{aligned}$$

In general for $k\delta \leq t \leq k\delta + \delta$ we either have $\tau_{\text{on}}(k\delta) < \tau_{\text{off}}(k\delta)$ which implies $v_i(k\delta) < v_{\text{on}}$ and zero discharge so

$$\begin{aligned} v_i(t) &= v_i(k\delta) + a_i \int_{\tau=k\delta}^{t} p'(\tau)\,d\tau \qquad (26)\\ &< a_i\rho\left(t - \tau + \Delta t_{\min} - \delta\right) + a_i r - (t-\tau)\,q_{\max,i}\\ &= v_{\max,i} \end{aligned}$$

or we have $\tau_{\text{on}}(k\delta) > \tau_{\text{off}}(k\delta)$ which implies $v_i(\tau_{\text{on}}(k\delta)) = v_{\text{on}}$ and maximum discharge from $\tau_{\text{on}}(k\delta) + \delta$ onwards so

$$\begin{aligned} v_i(t) &= v_i(\tau_{\text{on}}(k\delta)) + a_i \int_{\tau=\tau_{\text{on}}(k\delta)}^{t} p'(\tau)\,d\tau \qquad (27)\\ &< a_i\rho\left(t - \tau_{\text{on}}(k\delta) + \Delta t_{\min} - \delta\right)\\ &\quad + a_i r - \left(t - \tau_{\text{on}}(k\delta) - \delta\right) q_{\max,i}\\ &= v_{\max,i} \end{aligned}$$

which completes the proof.

Lemma 10 shows that local bang-bang control will have a performance close to the ideal controller as long as the switching delay is not too large and the volume needed in the wet well is small relative to the volume of the district. Also note that this controller does not use p'. This Lemma confirms both conjectures for systems consisting only of one leaf and a WWTP.

7. CHECK OF THE CONJECTURES FOR A HOMOGENEOUS SEWER SYSTEM DESIGN

Next we consider the conjectures for a non-trivial system.

Lemma 11. For a homogeneous sewer system with pump capacities

$$q_{\max,i} = \left(a_i + \sum_{j \in I_{\text{anc}}} a_j\right)\rho \tag{28}$$

and storage

$$v_{\max,i} = a_i r + q_{\max,i}\tau_{\min} \tag{29}$$

any homogeneous set of non-negative locally integrable input signals such that $p'(t) = a_i\left(p_i(t) + p_{\text{dwf},i}(t)\right)$ for which there exist times $t_0 < t_1$ such that

$$P(t_0, t_1) = \int_{t=t_0}^{t_1} p'(t)\,dt > \rho\left(t_1 - t_0 + \Delta t_{\min}\right) + r \tag{30}$$

will lead to

$$\frac{v_i(t_1) - v_{\max,i}}{a_i} > P(t_0, t_1) - \rho\left(t_1 - t_0 + \tau\right) - r \tag{31}$$

for at least one $i \in I_{\text{src}}$.

Proof. This follows immediately from Lemma 5.

Lemma 12. For a homogeneous sewer system with pump capacities

$$q_{\max,i} = \left(a_i + \sum_{j \in I_{\text{anc}(i)}} a_j\right)\rho \tag{32}$$

and storage

$$v_{\max,i} = a_i r + q_{\max,i}\Delta t_{\min} \tag{33}$$

for any $\delta > 0$ and any non-negative locally integrable input signal $p'(t) = a_i\left(p_i(t) + p_{\text{dwf},i}(t)\right)$ such that for all times $0 \leq t_0 < t_1$

$$\begin{aligned} P(t_0, t_1) &= \int_{t=t_0}^{t_1} p'(t)\,dt\\ &< \rho\left(t_1 - t_0 + \Delta t_{\min} - \delta\right) + r \qquad (34) \end{aligned}$$

the control action given by Eq. 7 will result in $0 \leq v_i(t) \leq v_{\max,i}$ for all $i \in I_{\text{src}}$ and all finite t.

Proof. According to Lemma 7 the rule will work for the leaves. For a non-leaf source i the control the control rule given by Eq. 7 is well defined on well defined on $[0, \delta]$ and by extension on $[k\delta, k\delta + \delta]$. Now suppose that

$$T_1 = \{\tau \geq 0 \,:\, v_i(\tau) \geq v_{\max,i}\} \tag{35}$$

is non-empty and therefore $t_1 = \inf T_1$ exists. In that case

$$T_0 = \{0 \leq \tau \leq t_1 \,:\, v_i(t) = 0\} \tag{36}$$

is non-empty and $\sup T_0$ exists. For $t_0 < \tau \leq t_1$ it follows that $v_i(\tau) > 0$ so for $t_0 + \delta < \tau \leq t_1$ the value of $q_i(t)$ is $q_{\max,i}$. This implies that for $t_0 + \delta < t \leq t_1$

