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Improvement of the Representativity of the Morris Method for Air-Launch-to-Orbit Separation

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Abstract:
This work is part of a technical study of the French space agency CNES (Centre National d'Etudes Spatiales) and the French aerospace laboratory Onera for the development of an air-launch-to-orbit system. The separation between the space launcher and the aircraft must be studied with a sensitivity analysis to estimate the role of the different uncertainties in the risk of collision and improve the robustness of the controller. As the number of factors is too large to apply a quantitative method, the qualitative Morris method is first used for factors fixing. The Morris method is generally used with limited calculations to roughly estimate the factors' impact, but the quality of its results is rarely discussed. The Morris method is based on the random sampling of so-called trajectories or, less frequently, radial points. The representativity of the trajectories and radial points is analyzed and improved with a new solution based on a discrete Latin hypercube. Extensive tests are used to compare the quality of the results of various procedures with five common test functions. A new quality indicator, more sensitive, is developed. It shows that the new solution greatly improves, at no additional cost, the estimation of the factors' impact by the Morris method. The classic radial points appear to be more efficient than the commonly used classic trajectories. From a more global perspective, this study raises the question of the convergence in the Morris method. The application of the new solution to the air-launch-to-orbit separation provides results with a much better quality indicator than in the previous campaign of sensitivity analysis.

Keywords: air-launch-to-orbit, store separation, sensitivity analysis, screening, elementary effects, sampling

1. INTRODUCTION

The French space agency CNES and the French aerospace laboratory Onera are developing an air-launch-to-orbit system in a program called Perseus to launch space rockets using an aircraft carrier rather than a land-based spaceport. A sensitivity analysis can help to understand the influence of the different factors of uncertainty during the separation of the aircraft and launcher. The separation is complex. It involves an important number of dynamics, it results in important mass and inertia variations, but it must be safe and accurate. Its sensitivity analysis will be used in the design of robust control laws. A simulation has been developed for this work. It integrates various problems faced in the history of store separation and outputs the minimum distance between the launcher and the carrier during the separation.

The number of uncertainty factors in this simulation is too large to apply a classic quantitative sensitivity analysis method like the Sobol's method (Sobol, 2001). In the case of the air-launch-to-orbit separation, the Sobol's method would typically require at least 50000 simulations, each of them lasting 15 seconds in average. In such case, a qualitative screening is used for factors fixing, that is fixing the less important factors. The Morris method is commonly used (Saltelli et al., 2004) (Ratto et al., 2007). This step is very sensitive as important factors can be underestimated and fixed, while negligible factors can be overestimated and included in the Sobol's method. The former case alters the Sobol's results and leads to wrong conclusions, while the latter case results in useless calculations.

The Morris method is classically based on random trajectories in the discretized factor hyperspace (Morris, 1991). The factors variations along each trajectory are used to assess the factors' importance. Because of the complete randomness they are based on, the trajectories may not properly cover the factor hyperspace. This issue is addressed in (van Griensven et al., 2006) with trajectories covering local areas of a continuous Latin hypercube. This idea of a Latin hypercube is adapted to the Morris' recommendation to use trajectories covering large areas of a discretized hypercube. In (Campolongo et al., 2007), the issue of the trajectories definition is addressed with a new norm used to estimate the distance between the trajectories. The improvements derived from this distance are tested for comparison purpose in this paper. Random radial points also appear as an alternative to random trajectories

(Campolongo et al., 2011). They are less frequently used but their benefit has been shown for the Sobol's method (Saltelli et al., 2010). This paper studies their benefit in the Morris method, both with a random sampling and the adaptation of the Latin hypercube sampling. A new indicator of the quality of the Morris method's results is defined in order to overcome the sensitivity limitations of the indicators based on the proportion of important factors correctly identified (Campolongo et al., 2011).

