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Temperature control of two interacting rooms with decoupled PI Control

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Abstract: Using a simulated model of a house consisting of two adjacent rooms, the temperature of the two rooms is controlled with a PI controller and a decoupled PI controller. This is compared to MPC control. In the simulation example here, both the MPC controller and the decoupled PI controller decreased the interaction between the temperature dynamics of the two rooms, as compared to an independent PI controller in each room. For the example in this paper, the control performance of the decoupled PI controller is comparable to that of MPC, regarding the interactions between the two rooms.

Keywords: Energy management systems; Building and home automation; PID control; Predictive control; dynamic decoupling;

1. INTRODUCTION

Buildings account for 40 % of total energy consumption in the European Union [European Parliament, 2010], in Sweden one third of the energy used is related to the building sector, and 60% of the energy used in buildings is used for heating and ventilation [Persson, 2002]. With a growing building sector, it is necessary to decrease the energy used by heating and ventilation in buildings, so the total energy used in the buildings sector is not increased. Therefore control of heating and ventilation systems in buildings has been an active research area for many years. Many different methods to control temperature and heat in a building have been proposed, model predictive control (MPC) being one of the most popular [Ma et al., 2012, Hazyuk et al., 2012].

Model predictive control is chosen many times as the most suitable control method for buildings, since it can consider predictions of disturbances such as weather, occupancy or outside temperature. On the other hand, MPC requires a model of the building with sufficiently good accuracy and accurate predictions of the disturbances to give a good control performance. Besides estimating a model describing the building dynamics, the tuning of the controller parameters is not necessarily intuitive and not always easy to achieve.

Another control methodology widely used in temperature control and building components control is PI control [Salsbury, 2005, Dounis and Caraiscos, 2009]. Building components using PI control areis thermostats and thermostatic valves on radiators [Peffer et al., 2011]. The temperature of a room or area is then controlled locally, without any knowlege about the temperature control in neighboring rooms.

When the temperature in several rooms in a building is controlled independently for each room, it can be expected that the influence of a temperature change in a neighboring room can degrade the controller performance. The approach in this paper is to connect the PI controllers for different rooms with a decoupling network [Gagnon et al., 1998]. Using such a network, temperature changes in one room are not affecting the other room, so that the local PI controllers are connected to a more centralized control solution. For comparison, the same building model is controlled with an MPC algorithm.

The building model used in this paper for simulation and controller design is presented in Section 2. Section 3 describes the decoupled PI control and the MPC algorithm. Next, the simulation results comparing PI controllers without a decoupling network, PI control with a decoupling network and an MPC controller are presented in Section 4. These results are discussed in Section 5. Finally, Section 6 concludes the paper.

2. A MODEL OF TWO NEIGHBORING ROOMS

The building to be controlled is shown in Fig. 1. It consists of two rooms with an interconnecting wall. The house is 5 meters long, 2.5 meters high and the two rooms have a width of 3 meters and 5 meters, respectively. It was assumed that the walls are 0.2 meters thick and have a heat conductivity of 0.04 [W/(m °C)]. Each room has a glass window area of 1 m^2 with a heat conductivity of 0.78 [W/(m °C)] and a thickness of 0.01 meter.

The temperature dynamics of the house were modeled through heat transfer, combining the effect of storage and conduction of heat in the building elements and the outside and inside air in a lumped parameter model [Felgner et al., 2002], where time delays were neglected. At the center of each room, a temperature node was placed to represent the average temperature in each room. Also, the outside temperature was represented though a temperature node outside of the two rooms. The resulting model describing the temperature dynamics of the house is shown in (1).

