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Robust Evolving Fuzzy Adaptive Control With Input-domain Clustering

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Abstract: The paper proposes a fuzzy model reference adaptive control approach with evolving antecedent part. The proposed algorithm has the possibility of controlling a plant with poorly known and/or time-varying nonlinearity which is an advantage over approaches with fixed antecedent part. It is intended for control of a large class of nonlinear plant models with the dominant dynamics of the first order. Such plants occur quite often in process industries.

Keywords: Adaptive neural and fuzzy control, adaptive control, model reference adaptive control, evolving systems, Takagi-Sugeno model, clusters

1. INTRODUCTION

Over the past decade a lot of work has been done in the field of adaptive fuzzy control. In general the schemes can be divided into direct and indirect adaptive schemes. The direct schemes approximate the ideal controller with fuzzy logic (Wang, 1993; Kim et al., 1996; Lee et al., 1996; Chen et al., 1997; Blažič et al., 2003; Labiod and Guerra, 2007; Tong et al., 2009, 2010; Blažič et al., 2012). The indirect schemes use a fuzzy system to approximate the plant dynamics (Wang et al., 2000, 2001; Chan et al., 2001; Golea et al., 2003; Qi and Brdys, 2008; Precup et al., 2009; Dovžan and Škrjanc, 2010). These approaches cope very well with nonlinear plant dynamics. They are also quite successful in controlling the plants with parasitic dynamics and disturbances due to their robustness properties. Even if slowly time varying systems are encountered, the results are usually quite satisfactory as a result of the parameter adaptation. The existing adaptive fuzzy schemes usually apply only the adaptation of the sub-models' parameters (consequent parameters) while the membership functions (premise parameters) are kept fixed. This results in two major shortcomings of these approaches: 1) they have problems with systems where the nonlinearity changes with time, and 2) quite often it is hard to partition the input space, i.e., to position the membership functions, with a limited plant knowledge.

Some papers apply the adaptation of both, the membership functions and the consequent parameters. Singh (1998) proposed a direct approach that uses triangular membership functions and the adaptation of its centres. Phan and Gale (2008) developed a direct algorithm that also uses triangular membership functions. The algorithm adds or replaces the membership functions depending on the error threshold. A direct approach is also proposed by Rojas et al. (2006). The approach uses triangular functions and tries to find the membership function configuration that distributes a certain performance criterion homogeneously throughout the operating regions. The approach presented uses the squared error as a performance criterion. The indirect adaptive approach is proposed by Qi and Brdys (2008). The approach uses Gaussian membership functions and adapts both the width and the centres. The adaptation of the centres and the widths of the membership functions is done using a gradient descent algorithm and the adaptation of sub-models' parameters is done using recursive least squares. Dovžan and Škrjanc (2010) proposed an adaptive fuzzy predictive control algorithm based on adaptive fuzzy model. The adaptation of the membership functions is done with the proposed recursive fuzzy c-means algorithm. The local model parameters are adapted using the recursive least squares method.

In recent years the on-line fuzzy learning has received a great amount of interest. The advantage of on-line fuzzy learning methods is that they are able to adapt a fuzzy model or a fuzzy control algorithm to the current process behaviour. This is useful especially for processes with changing dynamics or for the controller tuning. Depending on the learning abilities the on-line fuzzy learning methods can be divided into: *adaptive methods* where the initial structure of the fuzzy model must be given and the number of space partitions/clusters doesn't change over time, and therefore only the membership functions' and local models' parameters are adapted; *incremental methods* where only adding mechanisms are implemented; and *evolving methods* which, besides an adding mechanism, also implement removing and some of them also merging and splitting mechanisms. Some examples of these three groups are: adaptive – ANFIS (Shing and Jang, 1993), rFCM (Dovžan and Škrjanc, 2011b); incremental – DENFIS (Kasabov and Song, 2002), eTS (Angelov and Filev, 2004), FLEXFIS

(Lughofer and Klement, 2005); evolving – SAFIS (Rong et al., 2006), eTS+ (Angelov, 2010), etc.

