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Flatness-based Control of Torsional-Axial Coupled Drilling Vibrations

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Abstract: The main purpose of this contribution is the control of both torsional and axial vibrations occurring along a rotary oil well drilling system. The considered model consists of a system of wave equations with non-linear coupled boundary conditions. We propose a flatness-based control approach to tackle the trajectory tracking problem guaranteeing the suppression of harmful dynamics. The closed loop control design ensures the stability of the error dynamics. Moreover, numerical simulations illustrate the efficiency of the established control laws.

Keywords: Flatness property, infinite-dimensional systems, drilling vibrations, boundary control.

1. INTRODUCTION

The modelling, analysis and control of rotary drilling vibrations are topics whose economical interest has been renewed by recent oilfields discoveries leading to a growing literature, see for instance [6], [7], [12], [13], [14], [15] and [16].

Roughly speaking, a rotary drilling structure consists essentially of a rig, a drill string, and a bit. The essential components of the drill string are the bottom hole assembly, composed mainly of heavy steel tubes to provide a large downward force on the bit, and a set of drill pipes made of thinner tubes. The drill string is in particular subject to two main types of vibrations, each of them can at the least cause a premature wear of the various components. Torsional vibrations are responsible of the so-called stick-slip phenomenon which is essentially the cause of premature breakage of the drill pipes. Traction/compression vibrations, or shortly axial vibrations, mainly associated with the bit-bouncing phenomenon may cause premature wear of the bit.

These two types of vibrations are known to be coupled and present several non-linear phenomena. This combination makes the derivation of a model mathematically challenging and is the reason why the full system has been rarely considered so far. For the sake of reducing the complexity of the problem some works suggest that the axial vibrations are neglected with respect to torsional vibrations in order to avoid considering coupling dynamics (cf. [1]). Another approach is to neglect the infinite-dimensional aspects of those vibrations by using a lumped parameter model consisting of ordinary differential equations (cf. [12] and [13]). Finally, the non-linearity of the boundary conditions which comes essentially from the friction profile at the bit is approximated by a piecewise linear function (cf. [7]). Unfortunately, the adoption of such simplifications impoverishes the recovered dynamics.

Throughout this contribution we consider a model established in [2], which takes into account all the aforementioned aspects. More precisely, the propagation of each type of vibration is assumed to be governed by a wave equation with non-linear boundary conditions. Moreover, it is emphasized that the coupling terms in the boundary conditions are induced by the interface bit/rock friction.

The trajectory tracking problem considered in this contribution arises from the fact that the elimination of drilling vibrations requires the angular and axial velocities of the drilling system to follow a constant reference path. To tackle this problem, the flatness property of the system will be exploited. The design of a pair of effective flatness-based feedback controllers allowing the exponential convergence of the system trajectories guarantees the suppression of undesired dynamics. The originality of the present paper does not lie only in the proposed control strategy itself, but also in applying it to a relatively complete model.

This paper is organized as follows: In Section II, the drilling system modeling is presented; in Section III, the flatness property of the system is proved; Section IV presents the design of flatness-based controllers guaranteeing the drilling vibrations suppression; numerical simulations to highlight the effectiveness of the proposed approach are presented in Section V; concluding remarks are provided in the last Section.

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2. COUPLED AXIAL-TORSIONAL MODEL OF THE DRILLING SYSTEM

A sketch of a simplified drillstring system is shown in Figure 1.

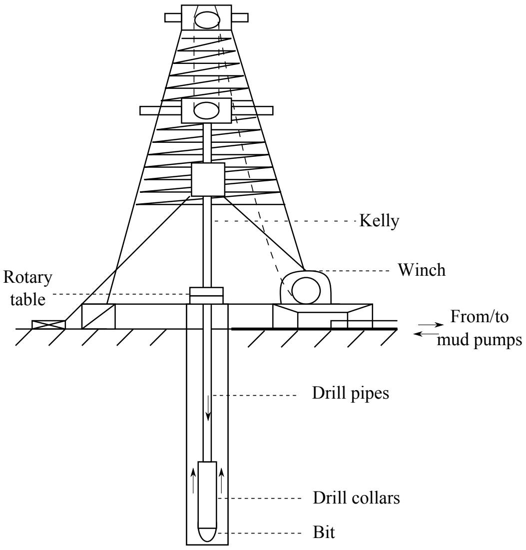

Figure 1. Basic scheme of a vertical drilling system.

