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Ideal, Simplified and Inverted Decoupling of Fractional Order TITO Processes

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Abstract: In recent years, more and more research work has demonstrated the advantages of using fractional order modeling and control techniques. However, many of these research focus on the single-input-single-output systems. In this paper, fractional order two-input-two-output processes are studied in terms of decoupling control. Three types of classic decoupling techniques for integer order processes are re-visited and are generalized to fractional order cases, which has not been addressed in the literature. This effort involves new problems other than those for integer order processes, such as the the properness, interaction analysis, and the frequency dependent relative gain array, etc. Simulation examples are given to illustrate these generalized decoupling methodologies and some notes on practical implementation are provided.

1. INTRODUCTION

Ideal, simplified, and inverted decoupling are some of the widely used classic methods for industrial process controls [1, 2, 3], as well as their variations [4, 5, 6]. The properness, realizability, causality, and stability of these decoupling techniques have been well studied. However, their application are investigated only on integer order process models. When facing a fractional order (FO) process model, will these methods lose vitality? The pursuit of the answer to this question is explored in this paper.

FO modeling and control have been proved capable to provide "better than the best" performance than integer order ones under fair comparisons [7, 8]. It is more and more convincing that FO models depict the physical world more closely to the nature, and FO controls are more powerful than integer order controls. As this research subject emerges and blooms, lots of classic methodologies for integer order systems have been extended to the fractional order cases, such as the $\text{PI}^\lambda\text{D}^\mu$ controller in [9], the FO sliding mode extremum seeking controller in [11] and the FO root locus in [12]. The needs for such efforts keep increasing. Upon viewing these works, it can be seen that although some methods seem simple for integer order cases, it is usually not straightforward to be extended to FO cases, especially for multi-input-multi-output (MIMO) FO systems. Some examples may be MIMO FO identification and minimum realizations, [10, 20, 23]. To explore the potential of de-centralized controllers and inject new perspectives to the development of FO controls, this paper investigates the extension of the decoupling techniques from integer order to fractional order, through which, some new issues are discussed, such as the properness of the fractional order decouplers, and the frequency-dependent relative gain array (RGA) for MIMO FO processes.

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The rest of this paper is organized as follows. First, the mathematical description of the FO two-input-two-output (TITO) processes are introduced in section 2, and three types of conventional decoupling techniques are applied to the model represented by the transfer function matrix. Then, simulation examples with distinctive characteristics are enumerated to illustrate the concepts described in section 3. Finally, some technical remarks are provided for implementation references.

2. DECOUPLING FRACTIONAL ORDER PROCESSES

2.1 Fractional order TITO processes

Fractional order models can be commonly found in the research field of biology, chemistry and physics. For example, the membrane charging model in [13], the fractional impedance in botanic elements [14], the ion channel gating model in [15] and the heat transfer process in [16]. Even in electrical engineering and motion control, FO models are found useful, to name a few, the analog FO control element, "fractor", in [17], the fractional order velocity model in [7], and the FO circuits in [18] and [19].

Besides the listed single-input-single-output (SISO) FO systems, MIMO fractional order systems also exist [21][22], with the TITO process being a particular case. In this paper, the system under investigation is a TITO process abstracted from a temperature control loop in the semiconductor manufacturing industry, as shown in figure 1. It is a linear time-invariant (LTI) system that can be depicted by the following fractional order differential equations,

$$ {}_0D_t^{\alpha}x(t) = Ax(t) + Bu(t) \tag{1}$$

$$ y(t) = Cx(t) \tag{2}$$

where ${}_0D_t^{\alpha}x(t)$ denotes the fractional differentiation with respect to time, and the fractional orders are $\alpha = [\alpha_{11}, \alpha_{12}; \alpha_{21}, \alpha_{22}] \in (0, 2)$. The system matrix A, input

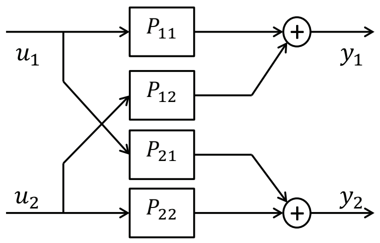

Fig. 1. The block diagram of a TITO process.