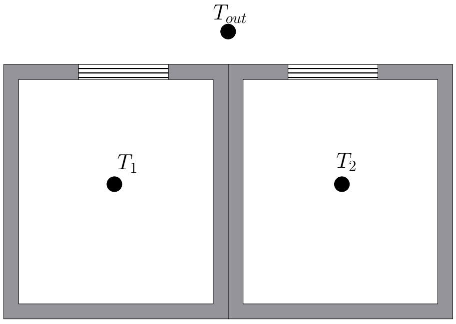

Fig. 1. Sketch of the two rooms to be controlled. The average temperature of room 1 is denoted by T_1 and the average temperature of room 2 by T_2. The outside temperature is denoted by T_{out}

$$
\begin{aligned}
M_1 c \frac{dT_1}{dt} &= \frac{A_1}{R_1}(T_{\text{out}} - T_1) + \frac{A_i}{R_i}(T_2 - T_1) + Q_1 \qquad (1) \\
M_2 c \frac{dT_2}{dt} &= \frac{A_2}{R_2}(T_{\text{out}} - T_2) + \frac{A_i}{R_i}(T_1 - T_2) + Q_2
\end{aligned}
$$

Here, the resistances R_1 and R_2 represent conduction of heat between the temperature nodes in the center of each room and the outside temperature through the room air, the outer walls and the windows. The resistance R_i represents the conduction of heat through the inner wall between the neighboring rooms. The area of the inside wall is denoted by A_i and the area of the outside walls of room 1 and room 2 by A_1 and A_2, respectively. The terms on the left-hand-side corresponds to storage of heat in the room air, where M_1 and M_2 are the air masses in the two rooms and c is the specific heat capacity of air. The heat flow into the two rooms is denoted by Q_1 and Q_2. This model was implemented in MATLAB/Simulink to simulate the temperature dynamics of the house shown in Fig. 1. It can be seen in (1), that the interconnecting wall between the two rooms introduces a coupling between the temperature dynamics.

For the use in PI and MPC control, the temperature dynamics need to be expressed as a transfer function model and a state-space model. Disregarding the outside temperature, which was assumed to be an unknown disturbance, the state-space representation of (1) is shown in (2) and the transfer function representation in (3).

$$
\begin{aligned}
\frac{d}{dt}\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} &= A \cdot \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} + B \cdot \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \qquad (2) \\
y(t) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} T_1 \\ T_2 \end{pmatrix}
\end{aligned}
$$

The states and the outputs of the state-space system (2) are the temperatures T_1 and T_2 in the two rooms. The inputs are the heat flows Q_1 and Q_2 into room 1 and room 2, respectively.

$$
\begin{aligned}
T_1(s) &= G_{11}(s)Q_1(s) + G_{12}(s)Q_2(s) \qquad (3) \\
T_2(s) &= G_{21}(s)Q_1(s) + G_{22}(s)Q_2(s)
\end{aligned}
$$

In the transfer function representation (3), $G_{12}(s)$ and $G_{21}(s)$ represent the interaction of the temperature dynamics between the two rooms.

3. TEMPERATURE CONTROL

The temperature of the house in Fig. 1 was controlled with an independent PI controller in each of the two rooms, PI controllers with a decoupling network and an MPC controller. Compared to the two single PI controllers the decoupled PI controllers take the interaction between the two rooms in the house into consideration and are expected to reduce the interaction in the temperature dynamics. Since the MPC is a controller with multiple inputs and outputs, it takes the interaction into account as well.

3.1 Decoupled PI control

For each of the rooms, a PI controller was designed to control the temperature of each room separately, disregarding the coupling of the temperature dynamics between the rooms. The parameters of the PI controllers were determined through Ziegler-Nichols step response method [Astrom and Hagglund, 2006].

To remove the effect of the coupling, a decoupling network was introduced between the PI controllers and the process. Here inverted decoupling was applied, which has both a simple realization and a diagonal decoupling matrix [Gagnon et al., 1998, Garrido et al., 2011]. Figure 2 depicts a control system with inverted decoupling, where the process inputs $u_1(s)$ and $u_2(s)$ are a combination of the respective controller output $c_1(s)$ or $c_2(s)$ and the other respective process input. The control signal for inverted decoupling is shown in (4).

$$
\begin{aligned}
u_1(s) &= c_1(s) + u_2(s)\frac{G_{12}(s)}{G_{11}(s)} \qquad (4) \\
u_2(s) &= c_2(s) + u_1(s)\frac{G_{21}(s)}{G_{22}(s)}
\end{aligned}
$$

With this, the transfer function from the controller outputs to the process outputs is a diagonal matrix where the effect of the coupling is removed.