$$v_{\max,i} \leq v_i(t_1) \tag{37}$$

which is equivalent to

$$\begin{aligned} a_i r + q_{\max,i}\Delta t_{\min} &\leq \int_{\tau=t_0}^{t_i} a_i p'(\tau)\,d\tau - \rho a_i\left(t_1 - t_0 - \delta\right)\\ &+ \delta \sum_{j \in I_{\text{par}}} \left(a_j + \sum_{m \in I_{\text{anc}(j)}} a_m\right)\rho \qquad (38) \end{aligned}$$

which we can write as

$$
\begin{aligned}
a_i r + q_{\max,i}\Delta t_{\min} &\leq \int_{\tau=t_0}^{t_i} a_i p'(\tau)\, d\tau \\
&\quad -\rho a_i (t_1 - t_0) + \delta q_{\max,i} \\
&< a_i \rho (t_1 - t_0 + \Delta t_{\min} - \delta) \\
&\quad + a_i r - \rho a_i (t_1 - t_0) + \delta q_{\max,i} \\
&= a_i \rho (\Delta t_{\min} - \delta) + a_i r + \delta q_{\max,i} \\
&< a_i r + a_i \Delta t_{\min} q_{\max,i}
\end{aligned}
$$

and this cannot be true.

Lemma 12 shows that at least one method of local control will keep the levels below the maximum levels for an upper bound on the inflow that differs from the condition on the lower bound on the inflow needed to force an excursion above at least one maximum level only be an arbitrarily small δ.

Lemma 13. Asume we have a homogeneous sewer system with pump capacities

$$
q_{\max,i} = \left(a_i + \sum_{j \in I_{\mathrm{anc}}(i)} a_j \right) \rho \tag{39}
$$

and storage

$$
v_{\max,i} = a_i r + q_{\max,i}\Delta t_{\min} \tag{40}
$$

Suppose there are $r_{\mathrm{off}}, r_{\mathrm{on}}, \delta$ such that $0 < 2\delta < a_i (r_{\mathrm{on}} - r_{\mathrm{off}})/q_{\max,i}$, $\delta < a_i r_{\mathrm{off}}/q_{\max,i}$, $\delta < \Delta t_{\min}$. For any a non-negative locally integrable input signal $p'(t) = a_i (p_i(t) + p_{\mathrm{dwf},i}(t))$ such that for all times $0 \leq t_0 < t_1$

$$
P(t_0, t_1) = \int_{t=t_0}^{t_1} p'(t)\, dt < \rho (t_1 - t_0 + \Delta t_{\min} - \delta) + r - r_{\mathrm{on}} \tag{41}
$$

and any initial condition such that $v_i(0) \leq a_i r_{\mathrm{off}}$ the control action defined by Eq. 7 will keep the stored volumes between zero and the specified maximum.

Proof. The control rule is well defined for $0 \leq t \leq \delta$. Therefore the solution is well defined for $0 \leq t \leq \delta$ and it is given by

$$
v_i(t) = v_i(0) + a_i \int_{\tau=0}^{t} p'(\tau)\, d\tau \tag{42}
$$

which satisfies

$$
\begin{aligned}
v_i(t) &< v_i(0) + a_i (\rho (t_1 - t_0 + \Delta t_{\min} - \delta) + r - r_{\mathrm{off}}) \\
&< a_i (\rho \Delta t_{\min} + r) = v_{\max,i}
\end{aligned} \tag{43}
$$

In general for $k\delta \leq t \leq k\delta + \delta$ we either have $\tau_{\mathrm{on}}(k\delta) < \tau_{\mathrm{off}}(k\delta)$ which implies $v_i(k\delta) \leq v_{\mathrm{on}}$ and zero discharge so

$$
\begin{aligned}
v_i(t) &= v_i(k\delta) + \int_{\tau=k\delta}^{t} a_i p'(\tau) + \sum_{j \in I_{\mathrm{anc}}(i)} a_j q_j(t)\, d\tau \quad (44) \\
&< v_{\mathrm{on}} + a_i \rho (\delta + \Delta t_{\min} - \delta) + \delta (q_{\max,i} - a_i \rho) \\
&\quad + a_i (r - r_{\mathrm{on}}) \\
&= a_i \rho \Delta t_{\min} + \delta (q_{\max,i} - a_i \rho) + a_i r < v_{\max,i}
\end{aligned}
$$

