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A PCA-based Modeling Approach for Estimation of Road-tire Forces by In-tire Accelerometers

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Abstract: The knowledge of the forces acting between tire and road is crucial for modern vehicle control systems. Typically, such forces are estimated on the basis of indirect measurements and rely on complex mathematical models. Sensors embedded in the tire (e.g. accelerometers) offer the possibility of directly reaching the area of the tire-road interaction, and of delivering signals during the wheel rotation. Such signals reflect the tire state and the force exchanged between tire and road.
In this paper an innovative approach to estimate such forces is presented. It exploits the signal delivered during the wheel rotation by an accelerometer glued to the inner side of the tire tread. The approach consists in processing the signal by means of the Principal Component Analysis (PCA). The results of experimental activities employing a real vehicle provide evidence to support the feasibility and the soundness of the proposed approach.

1. INTRODUCTION

In a wheeled vehicle, the tire contact patches transmit the forces between the road and the car. Almost every vehicle control system currently available (e.g. the Electronic Stability Control ESC) relies on the estimation of those acting forces. In general, due to technical and economic reasons, such forces are not measured directly. On the contrary, most of the approaches adopted in the industry rely on sensors to measure the car kinematics and dynamics (e.g. longitudinal and lateral accelerations) and estimate the tire forces by means of a model (see e.g. Guillaume Baffet and Lechner [2008]).

Since information about the contact forces is vital to improve vehicle control systems, extensive research has been carried out in the fields of "smart-tire". The main concept is to directly equip the tire with sensors in order to achieve better estimations. As a matter of fact, the closer the sensor to the source of the force, the less the spurious phenomena which contaminate the information on the force. In Tuononen [2009] optical sensors are employed to measure the carcass deflection, whereas Brusarosco et al. [2008] introduces the use of accelerometers. Yi [2008] relies on a PVDF-based sensing system. In Brusarosco et al. [2011] it is shown that correlation exists between the tire vibrations over 1 kHz and tread block dynamic behaviour which enables the recognition of sliding conditions. In Hong et al. [2013] an in-tire accelerometer allows measuring the deformations of the tire contact patch due to the acting forces. These deformations are used to estimate the lateral forces and the aligning moment of the tire by means of a lateral deflection model. Additionally, it is possible to estimate the tire-road friction coefficients in combination with a tire brush model. The works in Braghin et al. [2006] and Savaresi et al. [2008] introduce an approach to extract synthetic parameters (e.g. minimum, maximum and mean value of the signal or the min-max time lag) from the acceleration signals and exploit the correlation between those and several kinematic and dynamic quantities. The disadvantage of this method is that only a small part of the information contained in the signal is used instead of exploiting the whole signal.

The approach proposed and validated in this paper allows estimating the forces acting through the tire patch between the tire and the road by means of in-tire embedded accelerometers. It is shown that advanced signal processing techniques, such as the Principal Component Analysis (see e.g. Formentin et al. [2013] for another PCA-based estimation), applied to the signal of the in-tire accelerometer allow the identification of a model that correlates the "smart-tire" signal with the contact forces. Inspired by Savaresi et al. [2008], the development proposed in this work consists in the signal processing technique, which is proposed and applied to new measurement data. Also, the results are compared with a validated state-of-the art model, which was not the case of previous works. The innovative idea is, in contrast to Braghin et al. [2006] and Savaresi et al. [2008], to use a signal processing technique which exploits the whole signal delivered by the accelerometer during a wheel rotation. The complete signal is expected to contain more information than the few extracted condensed characteristics. Another advantage of this approach is that the identification of the complex or hard to estimate parameters, typical of physical models, is not needed. In this work, since no device for the measurement of the actual forces is available, the model proposed is validated against a reference car model.

The main idea behind this approach is presented in Section 2. The proposed approach is validated by means of experiments performed on a real car. In Section 3 the experimental setup and the experimental activity are presented. The Principal Component Analysis and the results of its application are presented in Section 4.1. Section 4.2 is devoted to the model of the car's longitudinal dynamics, which is subsequently used as reference for the force computation. The corresponding equations and methods to validate the car model are described. Section 4.3 proposes an alternative, independent method for the validation of the same car model. Several identifications and validations of the proposed PCA-based force models for the contact force estimation are presented in Section 4.4. The identifications differ in the number of used features and the length of the employed "smart-tire" signal, ranging from a small part of the rotation to the whole rotation. Finally, conclusion and future outlook are provided in Section 5.