It is well known that the wave equation is widely used to reproduce the oscillatory behavior of physical systems. Torsional and axial excitations of a drillstring described by the rotary angle $\phi(\sigma,t)$ and the longitudinal position $U(\sigma,t)$ can be described by the following normalized model consisting of a pair of coupled wave equations (cf. [2], [3], [9]):

$$\frac{\partial^2\phi}{\partial\sigma^2}(\sigma,t) = c_1^2\frac{\partial^2\phi}{\partial t^2}(\sigma,t), \quad c_1 = \frac{l}{\tilde{c}}, \quad \sigma\in(0,1), \quad (1)$$

$$\bar{G}\frac{\partial\phi}{\partial\sigma}(1,t) = \alpha\frac{\partial\phi}{\partial t}(1,t) - \Omega(t), \quad \bar{G} = \frac{G\Sigma}{l}, \quad (2)$$

$$J\frac{\partial^2\phi}{\partial t^2}(0,t) = -\bar{G}\frac{\partial\phi}{\partial\sigma}(0,t) - \tilde{p}F\left(\frac{\partial\phi}{\partial t}(0,t)\right), \quad (3)$$

and

$$\frac{\partial^2 U}{\partial\sigma^2}(\sigma,t) = c_2^2\frac{\partial^2 U}{\partial t^2}(\sigma,t), \quad c_2 = \frac{l}{c}, \quad \sigma\in(0,1), \quad (4)$$

$$\bar{E}\frac{\partial U}{\partial\sigma}(1,t) = \beta\frac{\partial U}{\partial t}(1,t) - H(t), \quad \bar{E} = \frac{E\Gamma}{l}, \quad (5)$$

$$M\frac{\partial^2 U}{\partial t^2}(0,t) = -\bar{E}\frac{\partial U}{\partial\sigma}(0,t) - pF\left(\frac{\partial\phi}{\partial t}(0,t)\right). \quad (6)$$

The spatial variable σ is chosen such that $\sigma = 1$ denotes the top of the drill string and $\sigma = 0$ the bottom extremity. The speeds of propagation $\tilde{c}$ and c can be computed from material parameters, namely shear modulus G, Young's modulus E and the density ρ, by means of $\tilde{c} = \sqrt{G/\rho}$ and $c = \sqrt{E/\rho}$. Apart from these, the model comprises geometrical parameters of the drill string, that are assumed to be spatially and timely constant: the length of the rod l, the drill string's cross-section Γ and its second moment of area Σ, the mass M and the moment of inertia J of the drill bit. The parameters $\tilde{p}$ and p together with the function F depending on the torsional velocity at the bottom end $\frac{\partial\phi}{\partial t}(0,t)$ account for the friction resulting from the interaction between the drill bit and the rock, it is the cause of growth of instabilities eventually leading to drilling oscillations. Some proposals for friction modeling can be found in [4], [8] and [17].

In [11] the friction F is modeled as follows:

$$F\left(\frac{\partial\phi}{\partial t}(0,t)\right) = W_{ob}R_b\mu_b\left(\frac{\partial\phi}{\partial t}(0,t)\right)\operatorname{sign}\left(\tfrac{\partial\phi}{\partial t}(0,t)\right),$$

$$\mu_b\left(\frac{\partial\phi}{\partial t}(0,t)\right) = \mu_{cb} + (\mu_{sb}-\mu_{cb})\mathrm{e}^{-\gamma_b\left|\frac{\partial\phi}{\partial t}(0,t)\right|},$$

where $R_b > 0$ is the bit radius, $W_{ob} > 0$ is the weight on the bit, $\mu_{sb}, \mu_{cb} \in (0,1)$ are the static and Coulomb friction coefficients and $0 < \gamma_b < 1$ is a constant defining the velocity decrease rate.