or we have $\tau_{\mathrm{on}}(k\delta) > \tau_{\mathrm{off}}(k\delta)$ which implies $v_i(\tau_{\mathrm{on}}(k\delta)) = v_{\mathrm{on}}$ and maximum discharge from $\tau_{\mathrm{on}}(k\delta) + \delta$ onwards so, with $t_0 = \tau_{\mathrm{on}}(k\delta)$

$$
\begin{aligned}
v_i(t) &= v_i(t_0) + \\
&\int_{\tau=t_0}^{t} a_i p'(\tau) + \sum_{j \in I_{\mathrm{anc}}(i)} a_j q_j(t)\, d\tau \\
&\quad - (t - t_0 - \delta) q_{\max,i} \\
&= v_{\mathrm{on}} + \int_{\tau=t_0}^{t} a_i p'(\tau) + \\
&\sum_{j \in I_{\mathrm{anc}}(i)} a_j q_j(t)\, d\tau - (t - t_0 - \delta) q_{\max,i} \\
&< v_{\mathrm{on}} + \int_{\tau=t_0}^{t} a_i p'(\tau)\, d\tau + \\
&+ (t - t_0)(q_{\max,i} - a_i \rho) - (t - t_0 - \delta) q_{\max,i}
\end{aligned}
$$

so

$$
\begin{aligned}
v_i(t) &< a_i \rho (t - t_0 + \Delta t_{\min} - \delta) + a_i r \\
&\quad + (t - t_0)(q_{\max,i} - a_i \rho) - (t - t_0 - \delta) q_{\max,i} \\
&= a_i \rho (\Delta t_{\min} - \delta) + a_i r + (t - t_0) q_{\max,i} - \\
&\quad (t - t_0 - \delta) q_{\max,i} \\
&= a_i \rho (\Delta t_{\min} - \delta) + a_i r + \delta q_{\max,i} \\
&\leq q_{\max,i} (\Delta t_{\min} - \delta) + a_i r + \delta q_{\max,i} = v_{\max,i}
\end{aligned}
$$

which completes the proof.

8. A COMPUTER EXPERIMENT

In van Nooijen et al. [2010] Linear Programming (LP) was combined with complete knowledge of the event beforehand to determine the best possible result of central control using prediction. Evidently, if even with perfect foreknowledge, an event leads to one or more CSO events, those events can only be prevented by additional investment in hardware. The results showed that local control of fixed speed pumps did considerably worse. Here we take the system from that paper and redistribute the storage to create a system satisfying our balance rule 4. We also redimension the pumps to satisfy 3. Table 1 gives the original

Node	a_i $10^3\mathrm{m}^2$	$v_{\max,i}/a_i$ $10^{-3}\mathrm{m}$	$q_{\max,i}$ $\mathrm{m}^3 \cdot \mathrm{s}^{-1}$	$q_{\mathrm{dwf},i}$ $\mathrm{m}^3 \cdot \mathrm{s}^{-1}$
CRKW-001	24.25	12.5	0.03194	0.002111
CRKW-002	176.24	9.4	0.03750	0.010000
CRKW-003	30.67	7.1	0.02361	0.000444
CRKW-004	21.76	12.4	0.06389	0.002528
CRKW-008	26.28	3.2	0.00381	0.001472

Table 1. Original system with balanced dwf

system (total installed pump capacity $= 0.1608\mathrm{m}^3 \cdot \mathrm{s}^{-1}$) and Table 2 the “balanced” system (total installed pump capacity $= 0.1288\mathrm{m}^3 \cdot \mathrm{s}^{-1}$) . We determined the CSO with LP and with local control. A one minute simulation time step was used and a five minute storage volume in the wet well was incorporated in the design. Over a period of 6 years the number of CSO events found for the LP solution

Node	a_i $10^3\mathrm{m}^2$	$v_{\max,i}/a_i$ $10^{-3}\mathrm{m}$	$q_{\max,i}$ $\mathrm{m}^3 \cdot \mathrm{s}^{-1}$	$q_{\mathrm{dwf},i}$ $\mathrm{m}^3 \cdot \mathrm{s}^{-1}$
CRKW-001	24.25	9.0	0.00555	0.002111
CRKW-002	176.24	9.0	0.04634	0.010000
CRKW-003	30.67	9.0	0.00702	0.000444
CRKW-004	21.76	9.8	0.06389	0.002528
CRKW-008	26.28	9.0	0.00601	0.001472

Table 2. Balanced system with balanced dwf

and the number found with local control differed by only one, local control had one additional $2.3\mathrm{m}^3$ event, which was an event where failure was to be expected as Eq. 41 did not hold. We also found that over a period of 7 years with 58 CSO's the difference in CSO size for events over $50\mathrm{m}^3$ (54 out of 58) was less than 10 percent.