2. MAIN CONCEPT

The principle behind the work presented in this paper is to exploit the relation between the deformation of the tire contact patch and the forces acting between the road and the tire (see Savaresi et al. [2008]. As a matter of fact, with reference to Figure 1, variation of the vertical force F_z results in change of the patch length $\Delta\phi$, whereas acceleration and braking, as well as the related variation of the horizontal force F_x (see Figure 2), affect the phase shift $\delta\phi$ of the patch. The main idea is that the signal from an in-tire embedded accelerometer contains information about these deformations and therefore, as presented in this work, an estimation of the acting forces by means of signal processing methods is possible.

Let us consider the typical form of the acquired "smart-tire" signal in Figure 3. Characteristic are the spikes, which indicate the instant when the accelerometer enters and leaves the patch, respectively. The parts outside the patch are in general related to the centripetal force. As it is shown in Figure 4, the ascending and descending fronts define the patch length and the phase shift. Investigations appearing in Savaresi et al. [2008] suggest that it is possible to find relationships like (1), (2) between the forces and the patch geometry.

$$F_z \approx F_z(\Delta\phi) = F_z(\alpha_2 - \alpha_1) \tag{1}$$

$$F_x \approx F_x(\delta\phi) = F_x\left(\frac{\alpha_1 + \alpha_2}{2}\right) \tag{2}$$

However, no parametrization of such a model is provided in Savaresi et al. [2008]. Also, the preliminary investigation and computations performed within the frame of this work have shown that very high precision is required in the measurements, which is hard to obtain with standard measurement devices and procedures. It also appears that, although the patch characteristics seem to be definitely related to the force acting on the patch, it is not a sound approach to condense all the information delivered by the accelerometers in just two values (the phase $\delta\phi$ and the length $\Delta\phi$). On the contrary, it is important to extract every useful piece of information from the signal delivered by the accelerometer. To this purpose, usage of the PCA is proposed in this work.

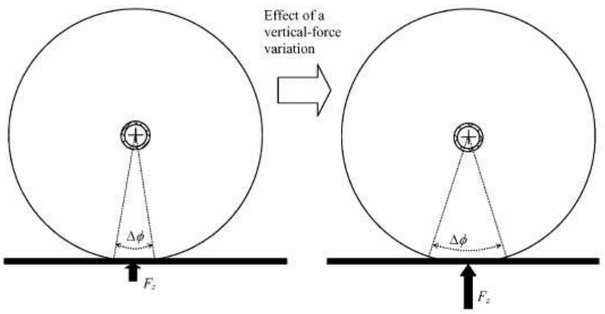

Fig. 1. Geometry of the patch with variation of the vertical force F_z (from Savaresi et al. [2008])

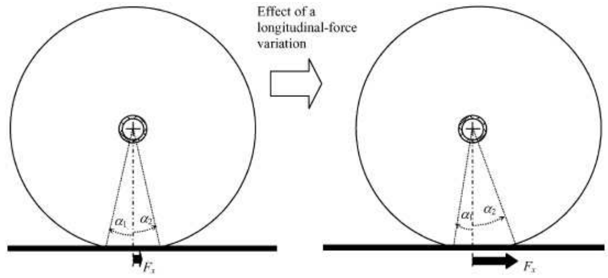

Fig. 2. Geometry of the patch with variation of the horizontal force F_x (from Savaresi et al. [2008])

