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Nonlinear Observer Design for the State of Charge of Lithium-Ion Batteries

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Abstract: As rechargeable lithium-ion batteries are widely used in many applications nowadays, how to accurately evaluate the battery's state of charge(SOC) becomes a more and more important issue. A new method for estimating the SOC of lithium-ion batteries based on an inclusive equivalent circuit model is proposed in this paper. To the best of our knowledge, the parameters in the model are usually considered as constants to simplify the problem of the SOC estimation, which may lead to some estimation error. In order to get more accurate estimation results, the capacitances and resistances in the battery model are considered as nonlinear functions of the SOC and the temperature of the battery. The resistances also depend on the battery's charging or discharging mode. Nonlinear relationship between the open circuit voltage(OCV) and the SOC is considered and a nonlinear observer is designed to estimate the inner characteristics of the battery. Lyapunov stability analysis is utilized to prove its performance and simulation results are provided to illustrate the performance of the proposed approach.

Keywords: lithium-ion battery, state of charge, equivalent circuit model, nonlinear observer, Lyapunov stability.

1. INTRODUCTION

Rechargeable Lithium-ion batteries are widely used in many applications in recent years, such as telecommunication and hybrid electric vehicles. The role they play in these technologies are more and more significant. The convenience, reliability, mobility and utility of lithium-ion battery can be enhanced, if the battery inner performance can be predicted accurately (Dubarry and Liaw (2007)).

The SOC of the battery, which is defined as the rate of the available capacity to its maximum capacity when the battery is completely charged(Hu and Yurkovich (2010)), is an important parameter of the battery management system for the battery optimized operation and extension of the battery's lifetime. But the SOC of batteries couldn't be obtained easily, as it can not be directly measured by sensors. How to estimate the SOC of the battery precisely has puzzled many researchers and has been investigated by many institutes. .

In order to evaluate the SOC of battery under different circumstances properly, an accurate battery model is indispensable. Lithium-ion battery models which are commonly used nowadays mainly include: electrochemical mechanism model(such as Boovargavan and Ramadesigan (2010)), equivalent circuit model(such as Pattipati et al. (2011)), neural network model(such as Chau et al. (2004)) and so on.

The internal and external characteristic reactions of the battery can be accurately described by the electrochemical model. It's often utilized for the battery mechanism analysis, the electrode/electrolyte materials selection and other aspects(Su et al. (2011)). However, as the parameters in the model are related to battery structure, dimensions and materials, it requires so many complicated calculations that it is seldom used in actual battery management systems. The equivalent circuit model is simple for analysis, with small restriction in battery materials and size. It needs small amount of computation and can be widely applied to many fields. But such models are just approximate simulation of the battery characteristics and can not describe the internal potential and temperature distribution of the battery. The neural network model, which has the basic characteristics of nonlinear and learning ability, can be utilized to simulate the characteristics of the battery, since the battery is a highly nonlinear system. It can get any relationship between the input and output of the battery, without the need to know the complicated internal mechanisms of the battery. Whereas, the internal characteristics of the battery couldn't be obtained, if they are needed to be validated.

Compared with other two kinds of models, the equivalent circuit model is the fittest model to estimate the SOC of lithium-ion batteries, due to its simplicity and good performance. So many schemes based on this kind of model have been presented by researchers to evaluate the battery's SOC estimation.

One of the most commonly used methods for SOC estimation is the ampere-hour counting(O. et al. (1998)), in which the time integral of the battery current is considered

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as a indication of SOC. Although it is simple and easily implemented, this method accumulates errors in measurements and may lead to large SOC errors in real-world applications. That the initial value of SOC can not be obtained accurately and it needs to be calibrated frequently. The open circuit voltage method(Lee et al. (2008)) is based on the identified relationship between the battery open-circuit voltage and SOC. The SOC of the battery can be obtained by measuring the OCV. But this method takes a lot of time, as the OCV can only be measured after a long rest of the battery. Kalman Filter is a recursive algorithm utilized for the estimation of the internal states of the battery system (Domenico et al. (2008)). A nonlinear extension of the Kalman Filter, known as Extended Kalman Filter(EKF), which takes account the nonlinear relation of OCV and SOC, has been investigated by many researchers(see He et al. (2011)). EKF is a widely used tool to extract the internal states of the battery model. However, the estimation error of the SOC may be very large, when there is no significant difference between the voltage measurement error and the discharge voltage drop. The performance of the EKF method can not be proved and its estimation performance can not be guaranteed.