9. ROOM FOR CONTROL

The following characteristic numbers can be defined.

$$\rho_i = \frac{q_{\max,i}}{a_i + \sum_{j \in I_{\mathrm{anc}}(i)} a_j} \tag{45}$$

$$r_i = \frac{v_{\max,i} - q_{\max,i}\Delta t_{\min}}{a_i} \tag{46}$$

Evidently the leaves of the system are the critical points. If there is a leaf i and a pair of times $t_0 < t_1$ such that

$$\begin{aligned} a_i P\left(t_0, t_1\right) &= a_i \int\limits_{t=t_0}^{t_1} p_i\left(t\right) + p_{\mathrm{dwf},i}\left(t\right) dt \\ &> \left(t_1 - t_0\right) q_{\max,i} + v_{\max,i} \end{aligned} \tag{47}$$

then the stored volume will exceed the maximum storage possible without a CSO. For homogeneous inflow room for central control and the use of forecasts will therefore exist only when leaves have higher ρ_i and/or r_i than their descendants, where the set of descendants of a leaf i is defined by

$$I_{\mathrm{des}}\left(i\right) = \left\{j : i \in I_{\mathrm{anc}}\left(j\right)\right\} \tag{48}$$

Similarly room for central control and the use of forecasts will only exist when the inflow load for at least one leaf is relatively lower than for one of its descendants.

10. CONCLUSIONS

It was established that for inflow into a sewer system which are according to a given definition homogeneous, neither forecasts nor central control can help to reduce the chance of a CSO. It was also established that any gains from implementation of central control and the use of forecasts can only be realized when the upstream most sub-catchments, the "leaves", have either more storage capacity or more pump capacity than at least one of their descendants.

REFERENCES

A. Campisano, W. Schilling, and C. Modica. Regulators' setup with application to the Roma-Cecchignola combined sewer system. *Urban Water*, 2(3):235 – 242, 2000.

A. Charron, D. Guthrie, and J. Gratzer. Real time control study to reduce combined sewer overflows in Louisville and Jefferson County, Kentucky. In *Proceedings of the 4th International Conference NOVATECH*, volume 1, pages 503–510, Lyon, France, 2001.

Yuri A. Ermolin. Mathematical modelling for optimized control of Moscow's sewer network. *Applied Mathematical Modelling*, 23(7):543 – 556, 1999.

M. Marinaki and M. Papageorgiou. *Optimal Real-time Control of Sewer Networks*. Advances in Industrial Control. Springer, London, 2005.

NLingenieurs Sewer Systems Workgroup. *Sewer Systems Module for Higher Professional Education*. KIVI-NIRIA, The Hague, The Netherlands, 2009. version 2 (Translated by M.C. de Geus).

C. Ocampo-Martinez. *Model Predictive Control of Wastewater Systems*. Advances in Industrial Control. Springer, 2010.

C. Ocampo-Martinez, V. Puig, J. Quevedo, and Ingimundarson. Fault tolerant model predictive control applied on the Barcelona sewer network. In *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC '05. 44th IEEE Conference on*, pages 1349 – 1354, December 2005.

ONRI-werkgroep riolering. *Module Riolering voor het HBO*. KIVI-NIRIA, The Hague, The Netherlands, 2009. maart 2009 / versie 2 (in Dutch).

M. Pleau, G. Pelletier, H. Colas, and R. Bonin. Global predictive real-time control of Quebec Urban Community's westerly sewer network. *Water Science and Technology*, 43(7):123–130, 2001.

M. Pleau, H. Colas, P. Lavallée, G. Pelletier, and R.Bonin. Global optimal real-time control of the Quebec urban drainage system. *Environmental Modelling and Software*, 20(4):401 – 413, 2005.

V. Puig, G. Cembrano, J. Romera, J. Quevedo, B. Aznar, G. Ramón, and J. Cabot. Predictive optimal control of sewer networks using CORAL tool: Application to Riera Blanca catchment in Barcelona. *Water Science and Technology*, 60(4):869–878, 2009.

Taakgroep Leidraad Riolering, editor. *Leidraad Riolering*. Stichting RIONED, Ede, The Netherlands, August 2004.

R. van Nooijen, A Kolechkina, and E. van Leeuwen. Input selection for sewer network flow controller evaluation. In *New Aspects of Automatic Control, Modelling & Simulation, 12th WSEAS Int. Conf. on Automatic Control, Modelling & Simulation (ACMOS 10)*, pages 53–58, Sofia, Bulgaria, 2010. EUROPMENT Press.

R. van Nooijen and A. Kolechkina. Speed of discrete optimization solvers for real time sewer control. *Urban Water Journal*, 10(5):354–363, Oct 2013. ISSN 1744-9006.