Since only heating of the rooms was considered, the control signals were limited to be positive. To cope with this input constraint, an anti-windup strategy was added to the decoupled PI controller. Because of the structure of inverted decoupling, an anti-windup method as employed for single PID controllers could be applied directly [Gagnon et al., 1998]. The anti-windup strategy used here is based on back-calculation [Astrom and Hagglund, 2006], where the difference between the saturated and the non-saturated signal is fed back around the integrator in the PI controller with a time constant T_t. When there is no saturation, the anti-windup scheme has no effect. Otherwise, it will try to drive the output of the integrator, such that the control signal is close to its saturation limit.

3.2 MPC control

A model predictive control (MPC) algorithm was designed to control the temperature of both rooms using the state-space model (2). Since the MPC algorithm operates in discrete time, (2) was discretized using zero-order-hold discretization with a sampling time of $Ts = 0.0028$ hours Astrom and Wittenmark [1997]. Furthermore, integral action was introduced into the MPC algorithm by introducing a constant disturbance on the input [Maciejowski and

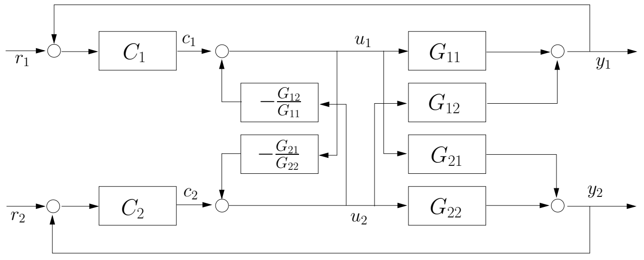

Fig. 2. A control system with inverted decoupling for a system with two inputs and two outputs.

Huzmezan, 1997], which was added as an additional state. The process model used for the MPC algorithm is shown in (5), where A_d, B_d, C_d are the discretized state-space matrices and $v(k)$ is a constant input disturbance.

$$\begin{bmatrix} x(k+1) \\ v(k+1) \end{bmatrix} = \begin{pmatrix} A_d & B_d \\ 0 & I \end{pmatrix} \begin{bmatrix} x(k) \\ v(k) \end{bmatrix} + \begin{pmatrix} B_d \\ 0 \end{pmatrix} u(k) \qquad (5)$$

$$y(k) = (C_d \ \ 0) \begin{bmatrix} x(k) \\ v(k) \end{bmatrix}$$

The state $x(k)$ consists of the room temperatures T_1 and T_2 at time k. Since the additional state $v(k)$ is not measured, a Kalman filter was used to estimate the state-vector of (5) [Astrom and Wittenmark, 1997].

For an MPC algorithm, predictions $\hat{y}(k)$ of the process output are needed. To determine these predictions for the prediction horizon H_p, the model (5) was used. These prediction are calculated as shown in (6).

$$\mathcal{Y}_{H_p} = \mathcal{S}_A \cdot x(k) + \mathcal{S}_u \cdot u(k-1) + \mathcal{S}_{\Delta u} \cdot \Delta\mathcal{U}_{H_u} \qquad (6)$$

where

$$\mathcal{Y}_{H_p} = [\hat{y}_1(k+1) \ \hat{y}_1(k+2) \ \ldots \ \hat{y}_1(k+H_p)]^T$$
$$\Delta\mathcal{U}_{H_u} = [\Delta u(k) \ \Delta u(k+1) \ \ldots \ \Delta u(k+H_u-1)]^T$$

and $\Delta u(k) = u(k) - u(k-1)$. The expressions for $\mathcal{S}_A$, $\mathcal{S}_u$ and $\mathcal{S}_{\Delta u}$ are presented in the Appendix A.

Using these predictions, the optimization problem to be solved for the MPC algorithm is (7).

$$\begin{aligned} \underset{\Delta u(k+m)}{\text{minimize}} \quad & \sum_{m=1}^{H_p} \|\hat{y}(k+m) - r(k+m)\|_Q^2 \\ & + \sum_{m=0}^{H_u} \|\Delta u(k+m)\|_R^2 \qquad (7) \\ \text{subject to} \quad & \\ & 0 \leq u(k+m) \leq u_{max}, \\ & m = 0, \ldots, H_u - 1. \end{aligned}$$

This optimization problem was solved using CVX, a package for specifying and solving convex programs [Grant and Boyd, 2013, 2008].