The following function, introduced in [9], allows to approximate the friction dynamics avoiding the complexity of most of the proposed model structures:

$$F\left(\frac{\partial\phi}{\partial t}(0,t)\right) = \frac{2k\frac{\partial\phi}{\partial t}(0,t)}{\left[\frac{\partial\phi}{\partial t}(0,t)\right]^2 + k^2}, \quad k > 0. \quad (7)$$

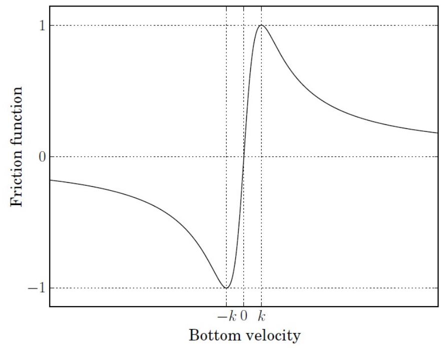

Figure 2. Graph of friction function F given in (7).

The system can be controlled by the boundary torque Ω coming from the motor drive at the surface and the boundary force H provided by the lifting hook at the drilling platform.

3. FLATNESS ANALYSIS OF THE DRILLING SYSTEM

In this section it will be shown that the previously introduced model can be considered flat. Even though flatness is a system property that has been originally defined for finite-dimensional models [5], its basic idea can be extended to the infinite-dimensional case. This idea consists in the possibility to parametrize all system variables by means of a so-called flat output, i.e., once a trajectory for the flat output has been prescribed, the trajectories of all system variables can be computed from it. The main difference between the finite and the infinite-dimensional case lies in the character of the relation between the flat output and the system variables: while in the first case this relation involves only finite-order derivatives of the flat output, the second case might comprise derivatives of arbitrary order or delays and predictions depending on the type of the underlying partial differential equation.

The flatness property of the system under consideration will be proved through the general solution to the wave equation: the D'Alembert formula.

3.1 Proof of the system flatness

Next, it will be shown that the flat output of the drilling system consist of the rotational and longitudinal variables $\phi(0,\cdot)$, $U(0,\cdot)$.

The parametrization of the axial variable $\phi(\sigma,t)$ is provided by the well known D'Alembert formula:

$$\begin{aligned}2\phi(\sigma,t) &= \phi(0,t+c_1\sigma)+\phi(0,t-c_1\sigma)\\ &\quad+\frac{1}{c_1}\int_{t-c_1\sigma}^{t+c_1\sigma}\frac{\partial\phi}{\partial\sigma}(0,\tau)d\tau.\end{aligned} \tag{8}$$

Notice that the solution depends on predicted and delayed values of the involved boundary trajectories.

Substituting (8) into the boundary condition (2) yields the parametrization of the torque $\Omega(t)$:

$$\begin{aligned}2\Omega(t) &= \omega_1\left[\frac{\partial\phi}{\partial t}(0,t+c_1)+\frac{1}{c_1}\frac{\partial\phi}{\partial\sigma}(0,t+c_1)\right]\\ &\quad+\omega_2\left[\frac{\partial\phi}{\partial t}(0,t-c_1)-\frac{1}{c_1}\frac{\partial\phi}{\partial\sigma}(0,t-c_1)\right]\end{aligned} \tag{9}$$

with

$$\omega_1=\alpha-c_1\bar{G},\quad \omega_2=\alpha+c_1\bar{G}.$$

The relation between $\frac{\partial\phi}{\partial\sigma}$ and the flat output $\phi(0,\cdot)$ can be established from the boundary condition (3):

$$\frac{\partial\phi}{\partial\sigma}(0,t)=-\frac{\tilde{p}}{\bar{G}}F\left(\frac{\partial\phi}{\partial t}(0,t)\right)-\frac{J}{\bar{G}}\frac{\partial^2\phi}{\partial t^2}(0,t). \tag{10}$$

Similarly, the axial variable $U(\sigma,t)$ is expressed as:

$$\begin{aligned}2U(\sigma,t) &= U(0,t+c_2\sigma)+U(0,t-c_2\sigma)\\ &\quad+\frac{1}{c_2}\int_{t-c_2\sigma}^{t+c_2\sigma}\frac{\partial U}{\partial\sigma}(0,\tau)d\tau.\end{aligned} \tag{11}$$