The control of a practically very important class of plants is treated in this paper that occur quite often in process industries. The class of plants consists of nonlinear time-varying systems of arbitrary order but where the control law is based on the first-order nonlinear approximation where the nonlinearity is allowed to slowly change with time. The dynamics not included in the first-order approximation are referred to as parasitic dynamics. The parasitic dynamics are treated explicitly in the development of the adaptive law to prevent the modelling error to grow unbounded. The class of plant also includes bounded disturbances.

2. THE CLASS OF NONLINEAR AND SLOWLY TIME-VARYING PLANTS

The proposed control approach is intended for a class of plants that include nonlinear time-invariant systems where the model behaves similarly to a first-order system at low frequencies. Such a class was treated in some earlier work (Blažič et al., 2003, 2012) where the TS fuzzy model of the first order was generalised by adding stable factor plant perturbations ($\Delta_u(p)$ and $\Delta_y(p)$, p is a differential operator d/dt) and disturbances (d), which results in the following model:

$$\begin{aligned}\dot{y}_p(t) = -(\boldsymbol{\beta}^T(t)\mathbf{a})y_p(t) + (\boldsymbol{\beta}^T(t)\mathbf{b})u(t) - \\ - \Delta_y(p)y_p(t) + \Delta_u(p)u(t) + d(t)\end{aligned} \quad (1)$$

where $\mathbf{a} = \left[\, a_1 \; a_2 \; \dots \; a_N \,\right]^T$ and $\mathbf{b} = \left[\, b_1 \; b_2 \; \cdots \; b_N \,\right]^T$ are vectors of unknown plant parameters in respective fuzzy domains ($\mathbf{a}, \mathbf{b} \in \mathbb{R}^N$), and $\boldsymbol{\beta}$ is a vector of normalised degrees of membership functions. An important step in obtaining a good TS model is the choice of antecedent variables $\mathbf{x}_f \in \mathbb{R}^p$ and the construction of the membership functions that map $\mathbf{x}_f$ to $\boldsymbol{\beta}$. Whether the membership functions are chosen suitably is critically affected by the position of nonlinearities in the system. In our earlier work (Blažič et al., 2003, 2012) the mapping from $\mathbf{x}_f$ to $\boldsymbol{\beta}$ was fixed, i.e., the membership functions were chosen taken into account some a priori knowledge about the plant.

The former approach is also suitable for slowly time-varying systems. As long as the plant variation is due to changes in unknown parameters $\mathbf{a}$ and $\mathbf{b}$, the frozen-time model is still suitable for the description of the plant, and the terms due to parameter changes can be treated as (bounded) disturbances. If the nature of changes is such that the position of nonlinearities changes with time, it is necessary that the membership functions also move accordingly. This can be done by applying some evolving mechanism to the antecedent part. Such a mechanism is also very useful if a designer lacks a priori knowledge about the position of a nonlinearity. In such case the proposed approach automatically decides the number and the position of membership functions.

3. ON-LINE CLUSTERING AND MEMBERSHIP IDENTIFICATION

For on-line cluster identification the eFuMo method for on-line learning of fuzzy model was used by Dovžan et al. (2012). The method was used in the incremental mode meaning that only the adding of clusters was enabled. The first cluster is created when the first data sample arrives. Then, if necessary, other clusters are added when new data arrive. The method has two adding conditions that must be satisfied in order for a new cluster to be added: the distance conditions and the consequent samples conditions. The consequent samples condition is to prevent a new cluster being created based on outliers. This condition means that several consecutive samples must satisfy the distance adding condition before a new cluster is added (Hartert et al., 2010). In our case at least 4 consecutive samples had to satisfy the distance adding condition.