4. SIMULATION

The model of the two rooms and the control algorithms were implemented in MATLAB/Simulink. The controller parameters for the PI controllers and the corresponding closed-loop damping and natural frequency for both rooms are shown in Table 4. The parameters for the MPC controller were prediction horizon $H_p = 10$, control horizon $H_u = 5$, and the weighting matrices for the cost function $R = I$ and $Q = \left[\begin{smallmatrix} 0.1 & 0 \\ 0 & 0.5 \end{smallmatrix}\right]$.

Table 1. PI controller parameters for the two rooms and the corresponding damping ξ and natural frequency ω.

	K	$\mathbf{T_i}$	ω	ξ
Room 1	0.95	0.02	61.01	0.68
Room 2	0.99	0.04	34.11	0.76

Table 2. Mean square deviation of the room temperaure from its reference value and heat used per hour in the case where the reference temperature in *room 1* changes. T_e: mean square error between room temperature and reference temperature. Q_h: heating power used per hour.

	Room 1		**Room 2**	
	$\mathbf{T_e}$[°C]	$\mathbf{Q_h}$ $[10^3 kW/h]$	$\mathbf{T_e}$[°C]	$\mathbf{Q_h}$ $[10^3 kW/h]$
PI	1.97	2159	0.06	3489
PI decoupled	1.97	2156	0.005	3530
MPC	1.95	2295	0.001	3523

To investigate the effect of interaction in the simulation, the temperature in one of the rooms was changed, while observing the effect of this change of temperature on the other room. Each room was controlled by either an independent PI controller, by PI controllers with inverse decoupling or by an MPC controller. The simulation results are shown in Fig. 3 and Fig. 4. In both figures, the first panel shows the temperature in room 1, the second panel the heat flow used to heat room 1, the third panel shows the temperature in room 2 and the fourth panel the heat flow needed to heat room 2. The deviation of the room temperature from its reference temperature and the heat used per hour are shown in Table 2 and Table 3. The deviation of the room temperature from its reference temperature T_e and the heat used per hour Q_h are calculated by $T_e = (1/N)\sum_{k=0}^{k=t_s}(T(k) - T_{ref}(k))$ and $Q_h = \sum_{k=0}^{k=t_s} Q(k)/t_s$. Here, the length of the simulation is denoted by t_s, the number of simulated data points by N, the temperature in a room by T, the corresponding reference temperature by T_{ref}, the heat used by Q and the time index in the simulation by k.

The results for the effect of changing the temperature in room 1 on the temperature in room 2 is shown in Fig. 3 and Table 2. The influence on the temperature in room 2 from a change of reference temperature in room 1 is largest with the independent PI controllers without decoupling. Both MPC control and PI control with inverse decoupling decrease the deviation of the temperature in room 2 from its 20,° C reference temperature.

The influence of a temperature change in room 2 on the temperature in room 1 is shown in Fig. 4 and Table 3. The PI controller without decoupling leads to a larger influence of temperature changes in room 2 on the temperature in room 1. Both the PI control with inverted decoupling and the MPC controller decrease the temperature change in room 1 after a change of temperature in room 2.