In this paper, a new scheme for estimating the SOC of lithium-ion batteries based on an inclusive equivalent circuit model (Chen and Rincon-Mora (2006)) is proposed. This model could estimate the remained runtime and V-I performance of lithium-ion batteries accurately and can be easily extended to other batteries, such as nickel-metal hydride batteries. The resistances and capacitances in the equivalent circuit model are often considered as constants to simplify the problem in many references(e.g. Gholizadeh and Salmasi (2013)). The temperature effect on the model is usually neglected. However, this will cause a lot of modeling and estimation error. In this paper, the equivalent capacitances in the model are considered as the functions of the SOC and the temperature of the battery. The resistances are considered as the functions of the SOC, the temperature and the battery's charging or discharging mode. As the battery model is a highly nonlinear system, a nonlinear observer has been designed to estimate the SOC of the battery. Then a Lyapunov based analysis is utilized to prove the observer's stability and convergence. Simulations are done to validate its estimation performance and an experimental facility will be designed to verify the proposed nonlinear SOC estimation scheme later.

This paper is organized in the following manner. In Section 2, an equivalent circuit model of lithium-ion batteries is provided. Section 3 details the process of nonlinear observer design for the SOC estimation and Lyapunov stability analysis is used to prove its stability and convergence. Related simulation results are provided in Section 4 and conclusions are provided in Section 5.

2. BATTERY MODEL DEVELOPMENT

In order to estimate the SOC of the battery accurately, an intuitive and comprehensive equivalent circuit model proposed by Chen and Rincon-Mora (2006) is selected in this paper. The nonlinear mapping from the battery's SOC to the open circuit voltage $V_{OC}(t)$ in this model is presented by a voltage-controlled voltage source. $0V - 1V$ of V_{SOC} corresponds to $0\% - 100\%$ of the SOC. As $V_{OC}(t)$ does not vary greatly with temperature(see Johnson and Pesaran (2000)), it can be considered only associated with $V_{SOC}(t)$ as follows

$$V_{OC} = f(V_{SOC}) \tag{1}$$

where $f(\cdot)$ is a nonlinear function between $V_{OC}(t)$ and $V_{SOC}(t)$.

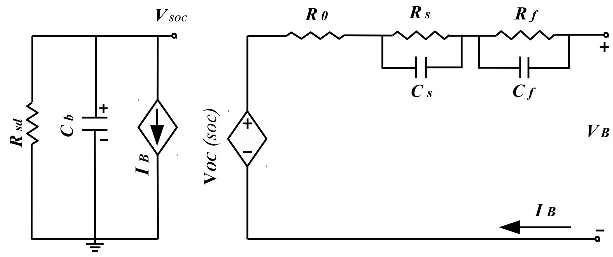

Fig. 1. Equivalent circuit model of the battery

Fig. 1(from Gholizadeh and Salmasi (2013)) illustrates the equivalent circuit model of the battery, with two inter-related subcircuits, which influence each other through a voltage-controlled voltage source and a current-controlled current source. The left circuit in Fig. 1 is used to simulate the SOC and remained runtime of the battery. C_b denotes the full-charge capacitor and R_{sd} denotes the self-discharge resistor. C_b and R_{sd} are used to denote the self-discharge character of the battery.

The circuit on the right in Fig. 1 represents the transient response and V-I curves of the battery(see Kim and Qiao (2011)). The resistor R_0 is used to characterize the charge and discharge energy losses of the battery. The RC networks (R_f, C_f) and (R_s, C_s) are used to characterize the short-term and long-term transient responses of the battery. $I_B(t)$ represents the charge/discharge current of the battery and $V_B(t)$ represents the terminal voltage of the battery.

If the battery is not used too many times, the effect of cycle number on the battery can be neglected. Ignoring the cycle number of the battery and other subordinate influence factors, all the parameters of the components in the model can be considered as the functions of the SOC and the temperature of the battery. In addition, the resistances also depend on the current direction(that is to say, the resistances are different when the battery is in charging or discharging mode). Practically, in order to simplify the battery model, C_b can be considered as its nominal capacity of the battery and R_{sd} can be simplified as a large constant resistor, if the temperature of the battery varies within a small range(see Chen and Rincon-Mora (2006)).

As C_f and C_s do not really exist in the battery, the effect of the change rate of capacitance on V-I performance can be neglected (refer to Sitterly et al. (2011)). The dynamics of the voltages across the capacitors $V_f(t)$, $V_s(t)$ and $V_{SOC}(t)$ can be expressed as follows

$$\begin{aligned}
\dot{V}_f &= -\frac{1}{R_f(V_{SOC}, T, \eta) C_f(V_{SOC}, T)} V_f + \frac{1}{C_f(V_{SOC}, T)} I_B \\
\dot{V}_s &= -\frac{1}{R_s(V_{SOC}, T, \eta) C_s(V_{SOC}, T)} V_s + \frac{1}{C_s(V_{SOC}, T)} I_B \\
\dot{V}_{SOC} &= -\frac{1}{R_{sd} C_b} V_{SOC} - \frac{1}{C_b} I_B
\end{aligned} \tag{2}$$

where T(t) is the battery temperature, η denotes the current direction, namely it denotes that the battery is in a charging or discharging mode with its value as follows

$$\eta = \begin{cases} 1, \; charging\ mode \\ 0, \; discharging\ mode. \end{cases}$$