The distance conditions are based on the normalised distance of the current data sample to the closest cluster. In our case we used the normalised Mahanalobis distance condition. The normalised Mahanalobis distance is given by the following equation:

$$d_{i_{norm}}(k) = \frac{((\mathbf{x}_f(k) - \mathbf{v}_i)^T \mathbf{F}_i^{-1} (\mathbf{x}_f(k) - \mathbf{v}_i))^{0.5}}{c_n(\mathbf{s}_{i_{norm}}^T \mathbf{F}_i^{-1} \mathbf{s}_{i_{norm}})^{0.5}} \quad (2)$$

where $\mathbf{x}_f(k) \in \mathbb{R}^p$ is the clustering vector (data vector) in k-th time instant, $\mathbf{v}_i \in \mathbb{R}^p$ is the i-th cluster centre vector, $\mathbf{F}_i \in \mathbb{R}^{p \times p}$ is the fuzzy covariance matrix, c_n is the normalisation constant (in our case set to 4) and $\mathbf{s}_{i_{norm}} \in \mathbb{R}^p$ is defined as:

$$\mathbf{s}_{i_{norm}} = \left[\sqrt{F_{i_{11}}} \;\; \sqrt{F_{i_{22}}} \;\; \dots \;\; \sqrt{F_{i_{pp}}} \right]^T \quad (3)$$

The $F_{i_{jj}}$ are diagonal elements of the fuzzy covariance matrix. If the normalized distance (2) to the closest cluster is higher than one, the clustering vector (data sample) satisfies the distance adding condition. The new cluster centre and fuzzy covariance matrix are initialised as:

$$\mathbf{v}_{new} = \mathbf{x}_f(k), \quad (4)$$

$$\mathbf{F}_{new} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_p^2 \end{bmatrix} \quad (5)$$

where σ_j is the variance calculated based on the distance of the newly added cluster to the others. In each space dimension the distance from the new cluster to the closest cluster in that dimension is calculated:

$$d_j^2 = (v_{new_j} - v_{i_j})^2 \;\; j = 1, ..., p \quad (6)$$

where i is the index of the closest cluster centre. The variance is then calculated as:

$$\sigma_j^2 = -\frac{d_j^2}{2\ln(\epsilon_\beta)} \;\; j = 1, ..., p \quad (7)$$

where ϵ_β represents the wanted membership degree of the closest cluster to new cluster for each space dimension. The value in our case was set to $\epsilon_\beta = 0.15$.

The cluster centre position and fuzzy covariance matrix are calculated and adapted on-line using the recursive Gustafson-Kessel (GK) algorithm (Dovžan and Škrjanc, 2011a). The equation for centre adaptation is given by the following equation:

$$\mathbf{v}_i(k+1) = \mathbf{v}_i(k) + \Delta\mathbf{v}_i(k) \quad (8)$$

$$\Delta\mathbf{v}_i(k) = \frac{\mu_i(k)^\eta \left(\mathbf{x}_f(k) - \mathbf{v}_i(k)\right)}{S_i(k)} \quad (9)$$

where η is fuzziness factor (usually set to 2), μ_i is membership degree of the current clustering vector to the i-th cluster, and $S_i(k)$ is the filtered sum of the past membership degrees of the i-th cluster:

$$S_i(k) = \lambda_c S_i(k-1) + \mu_i(k)^\eta \tag{10}$$

Factor λ_c was introduced as a forgetting factor to enable the adaptation of centres. In the case of GK clustering the membership degrees μ_i for all c clusters are calculated as:

$$\mu_i(k) = \begin{cases} \dfrac{1}{\sum_{j=1}^{c}\left(\frac{d_i(k)}{d_j(k)}\right)^{\frac{2}{\eta-1}}} & \text{if } \mathbf{x}_f(k) \neq \mathbf{v}_i; i=1,...,c \\ 1 & \text{if } \mathbf{x}_f(k) = \mathbf{v}_i \\ 0 & \text{if } \mathbf{x}_f(k) = \mathbf{v}_j; i \neq j \end{cases} \tag{11}$$

where $d_i(k)$ is defined as:

$$d_i(k) = \left((\mathbf{x}_f(k) - \mathbf{v}_i(k))^T \det(\mathbf{F}_i)^{\frac{1}{p}} \mathbf{F}_i^{-1} (\mathbf{x}_f(k) - \mathbf{v}_i(k))\right)^{0.5} \tag{12}$$