The parametrization of the control input $H(t)$ is obtained by substituting (11) into the boundary condition (5):

$$\begin{aligned}2H(t) &= h_1\left[\frac{\partial U}{\partial t}(0,t+c_2)+\frac{1}{c_2}\frac{\partial U}{\partial\sigma}(0,t+c_2)\right]\\ &\quad+h_2\left[\frac{\partial U}{\partial t}(0,t-c_2)-\frac{1}{c_2}\frac{\partial U}{\partial\sigma}(0,t-c_2)\right]\end{aligned} \tag{12}$$

where

$$h_1=\beta-c_2\bar{E},\quad h_2=\beta+c_2\bar{E}.$$

The relation between $\frac{\partial U}{\partial\sigma}$ and the flat output $U(0,\cdot)$ is established from the boundary condition (6) as follows:

$$\frac{\partial U}{\partial\sigma}(0,t)=-\frac{p}{\bar{E}}F\left(\frac{\partial\phi}{\partial t}(0,t)\right)-\frac{M}{\bar{E}}\frac{\partial^2 U}{\partial t^2}(0,t). \tag{13}$$

We conclude that the entire drilling system is flat, i.e., its solutions can be parametrized by the flat output $\mathbf{y}=(\phi(0,\cdot),U(0,\cdot))$.

4. CONTROL DESIGN: TRACKING PROBLEM

4.1 Open loop control

For the sake of notation simplicity, let us denote as $y_\phi(t)$ the rotational angle at the top end $\phi(0,t)$, and as $y_U(t)$ the axial displacement at the upper extremity of the drillstring $U(0,t)$.

The flatness-based parametrization obtained in the previous section can be directly used for the design of a feedforward controller (cf. [10]). To this end, it suffices to prescribe appropriate reference trajectories $\dot{y}_{\phi r}$, $\dot{y}_{Ur}$ for the bottom angular and axial velocities $\dot{y}_\phi$, $\dot{y}_U$ and compute the required control inputs from it. In view of (9)-(10) and (12)-(13), we get:

$$\begin{aligned}2\Omega(t) &= \omega_1\dot{y}_{\phi r}(t+c_1)+\omega_2\dot{y}_{\phi r}(t-c_1)\\ &\quad-\frac{\omega_1}{c_1\bar{G}}\left[\tilde{p}F\left(\dot{y}_{\phi r}(t+c_1)\right)+J\ddot{y}_{\phi r}(t+c_1)\right]\\ &\quad+\frac{\omega_2}{c_1\bar{G}}\left[\tilde{p}F\left(\dot{y}_{\phi r}(t-c_1)\right)+J\ddot{y}_{\phi r}(t-c_1)\right],\end{aligned} \tag{14}$$

$$\begin{aligned}2H(t) &= h_1\dot{y}_{Ur}(t+c_2)+h_2\dot{y}_{Ur}(t-c_2)\\ &\quad-\frac{h_1}{c_2\bar{E}}\left[pF\left(\dot{y}_{\phi r}(t+c_2)\right)+M\ddot{y}_{Ur}(t+c_2)\right]\\ &\quad+\frac{h_2}{c_2\bar{E}}\left[pF\left(\dot{y}_{\phi r}(t-c_2)\right)+M\ddot{y}_{Ur}(t-c_2)\right].\end{aligned} \tag{15}$$

4.2 Closed loop scheme

The open loop control laws (14) and (15) are designed under the supposition that the model under consideration is perfect, which, due to the uncertainties and time varying parameters not considered, is not the case. In order to overcome this problem, closed loop controllers ensuring the system stabilization around the reference trajectories must be designed. The main idea in designing the feedback controllers is to compute the control inputs such that the errors between the desired and the actual trajectories $e_\phi:=\dot{y}_\phi-\dot{y}_{\phi r}$ and $e_U:=\dot{y}_U-\dot{y}_{Ur}$ satisfy the stable dynamics: $\dot{e}_\phi=-\lambda e_\phi$, $\dot{e}_U=-\bar{\lambda}e_U$.