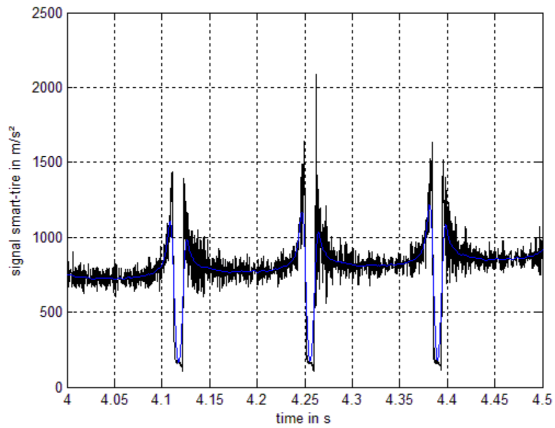

Fig. 3. Typical "smart-tire" signal

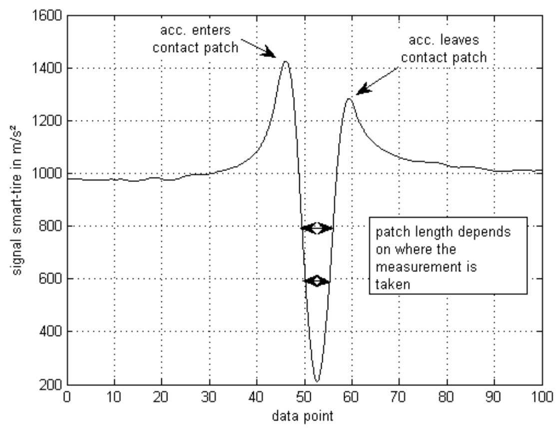

Fig. 4. Filtered "smart-tire" signal with highlighted characteristics (single rotation)

3. EXPERIMENTAL SETUP AND DATA COLLECTION

The required measurements are performed while driving in urban and extra-urban traffic conditions with a rear-wheel driven BMW 3-series. As it is not possible to measure the contact patch forces directly, they are estimated by means of the model presented in Section 4.2. The "smart-tire" is equipped with a piezoresistive accelerometer (Endevco 7264B-2000) glued to the inner side of the tire tread. The sensor's measurement range ± 2000 g is required to cover the high centripetal force and the higher spikes for entering and leaving the patch (see Figure 3). As the vehicle is rear-wheel-driven the "smart-tire" is mounted to the rear left. The signal acquired by the accelerometer is transmitted to the car by means of a 1-channel telemetry sender (Datatel dt1001-T-ST). An incremental encoder (Kübler 5820) is used to acquire information about the rotation of the wheel. In order to obtain the acceleration of the car an optical velocity sensor (Correvit Corrsys L-400) is attached as close as possible to the ”smart-tire”. The experimental setup is shown in Figure 5. The acquisition of the sensor signals is performed by a dSpace Autobox at a sampling rate of 0.1 ms. Such a high acquisition frequency is needed because of the high rotation speed of the wheels.

With the purpose of further improving the result of this work, an inertial measurement unit equipped with a 3-axis gyroscope and accelerometer is installed at the center of the car. The DSpace Autobox is also connected to the CAN-Bus of the car, offering access to many sensors already built in the vehicle by the manufacturer. In this work the torque and the revolution speed of the engine are recorded. In Section 4.3 it is shown that the torque acting on the tire can be computed from those signals.

The data acquisition itself is performed in urban as well as in extra-urban traffic. The track should at best fulfill the following requirements, being the model employed to estimate the contact forces valid only in such conditions: plain, straight and flat road (no potholes, bumps, ...), constant road properties (dry/wet, tarmac/gravel, temperature).

Fig. 5. Experimental setup

Multiple datasets were recorded, some static ones (at constant speed) and some dynamic ones (with aggressive acceleration).

4. DATA ANALYSIS

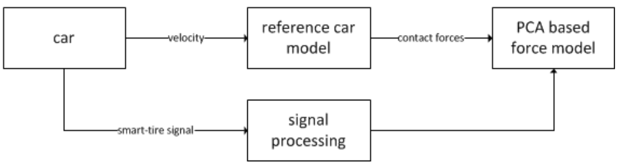

Fig. 6. Different stages in the flow of the signal processing

Since no measurement device for the direct measurement of the tire forces is available, such forces have been computed by means of a reference model (presented in Section 4.2) covering the vehicle longitudinal dynamics. The inputs of such a model are the vehicle speed and acceleration. Figure 6 gives a short overview of the different stages in the flow of the signal processing. The vehicle delivers the velocity and the ”smart-tire” signal. The velocity is employed in the longitudinal model of the car. Such forces and the processed ”smart-tire” signal are used for the model identification. In Section 4.4 the following investigations are proposed: firstly, in order to achieve the best possible fit values, the ”smart-tire” signal for the whole rotation in combination with a relatively high number of features is used for the identification. Next, it is checked whether a reduced number of features offers similar results. Lastly, the results obtained using only small parts of the signal during the rotation are presented.