In order to calculate the distance in (12), the fuzzy covariance matrix, its inverse and determinant must be calculated recursively. The recursive update equation for fuzzy covariance matrix is as follows:

$$\begin{aligned} \mathbf{F}_i(k+1) &= \gamma_c \frac{S_i(k-1)}{S_i(k)} \mathbf{F}_i(k) + \frac{\mu_i(k)^\eta}{S_i(k)} \mathbf{D}_{F_i}(k) \\ \mathbf{D}_{F_i}(k) &= (\mathbf{x}_f(k) - \mathbf{v}_i(k))(\mathbf{x}_f(k) - \mathbf{v}_i(k))^T \end{aligned} \tag{13}$$

where γ_c is the forgetting factor. The recursive update equation for the inverse matrix is obtained by using the Woodbury matrix identity lemma. The equation is as follows:

$$[\mathbf{F}_i(k+1)]^{-1} = \frac{1}{\gamma_c} \frac{S_i(k)}{S_i(k-1)} \left[[\mathbf{F}_i(k)]^{-1} - \frac{\mathbf{B}}{C} \right] \tag{14}$$

$$\mathbf{B} = [\mathbf{F}_i(k)]^{-1} \mathbf{D}_{F_i} [\mathbf{F}_i(k)]^{-1} \tag{15}$$

$$C = \gamma_c \frac{S_i(k-1)}{\mu_i(k)^\eta} + \mathbf{d}_{F_i}^T(k) [\mathbf{F}_i(k)]^{-1} \mathbf{d}_{F_i}(k) \tag{16}$$

$$\mathbf{d}_{F_i}(k) = \mathbf{x}_f(k) - \mathbf{v}_i(k) \tag{17}$$

The determinant is obtained using determinant lemma. The recursive equation for determinant can be written as:

$$\det(\mathbf{F}_i(k+1)) = \left(\gamma_c \frac{S_i(k-1)}{S_i(k)}\right)^p \det(\mathbf{F}_i(k))(1+A) \tag{18}$$

$$A = \frac{1}{\gamma_c} \frac{\mu_i(k)^\eta}{S_i(k-1)} \left(\mathbf{d}_{F_i}^T(k) [\mathbf{F}_i(k)]^{-1} \mathbf{d}_{F_i}(k)\right) \tag{19}$$

where p is the number of rows/columns of fuzzy covariance matrix. The detailed derivations of equations are given in Dovžan and Škrjanc (2011a).

4. ROBUST FUZZY ADAPTIVE ALGORITHM

The fuzzy model reference adaptive control is proposed in the paper to achieve tracking control for the class of plants described in the previous section. The control goal is that the plant output follows the output y_m of the reference model. The latter is defined by a first order linear system $G_m(p)$:

$$y_m(t) = G_m(p) w(t) = \frac{b_m}{p + a_m} w(t) \tag{20}$$

where $w(t)$ is the reference signal while b_m and a_m are the constants that define desired behaviour of the closed system. The tracking error

$$\varepsilon(t) = y_p(t) - y_m(t) \tag{21}$$

therefore represents some measure of the control quality. To solve the control problem simple control and adaptive laws are proposed in the following subsections.

4.1 Control law

The control law is the same as the one proposed in Blažič et al. (2003):

$$u(t) = \left(\boldsymbol{\beta}^T(t)\hat{\mathbf{f}}(t)\right) w(t) - \left(\boldsymbol{\beta}^T(t)\hat{\mathbf{q}}(t)\right) y_p(t) \tag{22}$$

where $\hat{\mathbf{f}}(t) \in \mathbb{R}^k$ and $\hat{\mathbf{q}}(t) \in \mathbb{R}^k$ are the control gain vectors to be determined by the adaptive law. This control law is obtained by generalising the model reference adaptive control algorithm for the first order linear plant to the fuzzy case.

4.2 Adaptive law

The adaptive law proposed by Blažič et al. (2012) is also used here:

$$\begin{aligned} \dot{\hat{f}}_i &= -\gamma_{fi} b_{sign} \varepsilon w \beta_i - \gamma_{fi} \sigma' w^2 \beta_i^2 (\hat{f}_i - f_i^*) \quad i = 1,2,\dots k \\ \dot{\hat{q}}_i &= \gamma_{qi} b_{sign} \varepsilon y_p \beta_i - \gamma_{qi} \sigma' y_p^2 \beta_i^2 (\hat{q}_i - q_i^*) \quad i = 1,2,\dots k \end{aligned} \tag{23}$$