The result on the stabilization of the drilling system regarding its torsional and axial dynamics is stated as follows:

Theorem 1. The controllers

$$\begin{aligned}\Omega(t) &= \left[\alpha+c_1\bar{G}\right]\frac{\partial\phi}{\partial t}(1,t)-c_1\bar{G}\left[\dot{y}_\phi(t-c_1)+\gamma(t)\right]\\ &\quad+Jv(t)+\tilde{p}F\left(\dot{y}_\phi(t-c_1)+\gamma(t)\right),\\ v(t) &= \frac{\bar{G}\chi}{\lambda}\ddot{y}_{\phi r}(t+c_1)-\bar{G}\chi I+\frac{J\chi}{c_1}\ddot{y}_\phi(t-c_1)\\ &\quad-\frac{\tilde{p}\chi}{c_1}\left[F\left(\dot{y}_\phi(t-c_1)+\gamma(t)\right)-F\left(\dot{y}_\phi(t-c_1)\right)\right],\\ I &= 2\frac{\partial\phi}{\partial t}(1,t)-\dot{y}_\phi(t-c_1)-\dot{y}_{\phi r}(t+c_1),\\ \gamma(t) &= \int_{t-2c_1}^{t}v(\tau)d\tau,\qquad \chi=\frac{\lambda c_1}{c_1\bar{G}+\lambda J},\end{aligned}$$

and

$$H(t) = \left[\beta + c_2\bar{E}\right] \frac{\partial U}{\partial t}(1,t) - c_2\bar{E}\left[\dot{y}_U(t-c_2) + \bar{\gamma}(t)\right] + M\bar{v}(t) + pF\left(\dot{y}_\phi(t-c_1) + \gamma(t)\right),$$

$$\bar{v}(t) = \frac{\bar{E}\bar{\chi}}{\bar{\lambda}}\ddot{y}_{Ur}(t+c_2) - \bar{E}\bar{\chi}\bar{I} + \frac{M\bar{\chi}}{c_2}\ddot{y}_U(t-c_2) - \frac{p\bar{\chi}}{c_2}\left[F\left(\dot{y}_\phi(t-c_1)+\gamma(t)\right) - F\left(\dot{y}_\phi(t-c_1)\right)\right],$$

$$\bar{I} = 2\frac{\partial U}{\partial t}(1,t) - \dot{y}_U(t-c_2) - \dot{y}_{Ur}(t+c_2),$$

$$\bar{\gamma}(t) = \int_{t-2c_2}^{t} \bar{v}(\tau)d\tau, \quad \bar{\chi} = \frac{\bar{\lambda}c_2}{c_2\bar{E} + \bar{\lambda}M}$$

lead to an exponential convergence of the torsional and axial trajectories $\dot{y}_\phi(t) = \frac{\partial\phi}{\partial t}(0,t)$ and $\dot{y}_U(t) = \frac{\partial U}{\partial t}(0,t)$ to the reference velocities $\dot{y}_{\phi r}(t)$ and $\dot{y}_{Ur}(t)$.

Proof. According to (8)-(10), the trajectories of the torsional subsystem can be parametrized as follows:

$$2\phi(\sigma,t) = y_\phi(t+c_1\sigma) + y_\phi(t-c_1\sigma) - \frac{1}{c_1\bar{G}}\int_{t-c_1\sigma}^{t+c_1\sigma}\left[\tilde{p}F\left(\dot{y}_U(\tau)\right) + J\ddot{y}_\phi(\tau)\right]d\tau. \tag{16}$$

Taking the time derivative of (16) and evaluating in $\sigma = 1$ yields

$$2\frac{\partial\phi}{\partial t}(1,t) = \dot{y}_\phi(t+c_1) + \dot{y}_\phi(t-c_1) - \frac{J}{c_1\bar{G}}\left[\ddot{y}_\phi(t+c_1) - \ddot{y}_\phi(t-c_1)\right] - \frac{\tilde{p}}{c_1\bar{G}}\left[F\left(\dot{y}_\phi(t+c_1)\right) - F\left(\dot{y}_\phi(t-c_1)\right)\right], \tag{17}$$

in view of (9) and (17), we can write

$$2\omega_2\frac{\partial\phi}{\partial t}(1,t) - 2\Omega(t) = (\omega_2-\omega_1)\dot{y}_\phi(t+c_1) - \frac{J}{c_1\bar{G}}(\omega_2-\omega_1)\ddot{y}_\phi(t+c_1) - \frac{\tilde{p}}{c_1\bar{G}}(\omega_2-\omega_1)F\left(\dot{y}_\phi(t+c_1)\right). \tag{18}$$

The introduction of a new variable $v(t)$ defined as $v(t) := \ddot{y}_\phi(t+c_1)$ implies

$$\dot{y}_\phi(t+c_1) = \dot{y}_\phi(t-c_1) + \int_{t-2c_1}^{t} v(\tau)d\tau. \tag{19}$$