matrix B and output matrix C are of the following forms respectively,

$$A = \begin{bmatrix} -\frac{1}{T_{11}} & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{12}} & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{21}} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_{22}} \end{bmatrix}, B = \begin{bmatrix} \frac{K_{11}}{T_{11}} & 0 \\ 0 & \frac{K_{12}}{T_{12}} \\ \frac{K_{21}}{T_{21}} & 0 \\ 0 & \frac{K_{22}}{T_{22}} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Similar to the way of manipulating integer order differential equations, by taking Laplace transforms, the state space representation of the above FO differential equations can be derived, with zero initial condition assumed,

$$s^{\alpha} X(s) = AX(s) + BU(s) \quad (3)$$

$$Y(s) = CX(s). \quad (4)$$

Furthermore, the state space representation can be converted to a transfer function matrix in the same manner for integer order models [20],

$$Y(s) = P(s) U(s), \quad (5)$$

and

$$P(s) = C(s^{\alpha} I - A)^{-1} B = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s). \end{bmatrix} \quad (6)$$

where each element P_{ij} is a transfer function with one fractional order pole,

$$P_{ij}(s) = \frac{K_{ij}}{T_{ij} s^{\alpha_{ij}} + 1}, \; i, j = 1, 2. \quad (7)$$

The off diagonal elements P_{12} and P_{21} are the cause of the interaction between two primary loops. For integer order processes, varieties of existing methods for decoupling the interaction are mentioned in Sec. 1. No matter which method is used, the goal is to eliminate or minimize the interaction, which is the same for fractional order processes. In the following subsections, the decoupling of FO processes will be presented, with the cases of zero dead time discussed first, and the cases of a non-zero dead time dealt with separately.

2.2 The ideal decoupling

With the ideal decoupling, the decoupled process is expected to have a diagonal transfer function matrix in the form below:

$$G(s) = P(s) D(s) = \begin{bmatrix} P_{11}(s) & 0 \\ 0 & P_{22}(s) \end{bmatrix}, \quad (8)$$

where $D(s)$ is the transfer function matrix of the decoupler,

$$D(s) = \begin{bmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{bmatrix}. \quad (9)$$

An illustration of the system connection with an ideal decoupler is shown in figure 2.

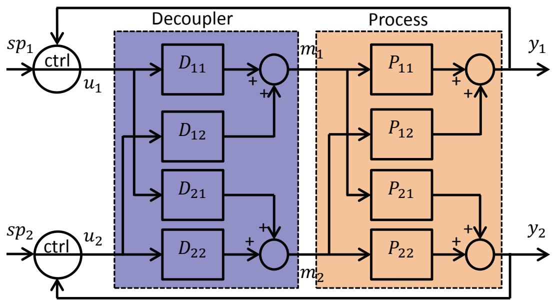

Fig. 2. The block diagram of the ideal decoupling.

Based on the decoupling requirement in equation (8), four equations can be established:

$$P_{11}(s) D_{11}(s) + P_{12}(s) D_{21}(s) = P_{11}(s)$$
$$P_{11}(s) D_{12}(s) + P_{12}(s) D_{22}(s) = 0$$
$$P_{21}(s) D_{11}(s) + P_{22}(s) D_{21}(s) = P_{22}(s)$$
$$P_{21}(s) D_{12}(s) + P_{22}(s) D_{22}(s) = 0.$$

The decoupler elements are then given by the solution:

$$\begin{aligned} D_{11}(s) &= \frac{P_{11}(s) P_{22}(s)}{P_{11}(s) P_{22}(s) - P_{12}(s) P_{21}(s)} \\ D_{12}(s) &= \frac{-P_{12}(s) P_{22}(s)}{P_{11}(s) P_{22}(s) - P_{12}(s) P_{21}(s)} \\ D_{21}(s) &= \frac{-P_{11}(s) P_{21}(s)}{P_{11}(s) P_{22}(s) - P_{12}(s) P_{21}(s)} \\ D_{22}(s) &= \frac{P_{11}(s) P_{22}(s)}{P_{11}(s) P_{22}(s) - P_{12}(s) P_{21}(s)}. \end{aligned} \quad (10)$$

Plugging equation (7) into the solutions will give the fractional order ideal decoupler.