The terminal voltage $V_B(t)$ can be expressed as

$$V_B = V_{OC} - R_0(V_{SOC}, T, \eta)I_B - V_f - V_s \quad (3)$$

As $\dot{I}_B(t)$ can be ignored compared with other time-constants in a sampling period(see Gholizadeh and Salmasi (2013)), based on (1), (2) and (3), the derivative of the terminal voltage is determined to be

$$\begin{aligned}
&\dot{V}_B = \frac{\partial V_{OC}}{\partial V_{SOC}}\dot{V}_{SOC} - \dot{V}_f - \dot{V}_s \\
&= \frac{V_f}{R_f(V_{SOC}, T, \eta)C_f(V_{SOC}, T)} - \frac{V_f}{R_s(V_{SOC}, T, \eta)C_s(V_{SOC}, T)} \\
&- \frac{V_B}{R_s(V_{SOC}, T, \eta)C_s(V_{SOC}, T)} - (\frac{R_0(V_{SOC}, T, \eta)}{C_s(V_{SOC}, T)R_s(V_{SOC}, T, \eta)} \\
&+ \frac{1}{C_f(V_{SOC}, T)} + \frac{1}{C_s(V_{SOC}, T)} + \frac{\dot{f}(V_{SOC})}{C_b})I_B \\
&+ \frac{f(V_{SOC})}{R_s(V_{SOC}, T, \eta)C_s(V_{SOC}, T)} - \frac{\dot{f}(V_{SOC})}{R_{sd}C_b}V_{SOC}.
\end{aligned} \quad (4)$$

The effect of the change of temperature on battery performance is usually neglected in many references to simplify the battery model. However, the temperature of the battery always increases because of the battery reactions and thermal generation factors. Therefore, the temperature control is an indispensable issue in lithium-ion battery management systems in reality.

The thermal generation $Q(t)$ can be decomposed to three elements: reaction heat value $Q_r(t)$, polarization heat value $Q_p(t)$ and Joule heat value $Q_J(t)$. The value of $Q_r(t)$ can be negligible in comparison with $Q_p(t)$, $Q_J(t)$ (see Sato (2001)). The complicated thermal distributions inside the battery is not considered in this paper. Using thermal energy balance (Gao et al. (2002)), the derivative of $T(t)$ can be obtained as follows

$$\dot{T} = \frac{\dot{Q}}{mc_p} - \frac{h_c S(T - T_a)}{mc_p} \quad (5)$$

where m represents the battery mass, c_p is the specific heat, S is the battery external surface area, h_c denotes heat transfer coefficient, T_a denotes the ambient temperature and it is considered as a constant.

The dynamics of $Q(t)$, $Q_J(t)$, $Q_p(t)$ can be expressed as follows

$$\begin{aligned}
&\dot{Q} = \dot{Q}_J + \dot{Q}_p \\
&\dot{Q}_J = I_B^2(R_0(\cdot) + R_s(\cdot) + R_f(\cdot)) + \frac{V_{SOC}^2}{R_{sd}} \\
&\dot{Q}_p = I_B^2 R_p
\end{aligned} \quad (6)$$

where R_p denotes the polarization resistance.

Remark 1. As high temperature is harmful to batteries, a heat sink is usually used to ensure the battery temperature not too high. The cooling performance will be reflected through influencing the value of h_c in this model.

Based on (2), (4) and (5), the battery model can be rewritten as follows

$$\begin{aligned}
\dot{x} &= A(x, \eta)x + g(x, u, \eta) \\
y &= Cx
\end{aligned} \quad (7)$$

where $x(t) \in \mathbb{R}^5$, $y(t) \in \mathbb{R}$, $u(t) \in \mathbb{R}$, $A(x, \eta) \in \mathbb{R}^{5\times5}$, $g(x, u, \eta) \in \mathbb{R}^5$ and $C \in \mathbb{R}^{1\times5}$ are defined as follow

$$\begin{aligned}
x(t) &\triangleq [x_1(t),\ x_2(t),\ x_3(t),\ x_4(t),\ x_5(t)]^T \\
&= [V_{SOC},\ V_f,\ V_s,\ V_B,\ T]^T
\end{aligned}$$

$$y(t) \triangleq V_B, \quad u(t) \triangleq I_B$$

$$A(x, , \eta) =$$

$$\begin{bmatrix}
-a & 0 & 0 & 0 & 0 \\
0 & -h_1(x_1, x_5, \eta) & 0 & 0 & 0 \\
0 & 0 & -h_2(x_1, x_5, \eta) & 0 & 0 \\
0 & h_3(x_1, x_5, \eta) & 0 & -h_2(x_1, x_5, \eta) & 0 \\
0 & 0 & 0 & 0 & -c
\end{bmatrix}$$