2. IMPROVEMENT OF THE MORRIS METHOD

2.1 Description of the Morris method

Let y be the output of a function f which depends on k input factors $\{x_1, x_2, ..., x_k\}$:

$$y = f(x_1, x_2, ..., x_k) \tag{1}$$

The objective of the sensitivity analysis is to estimate the influence of the factors uncertainty on the output. The factors are considered as scalar and their uncertainties are represented by probability distributions. In the Morris method, m discrete values are chosen for each factor. It is now frequent to consider the factors in the quantiles hyperspace (Saltelli et al., 2004). The values of the cumulative distribution function (c.d.f.) are typically chosen equally spaced, such that:

$$F_i(X_i^j) = \frac{2j-1}{2m} \tag{2}$$

where X_i^j is the j-th discrete value of the factor x_i, and F_i its c.d.f.. In the Morris method, the function is studied by varying one factor at a time. The *elementary effect* e_i associated to the factor x_i is defined by:

$$e_i = \frac{f(X_1^{(l)}, .., X_{i-1}^{(l)}, X_i^{(l+1)}, X_{i+1}^{(l)}, .., X_k^{(l)}) - f(X^{(l)})}{F_i(X_i^{(l+1)}) - F_i(X_i^{(l)})} \tag{3}$$

where $X_i^{(l)}$ and $X_i^{(l+1)}$ represent two different values of x_i, both in $\{X_i^1, .., X_i^m\}$, and where $X^{(l)} = \{X_1^{(l)}, .., X_k^{(l)}\}$. The denominator represents the distance separating them in the quantiles hyperspace. In this paper, m is chosen even and the following classic configuration is used:

$$F_i(X_i^{(l+1)}) - F_i(X_i^{(l)}) = \pm 0.5 \tag{4}$$

It derives from the recommendation of Morris to use a distance of $m/[2(m-1)]$ for discrete values equally spaced between 0 and 1, given that the values used are between $1/(2m)$ and $(2m-1)/(2m)$. The value $+0.5$ is used when $X_i^{(l)} < 0.5$, and -0.5 when $X_i^{(l)} > 0.5$. One elementary effect for each factor requires $2k$ computations of the function f. To reduce this number, Morris developed the concept of *trajectories* represented in the left part of Figure 1 for $k = 3$ factors. After the random definition of an initial point A, one of the factor, randomly chosen, is varied to obtain B. Another factor is then varied from B to obtain C, so forth so on. Another method, less common, is based on the concept of *radial points* represented in the right part of Figure 1. In this case, all the factors are varied

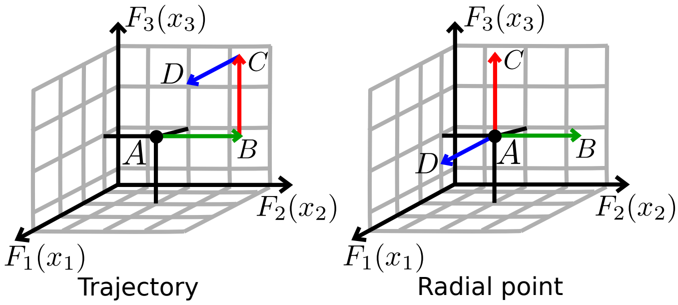

Fig. 1. Example of a trajectory and a radial point for three factors

from A. In this paper, A is called the initial point of the trajectory or the center of the radial point. One trajectory or one radial point provides an elementary effect for each of the k factors with $(k+1)$ computations of the function f. A number n of trajectories or radial points are used, with n typically equal to 4, 6, or 8. The importance of a factor is classically estimated with the average and the standard deviation of its n elementary effects. However, it has recently been shown that the average μ^* of the absolute values of the elementary effects could also be used alone and compared to the Sobol's total indexes S_t used as a reference (Campolongo et al., 2007). This paper focuses on the case $n = m$ which appears to be a good compromise between the coverage of each factor's interval and the inspection of the hyperspace. However, the method developed can easily be adapted to the case $n \neq m$.