At this stage, the properness of such decoupling elements needs to be examined. When the four channels have identical fractional orders, the decoupler is obviously proper (i.e. strictly proper or biproper) with the same highest order 2α on both the numerator and the denominator. When the fractional orders are different, it can be seen that the resulting decoupler is still proper. Take the first element as an example:

$$D_{11}(s) = \frac{\Phi}{\Phi - \Psi}, \quad (11)$$

where

$$\Phi = K_{11} K_{22} (T_{12} s^{\alpha_{12}} + 1)(T_{21} s^{\alpha_{21}} + 1)$$
$$\Psi = K_{12} K_{21} (T_{11} s^{\alpha_{11}} + 1)(T_{22} s^{\alpha_{22}} + 1.)$$

Thus, the highest order of the denominator is $\max(\alpha_{12} + \alpha_{21}, \alpha_{11} + \alpha_{22})$ while that of the numerator is $\alpha_{12} + \alpha_{21}$, and the relationship among the fractional orders does not affect the properness.

2.3 The simplified decoupling

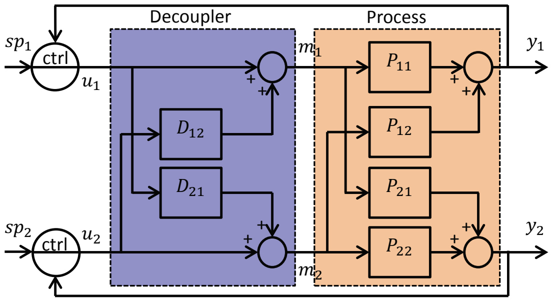

Fig. 3. The block diagram of the simplified decoupling.

Compared with the ideal decoupling, the simplified decoupling has less stringent requirements on the diagonal elements of the process. In other words, it does not emphasize much on what the primary loops become after decoupling. Instead, it assigns less task to the decoupler by setting the diagonal elements to be 1, as shown in figure 3,

$$D(s) = \begin{bmatrix} 1 & D_{12}(s) \\ D_{21}(s) & 1 \end{bmatrix}. \tag{12}$$

Thus, the following two equations are used to satisfy the decoupling condition, i.e. to make the process behave diagonal,

$$P_{11}(s) D_{12}(s) + P_{12}(s) D_{22}(s) = 0,$$
$$P_{21}(s) D_{12}(s) + P_{22}(s) D_{22}(s) = 0,$$

with the solution being:

$$\begin{aligned} D_{12}(s) &= -\frac{P_{12}(s)}{P_{11}(s)} \\ D_{21}(s) &= -\frac{P_{21}(s)}{P_{22}(s)} \end{aligned} \tag{13}$$

This leads to a simpler decoupler transfer function but a relatively more complex decoupled process,

$$G(s) = \begin{bmatrix} P_{11} - \dfrac{P_{12}P_{21}}{P_{22}} & 0 \\ 0 & P_{22} - \dfrac{P_{12}P_{21}}{P_{11}} \end{bmatrix}. \tag{14}$$

To evaluate the properness in this circumstance, different cases need to be considered. When the fractional orders are identical, i.e. $\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22}$, the decoupler will be in a fractional order filter form [21]. If the fractional orders are different, it might result in an improper decoupler that can not be realized. Specifically, for example, when $\alpha_{11} > \alpha_{12}$, the second decoupler element is improper,

$$D_{12}(s) = -\frac{K_{12}(T_{11}s^{\alpha_{11}} + 1)}{K_{11}(T_{12}s^{\alpha_{12}} + 1)}.$$