$$g(x, u, \eta) = \begin{bmatrix}
-bu \\
g_1(x_1, x_5)u \\
g_2(x_1, x_5)u \\
g_3(x_1, x_5, \eta)u + g_4(x_1, x_5, \eta) \\
h_4(x_1, x_5, \eta, u)
\end{bmatrix}$$

$$C = [0\ \ 0\ \ 0\ \ 1\ \ 0]$$

$$\begin{aligned}
&a = \tfrac{1}{R_{sd}C_b}, b = \tfrac{1}{C_b}, c = \tfrac{h_c S}{mc_p} \\
&h_1(x_1, x_5, \eta) = \tfrac{1}{R_f(x_1, x_5, \eta)C_f(x_1, x_5)} \\
&h_2(x_1, x_5, \eta) = \tfrac{1}{R_s(x_1, x_5, \eta)C_s(x_1, x_5)} \\
&h_3(x_1, x_5, \eta) = \tfrac{1}{R_f(x_1, x_5, \eta)C_f(x_1, x_5)} - \tfrac{1}{R_s(x_1, x_5, \eta))C_s(x_1, x_5)} \\
&h_4(x_1, x_5, , \eta, u) = \tfrac{\dot{Q}(x_1, x_5, \eta, u)}{mc_p} + \tfrac{h_c S T_a}{mc_p} \\
&g_1(x_1, x_5) = \tfrac{1}{C_f(x_1, x_5)} \\
&g_2(x_1, x_5) = \tfrac{1}{C_s(x_1, x_5)} \\
&g_3(x_1, x_5, \eta) = -\tfrac{R_0(x_1, x_5, \eta)}{C_s(x_1, x_5)R_s(x_1, x_5, \eta)} - \tfrac{1}{C_f(x_1, x_5)} \\
&\quad - \tfrac{1}{C_s(x_1, x_5)} - \tfrac{\dot{f}(x_1)}{C_b} \\
&g_4(x_1, x_5, \eta) = \tfrac{f(x_1)}{R_s(x_1, x_5, \eta)C_s(x_1, x_5)} - \tfrac{\dot{f}(x_1)}{R_{sd}C_b}x_1 \ .
\end{aligned}$$

The formulation in (7) illustrates a single-input single-output nonlinear system, which is complicated and hard to be analyzed because of its high nonlinearity.

Remark 2. For the problem of SOC estimation, the resistances, capacitances, charge/discharge current and temperature can be assumed positive and bounded. Then it can be obtained that a, m, c, $\dot{Q}(t)$, $h_1(\cdot)$ and $h_2(\cdot)$ are all bounded and positive. And $u(t)$, $h_3(\cdot)$, $h_4(\cdot)$, $g_i(\cdot)$ $(1 \le i \le 4)$ are all bounded.

3. NONLINEAR OBSERVER DESIGN FOR SOC ESTIMATION

3.1 Nonlinear Observer Design

As illustrated in (7), the battery model is a nonlinear system. In this model, the state $x_1(t)$, which is the SOC of the battery, can't be measured directly. In order to predict it, a nonlinear observer is designed.

Since $A(\cdot)$ and $g(\cdot)$ in (7) can not be perfectly known, $A^0(\cdot)$ and $g^0(\cdot)$, which are the nominal models (see Khalil (2002)) of $A(\cdot)$ and $g(\cdot)$, are used in the observer. The observer can be denoted as follows

$$\dot{\hat{x}} = A^0(\hat{x}, \eta)\hat{x} + g^0(\hat{x}, u, \eta) + L(y - C\hat{x}) \tag{8}$$

where $\hat{x}(t) \in \mathbb{R}^5$ is the estimation of $x(t)$, $L \triangleq [l_1,\ l_2,\ l_3,\ l_4,\ l_5]^T \in \mathbb{R}^5$ is the observer gain to be designed.

Based on (7), (8) can be rewritten as follows

$$\begin{aligned}
\dot{\hat{x}}_1 &= -a\hat{x}_1 - bu + l_1(x_4 - \hat{x}_4)\\
\dot{\hat{x}}_2 &= -h_1^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_2 + g_1^0(\hat{x}_1, \hat{x}_5)u + l_2(x_4 - \hat{x}_4)\\
\dot{\hat{x}}_3 &= -h_2^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_3 + g_2^0(\hat{x}_1, \hat{x}_5)u + l_3(x_4 - \hat{x}_4)\\
\dot{\hat{x}}_4 &= h_3^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_2 - h_2^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_4 + g_3^0(\hat{x}_1, \hat{x}_5, \eta)u\\
&\quad + g_4^0(\hat{x}_1, \hat{x}_5, \eta) + l_4(x_4 - \hat{x}_4)\\
\dot{\hat{x}}_5 &= -c\hat{x}_5 + h_4^0(\hat{x}_1, \hat{x}_5, \eta, u) + l_5(x_4 - \hat{x}_4)
\end{aligned} \tag{9}$$