In view of (18) and (19), the controller $\Omega(t)$ can be written as:

$$\Omega(t) = \left[\alpha + c_1\bar{G}\right]\frac{\partial\phi}{\partial t}(1,t) - c_1\bar{G}\left[\dot{y}_\phi(t-c_1) + \gamma(t)\right] + Jv(t) + \tilde{p}F\left(\dot{y}_\phi(t-c_1) + \gamma(t)\right),$$

where $\gamma(t) = \int_{t-2c_1}^{t} v(\tau)d\tau$. Now, substituting (19) into (17) yields:

$$2\frac{\partial\phi}{\partial t}(1,t) = \dot{y}_\phi(t+c_1) + \dot{y}_\phi(t-c_1) - \frac{J}{c_1\bar{G}}\left[v(t) - \ddot{y}_\phi(t-c_1)\right] - \frac{\tilde{p}}{c_1\bar{G}}\left[F\left(\dot{y}_\phi(t-c_1)+\gamma(t)\right) - F\left(\dot{y}_\phi(t-c_1)\right)\right],$$

then, the prediction term can be written as follows:

$$\dot{y}_\phi(t+c_1) = 2\frac{\partial\phi}{\partial t}(1,t) - \dot{y}_\phi(t-c_1) + \frac{J}{c_1\bar{G}}\left[v(t) - \ddot{y}_\phi(t-c_1)\right] + \frac{\tilde{p}}{c_1\bar{G}}F\left(\dot{y}_\phi(t-c_1)+\gamma(t)\right) - \frac{\tilde{p}}{c_1\bar{G}}F\left(\dot{y}_\phi(t-c_1)\right). \tag{20}$$

Regarding the velocity tracking problem under consideration, the error is defined as $e_\phi := \dot{y}_\phi(t+c_1) - \dot{y}_{\phi r}(t+c_1)$. The controller must guarantee stable closed loop error dynamics ($\dot{e_\phi} = -\lambda e_\phi$), to this end we set

$$v(t) = \ddot{y}_{\phi r}(t+c_1) - \lambda\left[\dot{y}_\phi(t+c_1) - \dot{y}_{\phi r}(t+c_1)\right],$$

which, in view of (20), is written as:

$$v(t) = \frac{c_1\bar{G}}{c_1\bar{G}+\lambda J}\ddot{y}_{\phi r}(t+c_1) - \frac{c_1\bar{G}\lambda}{c_1\bar{G}+\lambda J}I + \frac{\lambda J}{c_1\bar{G}+\lambda J}\ddot{y}_\phi(t-c_1) - \frac{\lambda\tilde{p}}{c_1\bar{G}+\lambda J}F\left(\dot{y}_\phi(t-c_1)+\gamma(t)\right) + \frac{\lambda\tilde{p}}{c_1\bar{G}+\lambda J}F\left(\dot{y}_\phi(t-c_1)\right),$$

with

$$I = 2\frac{\partial\phi}{\partial t}(1,t) - \dot{y}_\phi(t-c_1) - \dot{y}_{\phi r}(t+c_1).$$

Similarly, for the axial subsystem, we define the variable $\bar{v}(t)$ as $\bar{v}(t) = \ddot{y}_U(t+c_2)$, which implies

$$\dot{y}_U(t+c_2) = \dot{y}_U(t-c_2) + \int_{t-2c_2}^{t}\bar{v}(\tau)d\tau, \tag{21}$$

and

$$H(t) = \left[\beta + c_2\bar{E}\right]\frac{\partial U}{\partial t}(1,t) - c_2\bar{E}\left[\dot{y}_U(t-c_2) + \bar{\gamma}(t)\right] + M\bar{v}(t) + pF\left(\dot{y}_\phi(t-c_1)+\gamma(t)\right),$$

where $\bar{\gamma}(t) = \int_{t-2c_2}^{t}\bar{v}(\tau)d\tau$. As we have shown before, the flatness property allows the following parametrization:

$$2U(\sigma,t) = y_U(t+c_2\sigma) + y_U(t-c_2\sigma) - \frac{1}{c_2\bar{E}}\int_{t-c_2\sigma}^{t+c_2\sigma}\left[pF\left(\dot{y}_\phi(\tau)\right) + M\ddot{y}_U(\tau)\right]d\tau, \tag{22}$$

substituting (21) into the time derivative of (22) at $\sigma = 1$ yields:

$$\dot{y}_U(t+c_2) = 2\frac{\partial U}{\partial t}(1,t) - \dot{y}_U(t-c_2) + \frac{M}{c_2\bar{E}}\left[\bar{v}(t) - \ddot{y}_U(t-c_2)\right] + \frac{p}{c_2\bar{E}}F\left(\dot{y}_\phi(t-c_1)+\gamma(t)\right) - \frac{p}{c_2\bar{E}}F\left(\dot{y}_\phi(t-c_1)\right). \tag{23}$$

The error related to axial dynamics e_U is defined as $e_U := \dot{y}_U(t+c_2) - \dot{y}_{Ur}(t+c_2)$. In order to ensure stable closed loop error dynamics, we set

$$\bar{v}(t) = \ddot{y}_{Ur}(t+c_2) - \bar{\lambda}\left[\dot{y}_U(t+c_2) - \dot{y}_{Ur}(t+c_2)\right],$$

which, in view of (23), is written as:

$$\begin{aligned}\bar{v}(t) = & \frac{c_2\bar{E}}{c_2\bar{E}+\bar{\lambda}M}\ddot{y}_{Ur}(t+c_2) \\ & -\frac{c_2\bar{E}\bar{\lambda}}{c_2\bar{E}+\bar{\lambda}M}\bar{I} + \frac{\bar{\lambda}M}{c_2\bar{E}+\bar{\lambda}M}\ddot{y}_U(t-c_2) \\ & -\frac{\bar{\lambda}p}{c_2\bar{E}+\bar{\lambda}M}F\left(\dot{y}_\phi(t-c_1)+\gamma(t)\right) \\ & +\frac{\bar{\lambda}p}{c_2\bar{E}+\bar{\lambda}M}F\left(\dot{y}_\phi(t-c_1)\right),\end{aligned}$$

with

$$\bar{I} = 2\frac{\partial U}{\partial t}(1,t) - \dot{y}_U(t-c_2) - \dot{y}_{Ur}(t+c_2).$$

5. NUMERICAL SIMULATIONS

In this Section, the effectiveness of the proposed control approach is highlighted through simulations results. The numerical values of the physical parameters used in the following are given in Table 1.

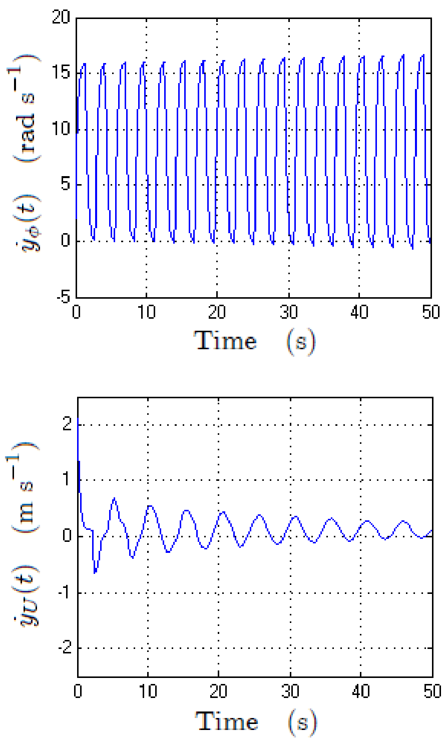

Figure 3. Drilling system trajectories without feedback control actions (stick-slip and bit-bounce phenomena).

System trajectories corresponding to the rotational and longitudinal velocities of the drilling rod at the bottom end ($\dot{y}_\phi$, $\dot{y}_U$) without feedback control actions are shown in Figure 3. The friction at the rock-bit interface is approximated by the function given in (7). Simulation results are in close agreement with field observations regarding axial and torsional vibrations.

Table 1. Physical parameters

G	80 GPa	E	200 GPa
ρ	8000 kg/m^3	l	3500 m
Σ	19 cm^4	Γ	35 cm^2
α	2000 Nms	β	200.025 kg/s
J	30000 kg m^2	M	40000 kg
$\tilde{p}$	210	p	3500
k	0.18		

As explained above, the flatness approach allows the design of feedback controllers to track prescribed reference trajectories ensuring stable error dynamics and consequently the drilling vibration elimination.