where γ_{fi} and γ_{qi} are positive scalars referred to as adaptive gains, $\sigma' > 0$ is the parameter of the leakage term, f_i^* and q_i^* are the a priori estimates of the control gains $\hat{f}_i$ and $\hat{q}_i$, respectively, and b_{sign} is the sign of all the elements in vector $\mathbf{b}$. If the signs of all elements in vector $\mathbf{b}$ are not the same, the plant is not controllable for some $\boldsymbol{\beta}$ ($\boldsymbol{\beta}^T\mathbf{b}$ is equal to 0 for this $\boldsymbol{\beta}$) and the control is not possible using this approach.

It is possible to rewrite the adaptive law (23) in the compact form if the control gain vectors $\hat{\mathbf{f}}$ and $\hat{\mathbf{q}}$ are defined. Then the adaptive law (23) takes the following form:

$$\begin{aligned} \dot{\hat{\mathbf{f}}} &= -\boldsymbol{\Gamma}_f b_{sign} \varepsilon w \boldsymbol{\beta} - \boldsymbol{\Gamma}_f \sigma' w^2 \operatorname{diag}(\boldsymbol{\beta}) \operatorname{diag}(\boldsymbol{\beta})(\hat{\mathbf{f}} - \hat{\mathbf{f}}^*) \\ \dot{\hat{\mathbf{q}}} &= \boldsymbol{\Gamma}_q b_{sign} \varepsilon y_p \boldsymbol{\beta} - \boldsymbol{\Gamma}_q \sigma' y_p^2 \operatorname{diag}(\boldsymbol{\beta}) \operatorname{diag}(\boldsymbol{\beta})(\hat{\mathbf{q}} - \hat{\mathbf{q}}^*) \end{aligned} \tag{24}$$

where $\boldsymbol{\Gamma}_f \in \mathbb{R}^k \times \mathbb{R}^k$ and $\boldsymbol{\Gamma}_q \in \mathbb{R}^k \times \mathbb{R}^k$ are positive definite matrices, $\operatorname{diag}(\mathbf{x}) \in \mathbb{R}^k \times \mathbb{R}^k$ is the diagonal matrix with the elements of vector $\mathbf{x}$ on the main diagonal, while $\hat{\mathbf{f}}^* \in \mathbb{R}^k$ and $\hat{\mathbf{q}}^* \in \mathbb{R}^k$ are the a priori estimates of the control gain vectors.

5. STABILITY ISSUES

In the previous work (Blažič et al., 2014) the stability analysis has been presented for the case where fixed membership functions are used while the adaptive law is almost the same as in this work. Using the approach there the following error model for the tracking error ε can be obtained:

$$\begin{aligned} \dot{\varepsilon} = &-a_m \varepsilon - \left[(\boldsymbol{\beta}^T\mathbf{b})(\boldsymbol{\beta}^T\hat{\mathbf{q}}) + (\boldsymbol{\beta}^T\mathbf{a}) - a_m\right] y_p + \\ &+ \left[(\boldsymbol{\beta}^T\mathbf{b})(\boldsymbol{\beta}^T\hat{\mathbf{f}}) - b_m\right] w + \Delta_u(p) u - \Delta_y(p) y_p + d \end{aligned} \tag{25}$$

If ideal parameters for the consequent part are defined, the total error can be defined as (Blažič et al., 2014):

$$E(t) = \eta_f(t)w(t) - \eta_q(t)y_p(t) + \\ + \Delta_u(p)u(t) - \Delta_y(p)y_p(t) + d(t) \quad (26)$$

where $\eta_f(t)$ and $\eta_q(t)$ are bounded functions that arise because of the TS modelling error. The error $E(t)$ represents a disturbance of the control system in the consequent part.

The extension in this paper results in two new sources that contribute to the error (26). The first one is due to changes in plant parameters $\mathbf{a}$ and $\mathbf{b}$. If $\dot{\mathbf{a}}(t)$ and $\dot{\mathbf{b}}(t)$ are bounded, the resulting terms in the error $E(t)$ are bounded also. The second source is due to changes in membership functions. Since the calculation of memberships does not have any role in the stability analysis (the elements of the vector $\boldsymbol{\beta}(t)$ are a priori bounded), it is possible to conduct a similar stability analysis as in the previous paper (Blažič et al., 2012). The proposed approach therefore guarantee global stability in the sense that all the signals are bounded. The tracking error converges to a residual set whose size depends on the size of disturbances, the norm of the parasitic dynamics, the size of the fuzzy modelling error, and the term due to the plant parameter changes.