4.1 Principal Component Analysis (PCA)

The Principal Component Analysis is a mathematical procedure which allows representing a multitude of statistical variables with fewer uncorrelated linear transformations. Hence, the PCA emerges as a useful tool to reduce the amount of data while still containing most of the information (see e.g. Moser et al. [2013]).

Given a signal $x \in \mathbb{R}^n$, with n being the number of data points per wheel rotation, it is possible to compute $\begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix}^\top = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}^\top x = C^\top x$, where C is an orthogonal matrix collecting the so called features c_i. p_i can be referred to as the projection on c_i. The main idea for the extraction of the features c_i from the available signals x_j is to first compute c_1 in order to obtain the highest possible variance for p_1. Then, c_2 so that $p_2 = c_2^\top x$ delivers the second highest variance and so on. With $\mathrm{Var}(p_1) = c_1^\top \Sigma_{xx} c_1$, where $\Sigma_{xx} = \mathrm{Cov}(x)$ is the covariance matrix, the computation can be expressed as the optimization problem $\max_{c_1 \neq 0} \frac{c_1^\top \Sigma_{xx} c_1}{c_1^\top c_1}$. The optimization problem is solved by computing the eigenvalue/eigenvector pairs of Σ_{xx} $\{(\lambda_1, c_1), (\lambda_2, c_2), \ldots, (\lambda_n, c_n)\}$ with $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$ and the eigenvectors defined as unit vectors. As a result, $\mathrm{Var}(p_1) = \mathrm{Var}(c_1^\top x) = \lambda_1$ is the linear transformation with the highest possible variance. Since the eigenvectors are orthogonal, the calculation already contains the results for the rest of the features as well. The interested reader is referred to Shumway and Stoffer [2006] and Jolliffe [2005] for detailed information about the PCA and its theoretical background.

In order to apply the PCA, the ”smart-tire” signal has to be prepared accordingly. Firstly, the signal must be truncated per wheel turn. The end of the wheel revolution is provided by the use of the incremental encoder signal. Since the only use of the encoder is to provide the end of each wheel rotation, it has to be considered that the encoders typically employed on common ABS system provide such an information with adequate precision. Secondly, the signals acquired when the vehicle is cornering have to be removed as they are not covered by the model ((1), (2)). The decision to neglect a wheel turn is made by means of a threshold that defines the maximum allowed yaw rate of the vehicle measured by the gyroscope. Thirdly, as the PCA requires a constant number of data points per dataset, the datasets are linearly interpolated over the time.

In Figure 7 the extracted $c_i, i = 1 \ldots 5$ by means of the PCA, the so called features, are shown. It appears that feature 1 represents the general form of the signal per turn, whereas features 2 and 3 cover the accelerometer entering and leaving the contact patch. The corresponding eigenvalues are shown in Table 1.

In the following Section 4.4 the projections of the "smart-tire" signal on those features are used to identify the tire force models. The effect of different input configurations (number of features, complete or partial usage of the "smart-tire" signal) on the estimation are investigated.

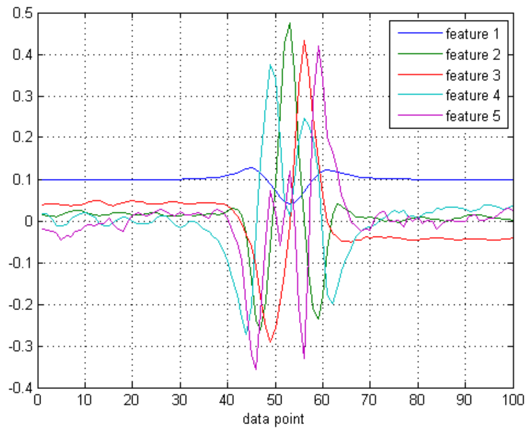

Fig. 7. Features 1-5

Table 1. Eigenvalues of features 1-5

feature(i)	λ_1	λ_2	λ_3	λ_4	λ_5
eigenvalue	2.69e7	3.69e4	2.63e4	3.23e3	8.04e2

4.2 Reference model for the patch forces

The equation of the vehicle longitudinal dynamic can be written as follows (Figure 8):

$$F_x = F_{aero} + F_{roll} + F_{acc} \tag{3}$$

$$F_{ZH} = \frac{m\,a\,h_S + m\,g\,l_V + F_{aero}\,h_S}{l} \tag{4}$$

where F_{aero} describes the aerodynamic effects on the car by means of a quadratic correlation with the velocity, namely

$$F_{aero} = \frac{c_w\,A\,\rho\,v^2}{2}. \tag{5}$$

For the sake of simplicity, it is assumed that the force is acting at the height of the center of gravity of the car.