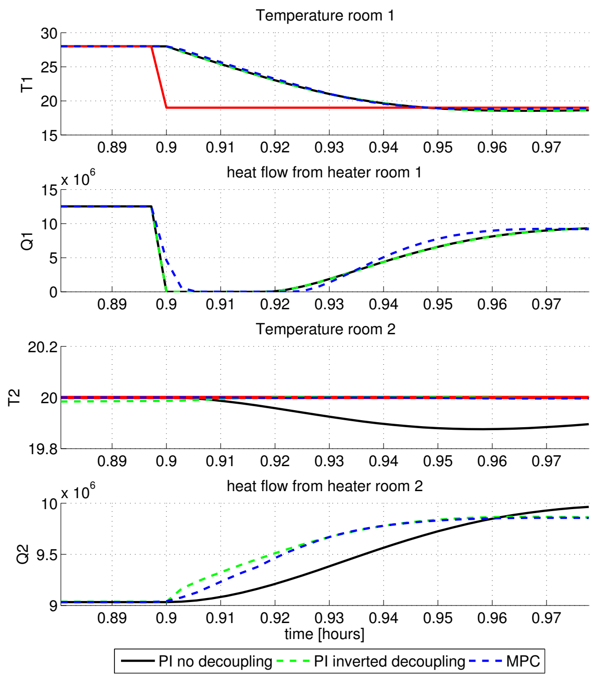

Fig. 3. Simlation results for a change of reference temperature in room 1 for PI control without decoupling (black solid), PI control with inverted decoupling (greed dashed) and MPC (blue dashed). Outside temperature $T_{\mathrm{out}} = -10\,°\mathrm{C}$. *first panel*: Temperature T_1 of room 1. The red dashed curve is the reference temperature. During the first 0.9 hours, the reference is 28 °C, afterwards the reference temperature is 19 °C. *second panel*: Heat flux Q_1 into room 1. *third panel*: Temperature T_2 in room 2. The red dashed curve is the reference temperature. *fourth panel*: Heat flux Q_2 into room 2.

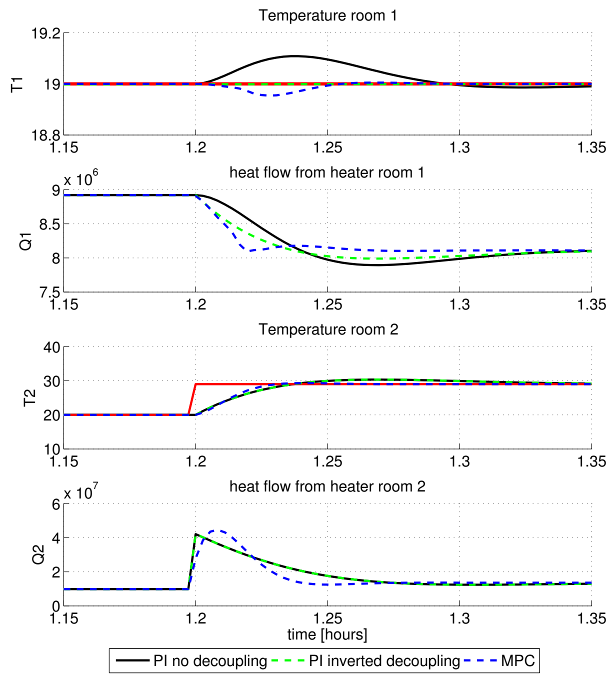

Fig. 4. Simulation results for a change of reference temperature in room 2 for PI control without decoupling (black solid), PI control with inverted decoupling (greed dashed) and MPC (blue dashed). *first panel*: Temperature T_1 of room 1. The red dashed curve is the reference temperature. *second panel*: Heat flux Q_1 into room 1. *third panel*: Temperature T_2 in room 2. The red dashed curve is the reference temperature. During the first 1.2 hours, the reference is 20 °C, afterwards the reference temperature is 29 °C. *fourth panel*: Heat flux Q_2 into room 2.

Table 3. Mean square deviation of the room temperaure from its reference value and heat used per hour in the case where the reference temperature in *room 2* changes. T_e: mean square error between room temperature and reference temperature. Q_h: heating power used per hour.

	Room 1		**Room 2**	
	$\mathbf{T_e}$[°C]	$\mathbf{Q_h}$ [$10^3 kW/h$]	$\mathbf{T_e}$[°C]	$\mathbf{Q_h}$ [$10^3 kW/h$]
PI	0.03	3044	1.15	5926
PI decoupled	0.001	3041	1.15	5930
MPC	0.006	3052	0.67	5775

5. DISCUSSION

Changing the temperature in one of two adjacent rooms affects the temperature in the second room as well. Having a seperate PI controller for each room, where each controller operates independent of the other one, this interaction between the temperature dynamics of the two rooms is not taken account of. Hence, a decoupling network was added connecting the PI controllers. The results in Fig. 3 and Fig. 4 show that with this decoupling network, the deviation of, e.g., the temperature in the second room from its reference temperature, when the reference temperature in the first room was changed, could be decreased.