where $h_i^0(\cdot)$ and $g_i^0(\cdot)$ are the nominal models of $h_i(\cdot)$ and $g_i(\cdot)$ $(1 \le i \le 4)$ respectively, and are all selected locally Lipschitz and bounded in its arguments over the domain of interest. Define $-h_1^0(\cdot) \le -d_1$, $-h_2^0(\cdot) \le -d_2$, $h_3^0(\cdot) \le d_3$ (for all $d_i > 0 (1 \le i \le 3)$).

Based on (7) and (9), the estimation error $\tilde{x}(t) \triangleq x(t) - \hat{x}(t)$ can be obtained as follows

$$\begin{aligned}
\dot{\tilde{x}}_1 &= -a\tilde{x}_1 - l_1\tilde{x}_4\\
\dot{\tilde{x}}_2 &= -h_1(x_1, x_5, \eta)x_2 + h_1^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_2 + (g_1(x_1, x_5)\\
&\quad - g_1^0(\hat{x}_1, \hat{x}_5))u - l_2\tilde{x}_4\\
\dot{\tilde{x}}_3 &= -h_2(x_1, x_5, \eta)x_3 + h_2^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_3 + (g_2(x_1, x_5)\\
&\quad - g_2^0(\hat{x}_1, \hat{x}_5))u - l_3\tilde{x}_4\\
\dot{\tilde{x}}_4 &= h_3(x_1, x_5, \eta)x_2 - h_3^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_2 - h_2(x_1, x_5, \eta)x_4\\
&\quad + h_2^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_4 + (g_3(x_1, x_5, \eta) - g_3^0(\hat{x}_1, \hat{x}_5, \eta))u\\
&\quad + (g_4(x_1, x_5, \eta) - g_4^0(\hat{x}_1, \hat{x}_5, \eta)) - l_4\tilde{x}_4\\
\dot{\tilde{x}}_5 &= -c\tilde{x}_5 + (h_4(x_1, x_5, \eta, u) - h_4^0(\hat{x}_1, \hat{x}_5, \eta, u)) - l_5\tilde{x}_4.
\end{aligned} \tag{10}$$

Based on simple mathematical deductions, the following expression can be easily obtained

$$\begin{aligned}
&-h_1(x_1, x_5, \eta)x_2 + h_1^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_2 = -h_1(x_1, x_5, \eta)x_2\\
&+h_1^0(\hat{x}_1, \hat{x}_5, \eta)x_2 - h_1^0(\hat{x}_1, \hat{x}_5, \eta)x_2 + h_1^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_2\\
&= -h_1^0(\hat{x}_1, \hat{x}_5, \eta)\tilde{x}_2 + (h_1^0(\hat{x}_1, \hat{x}_5, \eta) - h_1(x_1, x_5, \eta))x_2.
\end{aligned} \tag{11}$$

Similarly, the following expressions can also be obtained

$$\begin{aligned}
&-h_2(x_1, x_5, \eta)x_3 + h_2^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_3\\
&= -h_2^0(\hat{x}_1, \hat{x}_5, \eta)\tilde{x}_3 + (h_2^0(\hat{x}_1, \hat{x}_5, \eta) - h_2(x_1, x_5, \eta))x_3
\end{aligned} \tag{12}$$

$$\begin{aligned}
&h_3(x_1, x_5, \eta)x_2 - h_3^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_2\\
&= h_3^0(\hat{x}_1, \hat{x}_5, \eta)\tilde{x}_2 + (-h_3^0(\hat{x}_1, \hat{x}_5, \eta) + h_3(x_1, x_5, \eta))x_2
\end{aligned} \tag{13}$$

$$\begin{aligned}
&-h_2(x_1, x_5, \eta)x_4 + h_2^0(\hat{x}_1, \hat{x}_5, \eta)\hat{x}_4\\
&= -h_2^0(\hat{x}_1, \hat{x}_5, \eta)\tilde{x}_4 + (h_2^0(\hat{x}_1, \hat{x}_5, \eta) - h_2(x_1, x_5, \eta))x_4.
\end{aligned} \tag{14}$$