6. SIMULATION EXAMPLE

A simulation example will be given that illustrates the proposed approach. A plant model used in the paper is the same as in (Blažič et al., 2012). It is an extended form of the model in (Rohrs et al., 1985) that can be rewritten in state space form

$$\begin{aligned} \dot{y}_p &= -y_p + 2u_f \\ \dot{u}_f &= 229x_1 - 30u_f \\ \dot{x}_1 &= -u_f + u \end{aligned} \quad (27)$$

where the part of the system between the plant input u and u_f represents the parasitic dynamics while the first equation in (27) describes nominal plant (the one used for control design). The plant was made nonlinear by adding extra terms to (27). Some properties of the original system were preserved, namely linearised behaviour in the nominal operating point ($u = 0, y_p = 0$) and 'relative order' of the plant (meaning that u and x_1 do not influence $\dot{y}_p$ directly even in the form of higher powers). The resulting system used for simulations is

$$\begin{aligned} \dot{y}_p &= -y_p + 2u_f + (-0.5y_p + 0.1u_f)^2 + (-0.6y_p + 0.1u_f)^3 \\ \dot{u}_f &= 229x_1 - 30u_f + 6x_1^2 - 2x_1u_f - 0.1u_f^2 \\ \dot{x}_1 &= -u_f + u + 0.01u^2 - 0.01uu_f - 0.01u_f^2 \end{aligned} \quad (28)$$

The design objective is that the output of the plant follows the output of the reference model $3/(s+3)$ (Rohrs et al., 1985). Two experiments were conducted, one with the fuzzy adaptive model reference adaptive control with fixed membership functions (Blažič et al., 2012) and one with the proposed control algorithm. The reference signal was the same in both cases. It consisted of a periodic signal, followed by two big steps.

By analysing the plant (28) it can be seen that not only the plant but also the parasitic dynamics are nonlinear. The latter is a violation of the initial assumptions. This means that the example will also test the ability of the proposed control to cope with nonlinear parasitic dynamics. The coefficients of the linearised system in different operating points depend on u, x_1, u_f, and y_p. The approach here will be compared to the one in the previous paper (Blažič et al., 2012). This is why only y_p will be used as an antecedent variable in the control law. Although the proposed approach enables the positioning of membership functions in a higher dimensional space, we will use here a lower dimensional projection.

In the previous paper (Blažič et al., 2012) eleven triangular membership functions were used that were distributed evenly across all the operating region of the reference signal. In the current paper clusters define membership functions. Usually Gaussian membership functions are used since the parameters of the functions have straightforward connection to the obtained cluster properties. The cluster centres are used as centres of the Gaussian membership functions and diagonal elements of the fuzzy covariance matrix are used for defining the width of the membership functions. This means that the j-th element of the unnormalised degree of membership to the i-th cluster is calculated by the following formula:

$$\beta_{i_j}^0 = e^{-\frac{(x_{f_j} - v_{i_j})^2}{2\eta_m F_{i_{jj}}}} \quad j = 1, 2, ...p-1 \;\; i = 1, 2, ..., c \quad (29)$$

where η_m is the overlapping factor usually set to 1. Note that the vector $\boldsymbol{\beta}$ in (29) has only $p-1$ elements while the vector $\boldsymbol{\mu}$ in (11) has p elements. This is due to the fact that clustering is done in the space of two variables (u and y_p) while only y_p is used as an antecedent variable in the control law.

We used the following design parameters in both approaches: $\gamma_f = 0.03$, $\gamma_q = 0.03$, $\sigma' = 0.003$. Figures 1 and 2 show the results of the approach with fixed membership functions while the results of the proposed evolving approach are shown in figures 3 and 4. The position of clusters are shown by magenta dots in Fig. 5 where the measured data are plotted with cyan colour. After projection of the clusters to the 1-D input (y_p) unnormalised membership functions are obtained. They are plotted in Fig. 6 together with the normalised membership functions.