Although the resulting process is proper,

$$G_{11}(s) = \frac{\Psi - \Phi}{\Gamma},$$

where Φ and Ψ are the same as in equation (11), and

$$\Gamma = K_{22}(T_{11}s^{\alpha_{11}} + 1)(T_{12}s^{\alpha_{12}} + 1)(T_{21}s^{\alpha_{21}} + 1),$$

it cannot be achieved in practice because a fractional order differentiator $s^{\alpha_{11}-\alpha_{12}}$ can be factorized from $D_{12}(s)$ by the means we use for integer order systems, such as long division or partial fraction expansion. Similar to a pure differentiator in integer order control systems, such a fractional order differentiator will also amplify noise and result in divergent or singular solutions of system responses, which is not acceptable in practice. Some research on the existence of decouplers for integer order singular systems can be referred to such as [6, 24]. Thus, to guarantee the existence of a proper simplified decoupler, the TITO process model with one FO pole needs to satisfy the following condition,

$$\alpha_{11} \leq \alpha_{12}, \text{ and } \alpha_{22} \leq \alpha_{21}. \tag{15}$$

Otherwise, the conventional decoupling techniques do not apply. In order to still utilize them, a different model structure can be selected to approximate the process, such as FO transfer functions **with two poles**, either **commensurate or not**. This can be a topic for future exploration.

2.4 The inverted decoupling

Briefly, the inverted decoupling is to achieve the ideally-decoupled performance in equation (8), using simplified decoupling elements in equation (12). This is accomplished by subtly re-routing the decoupling block connections, as shown in figure 4, which is borrowed directed from integer order case for the FO case.

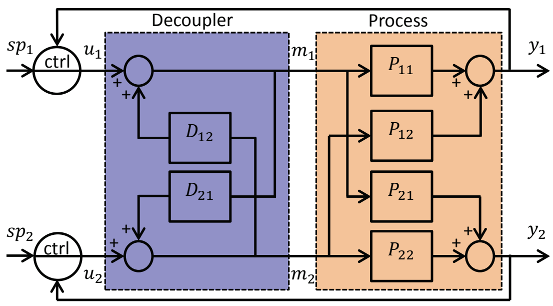

Fig. 4. The block diagram of the inverted decoupling.

Since the inverted decoupler uses the same decoupling elements with the simplified decoupler, the condition for the existence of a proper inverted decoupler is the same with equation (15).

2.5 Decoupling FO processes with dead time

The aforementioned discussion considers the process models with no dead time, which is too ideal to be true in practice. Nevertheless, it is not a problem when the models have dead time because the techniques for dealing with time-delayed integer order models already exist, which also can be used for FO processes. Specifically, denoting the dead time by L_{ij}, the model in equation (7) becomes the following:

$$\tilde{P}_{ij}(s) = \frac{K_{ij}}{T_{ij}s^{\alpha_{ij}}+1}e^{-sL_{ij}}. \tag{16}$$

Consequently, the simplified as well as the inverted decoupling elements in equations (13) become the forms below,

$$\tilde{D}_{12}(s) = -\frac{K_{12}(T_{11}s^{\alpha_{11}}+1)}{K_{11}(T_{12}s^{\alpha_{12}}+1)}e^{-(L_{12}-L_{11})s}, \tag{17}$$

$$\tilde{D}_{21}(s) = -\frac{K_{21}(T_{22}s^{\alpha_{22}}+1)}{K_{22}(T_{21}s^{\alpha_{21}}+1)}e^{-(L_{21}-L_{22})s}. \tag{18}$$

When $L_{12} < L_{11}$ and/or $L_{21} < L_{22}$, the decoupler is non-causal, which is to be avoided in the realization of transfer functions. This problem can be fixed by artificially by adding a time delay to the decoupler as described in Wang, *et, al's* work [25]. Thus, the refined decoupler $\tilde{D}(s)$ becomes the following form:

$$\tilde{D}(s) = \begin{bmatrix} e^{-v(L_{22}-L_{21})s} & D_{12}(s)e^{-v(L_{12}-L_{11})s} \\ D_{21}(s)e^{-v(L_{21}-L_{22})s} & e^{-v(L_{11}-L_{12})s} \end{bmatrix}, \tag{19}$$

where the function $v(L)$ is defined as:

$$v(L) = \begin{cases} L, & \text{if } L > 0, \\ 0, & \text{if } L \le 0. \end{cases} \tag{20}$$

Remark 1. We remark that the definition of $v(L)$ is inaccurate in the original proposed form. The value should be L when $L > 0$. The inaccurate use of this method in [26, 27] should be corrected.