The rolling resistance F_{roll}, due to the energy dissipation caused by the steady deformations of the patch, can be expressed as follows (del Re and Waschl [2013]):

$$f_R = f_{R,0} + f_{R,1}\,\frac{v}{v_0} + f_{R,4}\left(\frac{v}{v_0}\right)^4 \tag{6}$$

At low speed $f_{R,0}$ is the dominant part of the formula and is sufficient for a first approximation. The effects of tire pressure, temperature and road surface are not covered.

The force F_{acc} due to acceleration of the car can be computed on the basis of the car mass and acceleration, computed by derivation of the signal from the optical velocity sensor. The parameters in (3)-(6) have the usual meaning and their values are listed in Table 2.

A validation of this model and of its range of application is performed. To this purpose, the expected linear correlation (for small slip values, see e.g. Ammon [1997]) between the slip and the contact forces is verified. In Figure 9 it is shown that the longitudinal model delivers consistent results.

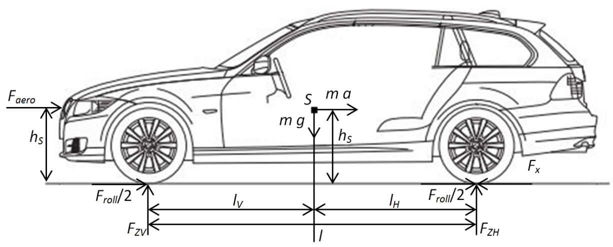

Fig. 8. Longitudinal model of the car

Table 2. Vehicle parameters and values

parameter	value	parameter	value
mass	1490kg	h_S	0.7m
l	2.735m	l_V	1.3675m
A	2m^2	c_w	0.32
ρ	1.29 $\frac{kg}{m^3}$	r_0	0.3064m
$f_{R,0}$	0.01	g	9.81 $\frac{m}{s^2}$

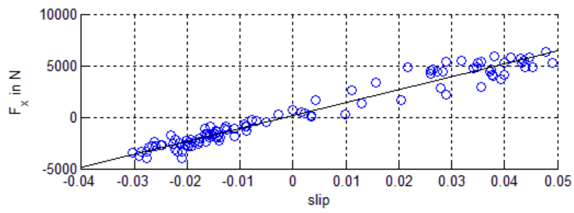

Fig. 9. Relation between slip and tire forces

4.3 Torque computation

An alternative approach to the force computation is based on the engine torque. It relies on the signals obtained from the Engine Control Unit (ECU) via the CAN-Bus. The revolution speed of both the motor (ECU) and the wheel (incremental encoder) are used to calculate the current gear ratio. Hence, from the motor torque (ECU) the torque acting on the tire can be evaluated. As the described method is independent from the aforementioned force computation based on the optical velocity sensor in Section 4.2, it can be used to further validate the model of the car for positive acceleration.

In Figure 10 the signals of the force model and of the motor torque are shown (note that the motor is only able to generate positive torque and, as a result, braking is not covered). The good fit is, together with the results in Figure 9, a further prove of the soundness of the approach employed to compute the patch contact force.

4.4 Identification and validation of tire force models

The whole data is divided into an identification and validation dataset. The identification dataset has been chosen so as to cover uniformly the data available, expressed in terms of acceleration and velocity. In Figure 11 the points are shown in blue circles. As a first attempt, the projections on the features 1-7 are used as inputs for the linear models $F_x = a_0 + \sum_{i=1}^{7} a_i\,p_i$, $F_z = b_0 + \sum_{i=1}^{7} b_i\,p_i$.

where $p_i = c_i^\top x$ is the projection of the "smart-tire" signal $x \in \mathbb{R}^n$ on the i-th feature c_i. The unknown parameters $a_i, b_i, i = 0 \dots 7$ have been computed by the least squares algorithm.