The results of both approaches are almost the same. We should emphasise that the evolving approach was initialised with cluster data empty. The system was able to position the clusters correctly.

7. CONCLUSION

The paper proposed a fuzzy adaptive approach with evolving antecedent part which guarantees global stability. It was shown in the example that the absence of any a priori knowledge about the position of the nonlinearity can be overcome since the approach is able to position the membership functions properly. The usefulness of the approach could be even more explicit if membership functions of higher dimensions are used and/or the nonlinearity changes with time.

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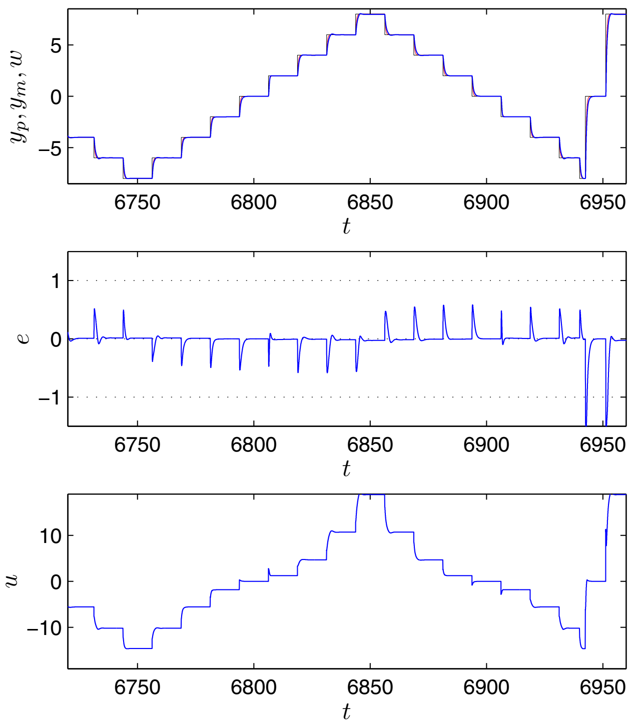

Fig. 1. The approach with fixed membership functions – time plots of y_p, y_m, w (upper figure), tracking error (middle figure), and the control signal (lower figure)

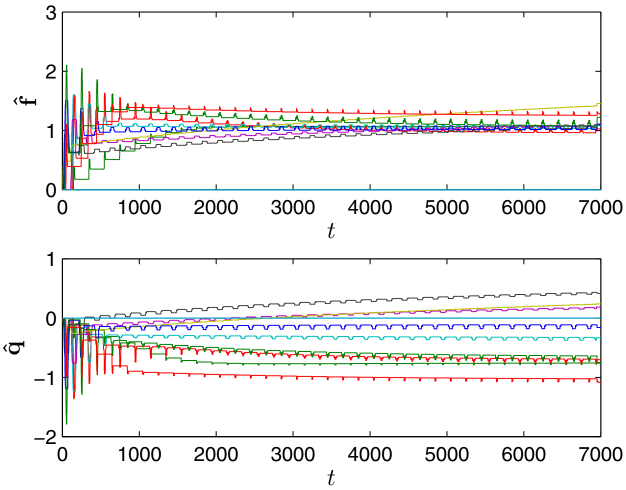

Fig. 2. The approach with fixed membership functions – time plots of feedforward and feedback control gains