2.6 The relative gain array for FO processes

RGA is a useful tool to characterize the loop interactions in MIMO processes, from which the advises for suitable input-output pairing can be drawn [28]. While the static RGA only evaluates the steady-state gains, the frequency dependent RGA evaluates the process gains at the corresponding operational frequencies of interest. For the LTI model with one FO pole, as in equation (7), the gain depends not only on the traditional model parameters K, T and L, but also on the FO order α,

$$\begin{aligned} |G_{ij}(j\omega)| &= \left|\frac{K_{ij}}{T_{ij}(j\omega)^{\alpha_{ij}}+1}e^{-L_{ij}j\omega}\right| \\ &= \left|\frac{K_{ij}}{T_{ij}\omega^{\alpha_{ij}}e^{j\frac{\pi}{2}\alpha_{ij}}+1}\right| \\ &= \frac{|K_{ij}|}{\left|T_{ij}\omega^{\alpha_{ij}}\left[\cos\left(\frac{\pi}{2}\alpha_{ij}\right)+j\sin\left(\frac{\pi}{2}\alpha_{ij}\right)\right]+1\right|} \\ &= \frac{|K_{ij}|}{\sqrt{(T_{ij}\omega^{\alpha_{ij}})^2+2T_{ij}\omega^{\alpha_{ij}}\cos\left(\frac{\pi}{2}\alpha_{\text{ij}}\right)+1}}. \end{aligned} \tag{21}$$

Hence, the frequency dependant RGA is:

$$RGA = G(j\omega)\cdot(G(j\omega)^{-1})^T, \tag{22}$$

where $G(j\omega)$ takes the form in equation (21). This will be illustrated through simulation example 3 in Sec. 3.

3. SIMULATION EXAMPLES

Example 1: Consider the FO TITO process model abstracted from a thermo-electric temperature control test bed, [29],

$$P(s) = \begin{bmatrix} \frac{1.2}{2s^{0.5}+1} & \frac{0.6}{3s^{0.7}+1} \\ \frac{0.5}{s^{0.8}+1} & \frac{1.5}{3s^{0.6}+1} \end{bmatrix}.$$

An output noise is added to emulate the measurement noise with the signal-to-noise ratio (SNR) of about 30dB. The step responses of the individual channels before and after decoupling are plotted in figure 5, from which it can be observed that the three decoupling methods are valid for different fractional orders as long as the condition in equation (15) is satisfied. The output signals from the inverted decoupler are plotted in figure 6 for reference and later discussion.

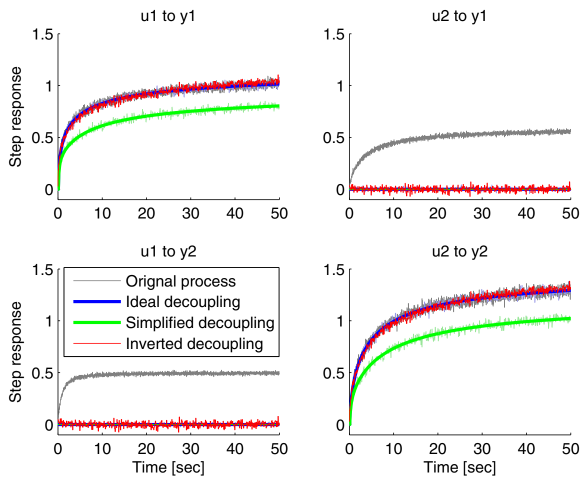

Fig. 5. The open-loop step responses of the system in Example 1, before and after decoupling.