Figure 4 shows the closed loop response of the drilling system subject to the proposed flatness-based control approach. Angular and axial velocities at the bottom end of the drillstring follow the references $\dot{y}_{\phi r} = 10$rad s^{-1} and $\dot{y}_{Ur} = 0.1$m s^{-1}. The exponential decay rates considered are $\lambda = \bar{\lambda} = 2.5$.

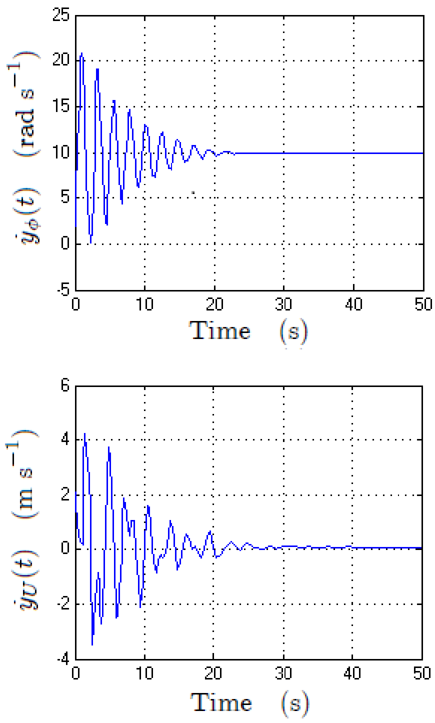

Figure 4. Trajectories of the drilling system under the controllers stated in Theorem 1.

Figures 5-6 show the system trajectories in the torsional space phase with the relative variable $\dot{y}_\phi(t)$. In presence of torsional oscillations, the bit motion converges to a limit cycle called stick-slip (Figure 5). The stick-slip phenomenon is characterized by stick phases, during which the rotation stops completely, and slip phases, during which the angular velocity of the tool increases up to two times the nominal angular velocity. This phenomenon occurs when a section of the rotating drillstring is momentarily caught by friction against the borehole, then releases. The bit might eventually get stuck and then, after accumulating energy in terms of torsion, be suddenly released, the string rotation speeds up dramatically and large centrifugal accelerations occur. This behavior is the major source of failures in oil fields.

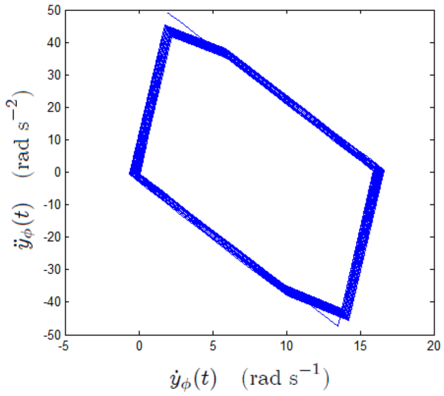

Figure 5. Stick-slip limit cycle.

With the implementation of the feedback controllers given in Theorem 1, the stick-slip motion is suppressed and the system trajectories converge to the stationary solution (Figure 6).

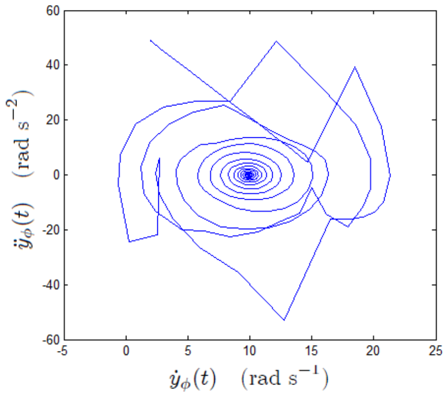

Figure 6. Phase plane torsional trajectories of the drilling system under the controllers stated in Theorem 1.

6. CONCLUDING REMARKS

The flatness property of the system has been shown to be useful in designing feedback controllers to tackle the drilling vibration problem. The trajectory tracking framework leads the axial and angular velocities of the system to follow a constant reference, thereby ensuring the elimination of undesirable oscillations. Numerical simulations confirm the accuracy of the proposed approach.

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