As $\|h_i(\cdot)\|$, $\|h_i^0(\cdot)\|$, $\|g_i(\cdot)\|$, $\|g_i^0(\cdot)\|$ are all bounded, it can be deduced that $\|h_i(\cdot) - h_i^0(\cdot)\|$, $\|g_i(\cdot) - g_i^0(\cdot)\|$ $(1 \le i \le 4)$ are bounded. So it can be denoted that

$$\begin{aligned}
&\|(h_1^0(\cdot) - h_1(\cdot))x_2\| + \|(g_1(\cdot) - g_1^0(\cdot))u\| \le M_1\\
&\|(h_2^0(\cdot) - h_2(\cdot))x_3\| + \|(g_2(\cdot) - g_2^0(\cdot))u\| \le M_2\\
&\|(h_3(\cdot) - h_3^0(\cdot))x_2\| + \|(h_2^0(\cdot) - h_2(\cdot))x_4\| +\\
&\quad \|(g_3(\cdot) - g_3^0(\cdot))u\| + \|g_4(\cdot) - g_4^0(\cdot)\| \le M_3\\
&\|h_4(\cdot) - h_4^0(\cdot)\| \le M_4
\end{aligned} \tag{15}$$

where $M_i (1 \le i \le 4)$ are all positive bounded constants.

3.2 Stability Analysis

Theorem 1. The nonlinear observer designed in (8) can estimate the states of the battery model (7) with uniformly bounded estimation error, if the observer gains $l_i (1 \le i \le 4)$ are selected properly.

Proof. To prove Theorem 1, a Lyapunov function $V(t) \in \mathbb{R}$ is chosen

$$V = \frac{1}{2}\tilde{x}_1^2 + \frac{1}{2}\tilde{x}_2^2 + \frac{1}{2}\tilde{x}_3^2 + \frac{1}{2}\tilde{x}_4^2 + \frac{1}{2}\tilde{x}_5^2. \tag{16}$$

Based on (10) to (15), the derivative of $V(t)$ can be obtained as follows

$$\begin{aligned}
\dot{V} &= \tilde{x}_1\dot{\tilde{x}}_1 + \tilde{x}_2\dot{\tilde{x}}_2 + \tilde{x}_3\dot{\tilde{x}}_3 + \tilde{x}_4\dot{\tilde{x}}_4 + \tilde{x}_5\dot{\tilde{x}}_5\\
&\le \tilde{x}_1(-a\tilde{x}_1 - l_1\tilde{x}_4) + \tilde{x}_2(-d_1\tilde{x}_2 + M_1 - l_2\tilde{x}_4) +\\
&\quad \tilde{x}_3(-d_2\tilde{x}_3 + M_2 - l_3\tilde{x}_4) + \tilde{x}_4(d_3\tilde{x}_2 - d_2\tilde{x}_4 + M_3\\
&\quad - l_4\tilde{x}_4) + \tilde{x}_5(-c\tilde{x}_5 + M_4 - l_5\tilde{x}_4).
\end{aligned} \tag{17}$$

Based on Lemma A.17 in Queiroz et al. (2000), which can be illustrated in the Appendix A, the following inequations can be determined

$$\begin{aligned}
&-a\tilde{x}_1^2 - l_1\tilde{x}_1\tilde{x}_4 \le -\tfrac{a}{2}\tilde{x}_1^2 + \tfrac{2l_1^2\tilde{x}_4^2}{a}\\
&-d_1\tilde{x}_2^2 + M_1\tilde{x}_2 + (d_3 - l_2)\tilde{x}_2\tilde{x}_4 \le -\tfrac{d_1}{3}\tilde{x}_2^2 + \tfrac{3M_1^2}{d_1} + \tfrac{3(d_3 - l_2)^2}{d_1}\tilde{x}_4^2\\
&-d_2\tilde{x}_3^2 + M_2\tilde{x}_3 - l_3\tilde{x}_3\tilde{x}_4 \le -\tfrac{d_2}{3}\tilde{x}_3^2 + \tfrac{3M_2^2}{d_2} + \tfrac{3l_3^2}{d_2}\tilde{x}_4^2\\
&-d_2\tilde{x}_4^2 + M_3\tilde{x}_4 - l_4\tilde{x}_4^2 \le -l_4\tilde{x}_4^2 + \tfrac{M_3^2}{d_2}\\
&-c\tilde{x}_5^2 + M_4\tilde{x}_5 - l_5\tilde{x}_4\tilde{x}_5 \le -\tfrac{c}{3}\tilde{x}_5^2 + \tfrac{3l_5}{c}\tilde{x}_4^2 + \tfrac{3M_4^2}{c}\ .
\end{aligned}$$

Based on (17) and the inequalities above, the following inequation can be obtained

$$\dot{V} \le -\frac{a}{2}\tilde{x}_1^2 - \frac{d_1}{3}\tilde{x}_2^2 - \frac{d_2}{3}\tilde{x}_3^2 - \alpha\tilde{x}_4^2 - \frac{c}{3}\tilde{x}_5^2 + M \tag{18}$$

where $\alpha = l_4 - \frac{2l_1^2}{a} - \frac{3(d_3 - l_2)^2}{d_1} - \frac{3l_3^2}{d_2} - \frac{3l_5^2}{c}$ and $M = \frac{3M_1^2}{d_1} + \frac{3M_2^2}{d_2} + \frac{M_3^2}{d_2} + \frac{3M_4^2}{c}$. An appropriate p can be selected which satisfies $-px_4^2 + M \le 0$ for some region of x_4, then it can be determined

$$\dot{V} \le -\frac{a}{2}\tilde{x}_1^2 - \frac{d_1}{3}\tilde{x}_2^2 - \frac{d_2}{3}\tilde{x}_3^2 - (\alpha - p)\tilde{x}_4^2 - \frac{c}{3}\tilde{x}_5^2. \tag{19}$$