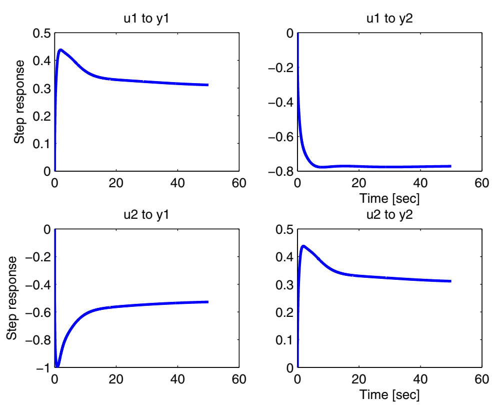

Fig. 6. The output signals of the inverted decoupler. Left: u_1 step, u_2 zero; right: u_1 zero, u_2 step. Top plots are from D_{12} and bottom are from D_{21}.

Example 2: To illustrate the concept in section 2.5, consider the following FO process with dead time, which is modified from the Wood-Berry distillation process in [30], by changing the integer order to half order and swapping the dead time of the primary and the interactive loops,

$$P(s) = \begin{bmatrix} \frac{12.8e^{-3s}}{16.7s^{0.5}+1} & \frac{-18.9e^{-s}}{21.0s^{0.5}+1} \\ \frac{6.60e^{-s}}{10.9s^{0.5}+1} & \frac{-19.4e^{-7s}}{14.4s^{0.5}+1} \end{bmatrix}.$$

Since $L_{12} < L_{11}$ and $L_{21} < L_{22}$ in this example, the manipulation of dead time needs to be included into the decoupler design. Following equation (19), the simulation result is shown in figure 7. While the artificial time delays ensure the causality of the decoupler, the advantage of being able to derive the input to decoupling element from the secondary loop actuator is lost [2]. It can be seen that although both decouplers achieve "perfect control" at steady state, [28], there are differences in the transients. The simplified decoupling (red line) with the artificial time delay can completely eliminate the interaction, although it changes the primary loop (this change can be compensated by controllers). In contrast, the inverted decoupling (black line) keeps the primary loop unchanged, but the decoupling effect at the initial part is a little off from expectation. In practical implementation, the selection of which decoupling to be used can be determined by control performance specifications.

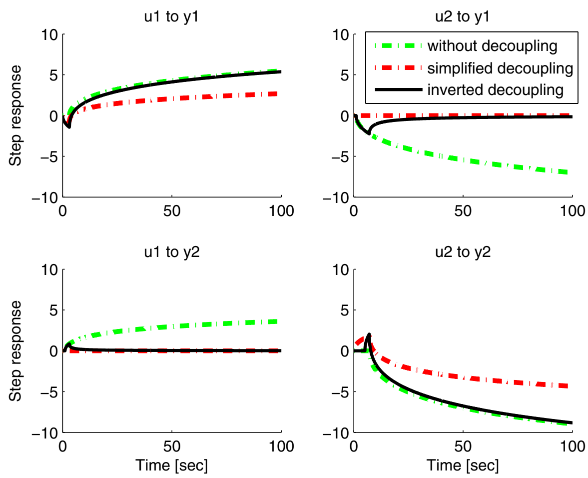

Fig. 7. The open-loop step responses of the system in Example 2, using different decoupling methods.

Example 3: Consider again the process model in example 2, but change the input u_2 to a periodic signal,

$$u(t) = \begin{bmatrix} u1 \\ u2 \end{bmatrix} = \begin{bmatrix} \text{unit step} \\ 0.5\sin(10t) \end{bmatrix}.$$

Such input signal combinations are usually used in chemical reaction processes where one reaction species is kept at a constant supply rate while the other is injected periodically. In this case, the frequency dependant RGA will play a more important role than the static RGA. For comparison, the RGAs of the original and the modified Wood-Berry process are plotted in figure 8 as an illustration of section 2.6. In this example, although the frequency dependant RGA differs from the integer order model, the paring does not change. For some practical processes, the paring may even change across broad band.