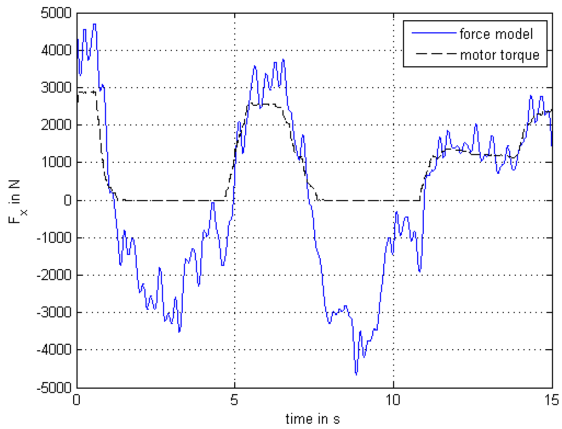

Fig. 10. Comparison of the force and the torque computation

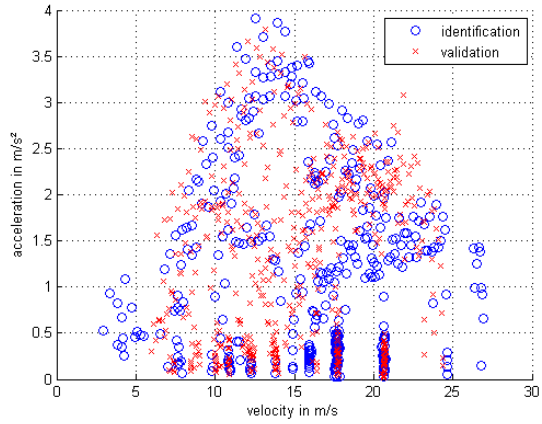

Fig. 11. Identification and validation data in the plane speed-acceleration

The comparison between the result of the identification and the real force F_x is shown in Figure 12. Similar results are delivered for F_z but are omitted for the sake of brevity. The corresponding fit values are listed in the third and in the fourth column of the second row of Table 3.

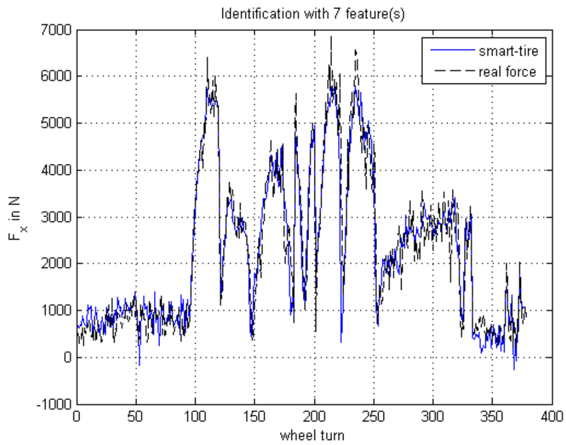

Fig. 12. Identification of the F_x model with features 1-7

The results of the validation of the F_x model are shown in Figure 13. Similar have been obtained for the force F_z. The high

Table 3. Fit values R^2. The *employed data* are referred to each tire rotation and are defined with reference to Figure 3.

features	employed data	$F_x ident$	$F_z ident$	$F_x val$	$F_z val$
1-7	1-100	0.91326	0.91325	0.89335	0.89350
2-3	1-100	0.88363	0.88347	0.86267	0.86325
1-7	25-75	0.91238	0.91238	0.89356	0.89372
1-7	1-25,75-100	0.89054	0.89050	0.82003	0.82023

R^2 values listed in the fifth and in the sixth column of the second row of Table 3 confirm that an estimation of the contact forces by means of an accelerometer glued to the inner side of the tire tread is possible.