From the previous analysis, it shows that $\frac{a}{2}$, $\frac{d_1}{3}$, $\frac{d_2}{3}$, $\frac{c}{3}$ are all positive. If the observer gain l_4 is chosen large enough and l_1, l_2, l_3 and l_5 are chosen properly, $(\alpha - p)$ can also be guaranteed as positive. Then a constant $\varepsilon > 0$ can be chosen which satisfies $\varepsilon \le min(\frac{a}{2}, \frac{d_1}{3}, \frac{d_2}{3}, \alpha - p, \frac{c}{3})$. Based on (19), it can be obtained

$$\dot{V} \le -\varepsilon \|\tilde{x}\|^2 . \tag{20}$$

Based on (19), (20) and the Lyapunov stability, it can be determined that if the solution of the error system enters

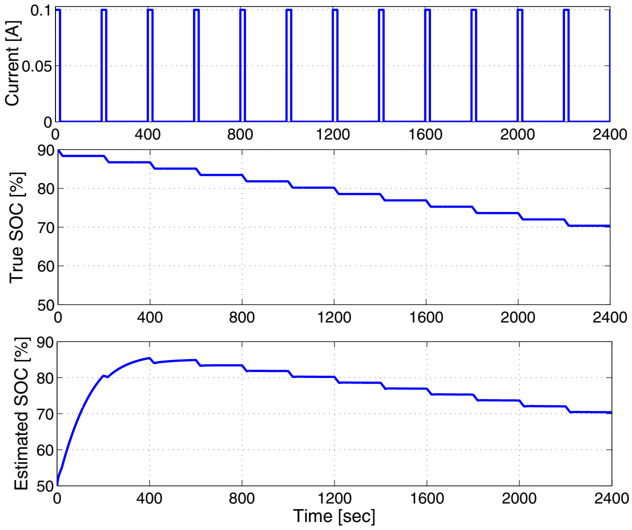

Fig. 2. Actual SOC and estimated SOC in a discharging mode

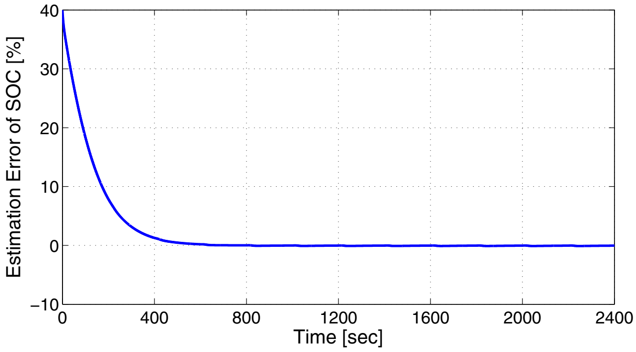

Fig. 3. SOC estimation error in a discharging mode

the bound $\{\|\tilde{x}(t)\| \leq \sqrt{\frac{M}{p}}\}$, it will remain in it. That is to say, the solution of the error system is uniformly bounded. if l_4 is selected big enough, then p can be chosen big enough and the bound can be arbitrarily small. So it can be obtained that the states of the battery model can be estimated by the nonlinear observer, if the observer gains are selected properly. □

4. SIMULATION RESULTS

Most of the parameters in the proposed model are based on Chen and Rincon-Mora (2006). For the observer, a constant model error about 5% of these parameters is chosen. The situation that the battery is just in discharging mode is simulated, in which the discharge current $I_B(t)$ is set to simple pulse current, whose amplitude is $0.1A$, period is 200 seconds and duty ratio is 10%. The battery's initial SOC is set to 90% and the observer's initial SOC is set to 50%. Fig. 2 and 3 illustrate that the SOC estimation error becomes almost zero after about 400 seconds. As shown in Fig. 4 and 5, the terminal voltage and temperature of the battery can be estimated by the nonlinear observer accurately.

In actual hybrid power systems, the battery is not always in discharging mode until the SOC of the battery goes to a lower value or even to zero. When the SOC of the battery is below a pre-reset value, the battery starts to be charged and the charge/discharge current is much more complicated than a simple pulse current. So simulation below is utilized to simulate a continuous power cycling test. An asymmetry and complicated current is set as Fig. 6. The initial SOC of the battery and the observer are also set to 90% and 50%, respectively. In the situation that the battery is in an alternately charging and discharging mode, the actual and estimated SOC of the battery are depicted in Fig. 7, which shows that the SOC of the battery can be estimated by the nonlinear observer accurately.