Furthermore, the two adjacent rooms were controlled using a central MPC controller, controlling the temperature of both rooms at the same time. In this way, interactions of the the temperature dynamics of both rooms are taken into account as well. In the simulation here (see Fig. 3 and Fig. 4), the deviation of the room temperature from the reference temperature of the room with constant reference temperature was less with the MPC control than with two independent PI controllers.

Often MPC is viewed as the preferable control method in building contexts [Hazyuk et al., 2012, Ma et al., 2012, Dounis and Caraiscos, 2009], since it can take into account predictions of disturbances such as weather conditions or occupant behavior. Also, constraints on, e.g., control signals can be included in the MPC formulation. The MPC used here did not have predictions of the reference signals or other disturbances available.

On the other hand, PI controllers are already used in thermostats and thermostatic valves on radiators to control the the temperature of a single room [Peffer et al., 2011]. Furthermore, the tuning for a PI controller is more intuitive than for an MPC controller and simpler to implement.

The fact that here the MPC controller performs worse than the decoupled PI controller concerning reducing the influence of the temperature of one room on the other could be due to tuning of the parameters or that the MPC did not take into account any disturbances. Nevertheless, the perspective that PI control with a decoupling network can have a performance comparable to that of an MPC controller regarding this coupling gives the chance to build on already used technology to improve temperature control.

Increasing the amount of rooms adjacent to each other, the structure of the decoupled PI controller will also get more complex. It remains to be investigated how this will affect the controller performance. Moreover, the a question is how the performance of a decoupled PI controller compares to that of an MPC in case of a more complex representation of a building, with disturbances from weather conditions or solar radiation, and when the control signals reach the saturation limits.

6. CONCLUSION

The connection of two adjacent rooms through a common wall introduces and interaction between the temperature dynamics of these rooms. A PI control strategy with and without a decoupling network and an MPC controller were used to control the temperature in the two rooms. It was observed how the change of temperature in one of the rooms affects the change of temperature in the other room. The three control strategies were compared with respect to this interaction. It was found that both MPC control and PI control with a decoupling network reduce the effect of a temperature change in the first room on the temperature of the second room. In this simulation PI control with a decoupling network lead to a smaller effect on the temperature of the second room than MPC control.

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Appendix A. PREDICTION MATRICES

$$\mathcal{S}_A = \left[C_I A_I \ C_I A_I^2 \ \ldots \ C_I A_I^{H_u} \ \ldots \ C_I A_I^{H_p} \right]^T$$

$$\mathcal{S}_u = \begin{bmatrix} C_I B_I \\ C_I \left[A_I + I \right] B_I \\ \vdots \\ C_I \left[\sum_{m=0}^{H_u - 1} A_I^m \right] B_I \\ C_I \left[\sum_{m=0}^{H_u} A_I^m \right] B_I \\ \vdots \\ C_I \left[\sum_{m=0}^{H_p - 1} A_I^m \right] B_I \end{bmatrix}$$

$$\mathcal{S}_{\Delta u} = \begin{bmatrix} C_I B_I & 0 & \cdots \ \cdots & \cdots & 0 \\ C_I \left[A_I + I \right] B_I & C_I B_{uc} & 0 \ \cdots & \cdots & 0 \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & 0 \\ C_I \left[\sum_{m=0}^{H_u - 1} A_I^m \right] B_I & \cdots & \cdots \ \cdots & C_I \left[\bar{A} + I \right] B_I & C_I B_I \\ C_I \left[\sum_{m=0}^{H_u} A_I^m \right] B_I & C_I \left[\sum_{m=0}^{H_u - 1} A_I^m \right] B_I & \cdots \ \cdots & C_I \left[\sum_{m=0}^{2} A_I^m \right] B_I & C_I \left[A_I + I \right] B_I \\ \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & & \vdots & \vdots \\ C_I \left[\sum_{m=0}^{H_p - 1} A_I^m \right] B_I & C_I \left[\sum_{m=0}^{H_p - 2} A_I^m \right] B_I & \cdots \ \cdots & \cdots & C_I \left[\sum_{m=0}^{H_p - H_u} A_I^m \right] B_I \end{bmatrix}$$

Appendix B. SOME LATIN VOCABULARY