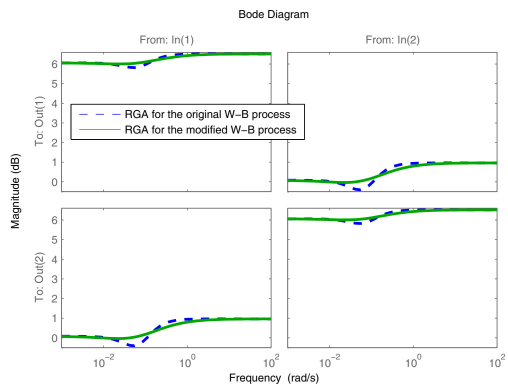

Fig. 8. The RGA of the original and modified Wood-Berry processes.

The simulation result using unit feedback with inverted decoupling is plotted in figure 9. The green line shows that the two primary loops interfere each other significantly before decoupling, which appears in the form of fluctuations for channel 1 and a bias for channel 2. The blue line shows that the interaction is well decoupled by the fractional order inverted decoupler.

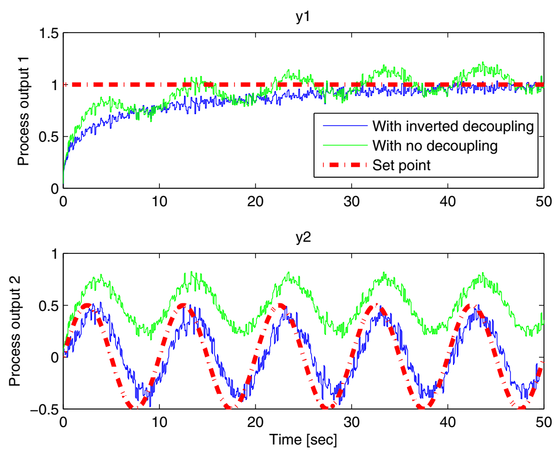

Fig. 9. The closed-loop step response for Example 3, with both inputs on.

The above simulations are performed in Matlab with the help of "Ninteger" toolbox [31] for solving the fractional integration and differentiation.

4. REMARKS

The decoupling results in the above simulations look encouraging; yet, an important remark is commented here with regards to some realistic aspects in industrial process controls, i.e. the control authority limitation.

In motion control, the inverse actuation can almost always be achieved by applying brake force onto wheels driven by engines, or applying revered voltage to a motor. In circuit control, the charge and discharge of a capacitor can also be performed bi-directionally. Unlike these circumstances, process controls often have no inverse actuation capability in quite a lot of applications, i.e. only on/off actuation or some status in between is allowed, but the actuation in the opposite direction is not available. For instance, a water tank with no draining pump will have a fixed draining speed determined by the liquid level and tank specifications; an oven with only heaters has no way to cool down faster than natural dissipation; sheet metal stretch bed are usually not equipped with compressing capabilities, etc. Dealing with these processes is much more challenging not only due to their nonlinear behavior in nature, but also because of the fundamental limitations in mechanical configuration [28]. In such cases, the negative control signals shown in figure 6 is not applicable, and the decoupling techniques based on approximated linear models may not give satisfactory results. Multi-variable control or predictive control strategies need to be employed instead of decentralized control.

To overcome this disadvantage, it is sometimes possible to adjust the operational point of the actuators so as to make them behave as if having the bi-directional actuation ability. For example, in a temperature control scenario, by assigning a higher duty cycle to the actuator at fundamental temperature can enable the process to cool down faster than natural dissipation from a high temperature. Similar tricks can be employed to confine the decoupler output signals within the permissible range. All in all, the key point is that decoupling may not be omnipotent, and the applicable scenario needs to be evaluated before attempt such effort.

5. CONCLUSION

In this work, the conventional decoupling techniques for integer order TITO processes are extended to fractional order cases, which results in the so-called fractional order decouplers. The effectiveness of such attempt is verified through both theoretical analysis and case studies. It is revealed that the decoupling techniques including the decoupler design and input-output paring need to be re-evaluated for fractional order MIMO systems. Our future work would be in performing experimental studies on our test-bench TITO system.

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