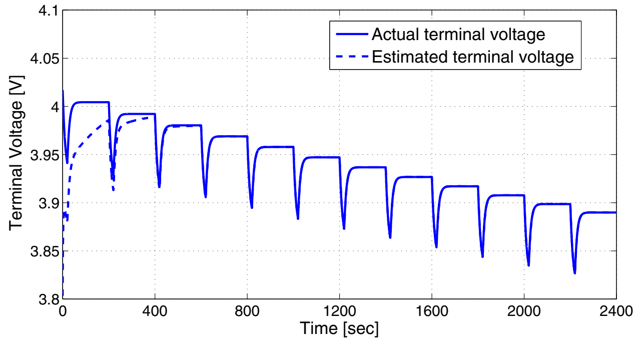

Fig. 4. Actual and estimated terminal voltages in a discharging mode

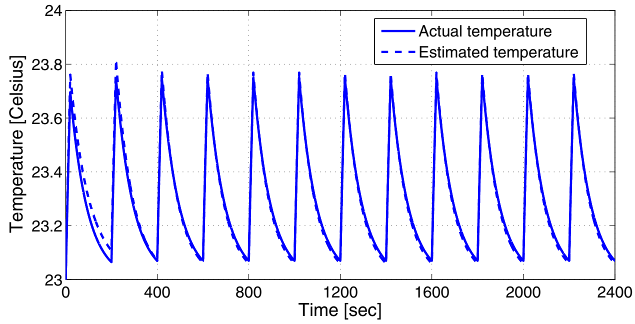

Fig. 5. Actual and estimated temperatures in battery in a discharging mode

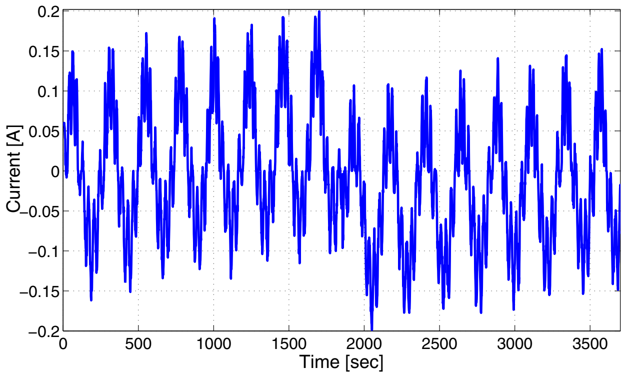

Fig. 6. Current profile of the battery

5. CONCLUSIONS

A new method for estimating the SOC of lithium-ion batteries based on an inclusive equivalent circuit model

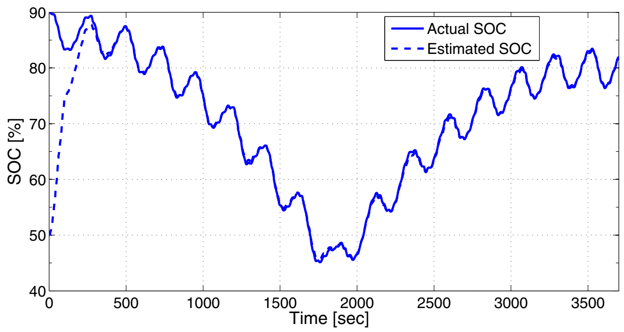

Fig. 7. Actual and estimated SOC in an alternately charging and discharging mode

is proposed in this paper. The influence of SOC ,temperature, charging or discharging mode on the resistances and capacitances in the equivalent circuit are considered. The nonlinear relationship between $V_{OC}(t)$ and the SOC is considered and a nonlinear observer is designed to estimate the inner characteristic of the battery model. Then the Lyapunov stability analysis is utilized to prove its stability and convergence. Simulations are done to estimate the SOC when the battery is in a discharging mode and in an alternately charging and discharging mode, with high accuracy in estimating the SOC being demonstrated. The results of the simulations show that the nonlinear observer is a promising method to estimate the SOC of the battery.

Appendix A

If a scalar function $N_d(x, y)$ is given by (Queiroz et al. (2000))

$$N_d = \Omega(x)xy - k_n\Omega^2(x)x^2 \tag{A.1}$$

where $x, y \in R$, $\Omega(x) \in R$ is a function dependent only on x, and k_n is a positive constant, then $N_d(x, y)$ can be upper bounded as follows

$$N_d \leqslant \frac{y^2}{k_n} \tag{A.2}$$

The bounding of $N_d(x, y)$ in the above manner is often referred to as nonlinear damping since a nonlinear control function can be used to "damp-out" an unmeasurable quantity multiplied by a known measurable nonlinear function.

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