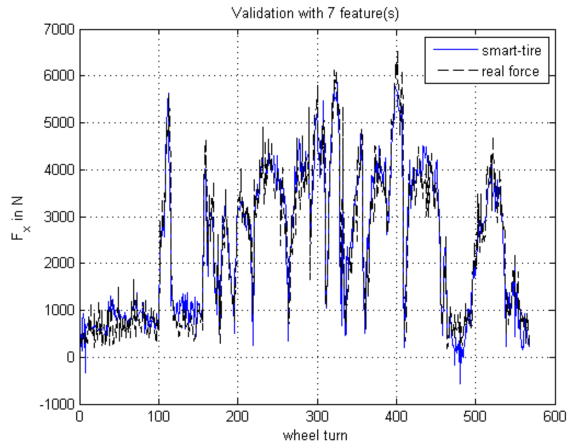

Fig. 13. Validation of the F_x model with features 1-7

Interesting is whether all of the 7 features are necessary to obtain such high R^2 values or if fewer projections offer similar results. As a matter of fact, the eigenvalues shown in Table 1 suggest that a low amount of "eigensignals" represent most of the signal characteristics. The results of the identification and validation are plotted in in Figure 14 and 15. Similar results are obtained for the model of the force F_z. The R^2 values in the third row of Table 3 show that, as already suspected, the degradation of quality of the model based only on the features 2-3 is minimal. Investigations have shown that inclusion of feature 1 do not provide considerable improvement. This is due to its dominant correlation with the vehicle speed and, hence, with the sole forces related to it.

As a further improvement, it is investigated if the usage of only a part of the signal can deliver high quality estimation. As a matter of fact, Figure 4 shows that the data points 25-75 represent the part of the signal, when the accelerometer is inside the contact patch, and that the two subsets 1-25 and 75-100 are principally determined by the centripetal acceleration. Therefore, Figure 4 suggests that the main information of the patch forces is included in the part 25-75.

The results of the identification performed on the subset 25-75 (fourth row of Table 3) confirm the validity of such an intuition. As a matter of fact, the validation fits in the fourth row of Table 3 are as good as those in the second row of Table 3.

Interestingly, also the signal parts outside the contact patch, although deemed to correlate mainly with the centripetal acceleration, still deliver promising fit (last row of Table 3).

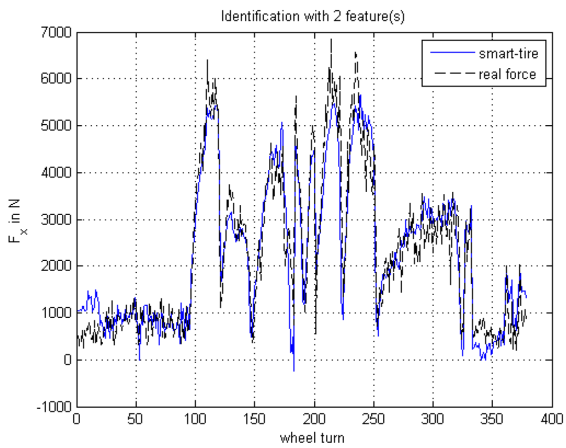

Fig. 14. Identification of the F_x model with features 2-3

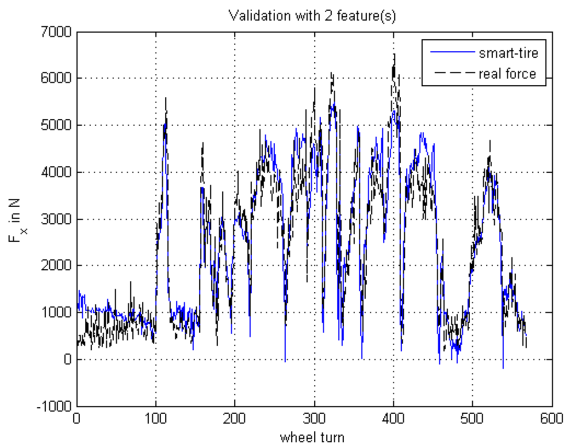

Fig. 15. Validation of the F_x model with features 2-3

5. CONCLUSION AND OUTLOOK

The experimental investigations in this work confirm that a piezoresistive accelerometer glued to the inner side of the tire tread in combination with advanced signal processing is capable of estimating the vertical and longitudinal contact forces between tire and road. A key element in this approach is the usage of Principal Component Analysis, as it describes most of the "smart-tire" signal variance by means of a few linear projections. Such projections correlate linearly with the contact forces. Several force models are identified and validated on the basis of the collected experimental data. Such models differ in the number of features considered as well as in the usage (complete or partial) of the smart-tire signal.

Future work will be devoted to the investigation of the robustness of the proposed approach with respect to the road conditions (dry/wet, tarmac/gravel, ...). The cornering and the lateral dynamics will also be tested in further experiments. Test bench experiments will allow further improvement compared to the approach stated in this paper.

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