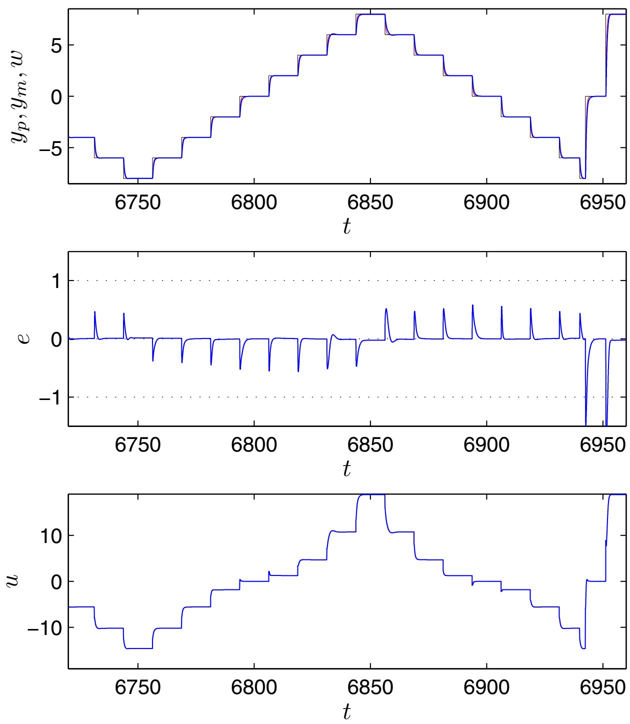

Fig. 3. The proposed evolving approach – time plots of y_p, y_m, w (upper figure), tracking error (middle figure), and the control signal (lower figure)

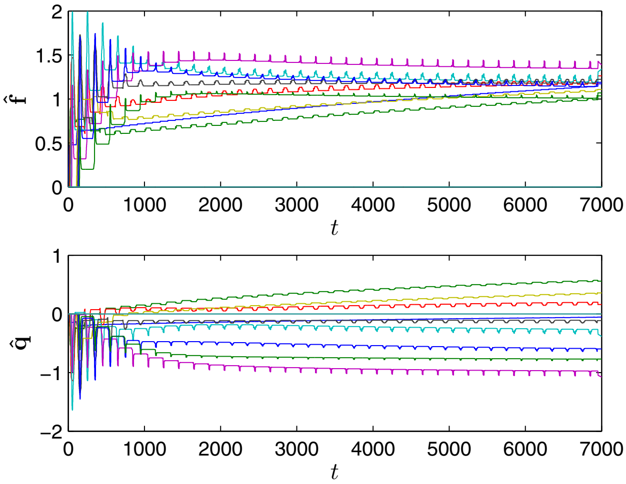

Fig. 4. The proposed evolving approach – time plots of feedforward and feedback control gains

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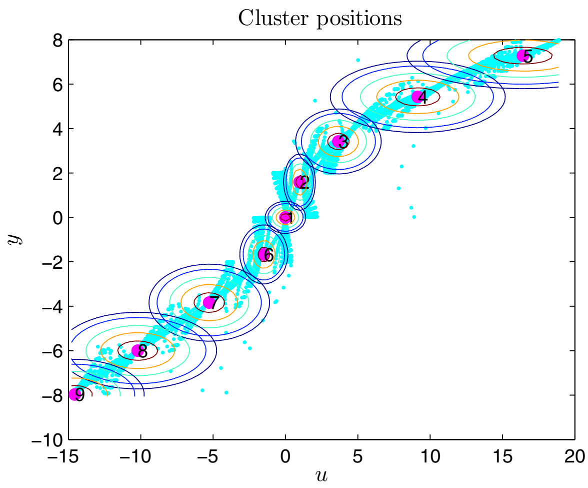

Fig. 5. The position of clusters

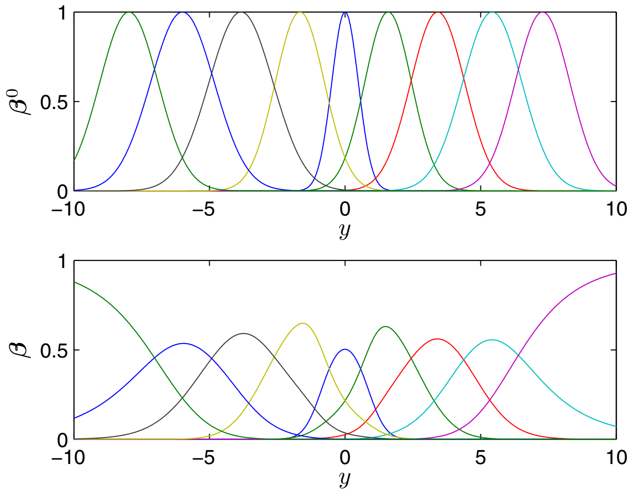

Fig. 6. Membership functions

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