2.2 Improvement of the Morris method

Because of the randomness of the trajectories and radial points, the elementary effects of a factor may be calculated at similar locations of the factor hyperspace and result in a non-representative μ^*. In (van Griensven et al., 2006), the trajectories are built by applying small variations to initial points sampled in a continuous Latin hypercube. If larger variations are used, as recommended by Morris, the trajectories reach other volume divisions of the Latin hypercube and get close to other trajectories. This problem is reinforced if the space is discretized, as also recommended by Morris. In this case, the variations cannot be smaller than the division of the Latin hypercube. If $A[x, y]$ and $B[x + \Delta, y + \Delta]$ are two initial points, the application of the variations $[\Delta, 0]$ and $[0, \Delta]$ to A results in the same elementary effects as the application of the variations $[0, -\Delta]$ and $[-\Delta, 0]$ to B. If the factors are changed in the same order, the application of $[\Delta, 0]$ and $[0, \Delta]$ to $A[x, y]$ and the application of $[-\Delta, 0]$ and $[0, -\Delta]$ to $B[x+2\Delta, y+\Delta]$ result in one common elementary effect. In (Campolongo et al., 2007), a different method is used. A distance between the trajectories is defined with the Euclidian norm. For example, the distance $d_{p,q}$ between two trajectories p and q is:

$$d_{p,q} = \begin{cases} \sum_{l=1}^{k+1} \sum_{h=1}^{k+1} \sqrt{\sum_{i=1}^{k} \left[X_i^{(l)}(p) - X_j^{(h)}(q) \right]^2} \\ \text{for } p \neq q \\ 0 \text{ otherwise} \end{cases} \tag{5}$$

where $X_i^{(l)}(p)$ is the value of the i-th factor at the l-th point of the trajectory p. N trajectories are gener-

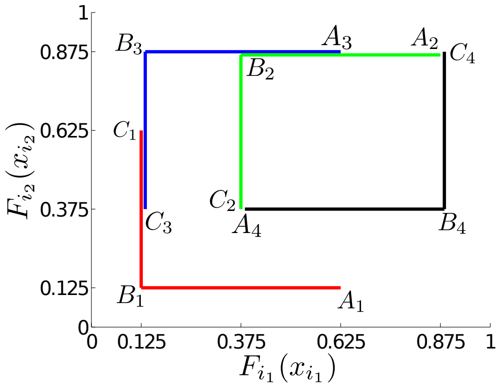

Fig. 2. Example of problematic trajectories or radial points

ated and the group of n trajectories with the largest $D = \sqrt{\sum_p \sum_{q>p} d_{p,q}^2}$ is used. This method requires the calculation of $N!/(n!(N-n)!)$ different D, which can be long. For example, selecting $n = 8$ trajectories out of $N = 25$ already requires the calculation of more than 1 million D. Test are carried out in Section 3 with both the Euclidian norm and the Manhattan (L_1 distance) norms, as suggested in (Campolongo et al., 2007).

Further improvement of the Morris method requires a better understanding of the possible problems faced with classic trajectories and radials points. Figure 2 represents the projection of 4 trajectories or radial points in a plane of the quantiles hyperspace corresponding to two of the k factors studied, x_{i_1} and x_{i_2}. This figure reveals two kinds of problems. The first one is related to the couples of points (B_1, C_1) and (B_3, C_3). These couples, used to calculate the elementary effects of the factor x_{i_2}, are both such that $F_{i_1}(x_{i_1}) = 0.125$. The representativity of the results could be improved if two different values of $F_{i_1}(x_{i_1})$ were used. Indeed, if $n = m$, the number of couples of points used to calculate the elementary effects of x_{i_2} is equal to the number of possible values for $F_{i_1}(x_{i_1})$. The second kind of problem is related to the four couples (B_1, C_1), (B_2, C_2), (B_3, C_3), and (B_4, C_4). In each couple, the two values of $F_{i_2}(x_{i_2})$ can either be $\{0.125, 0.625\}$ like for (B_1, C_1) or $\{0.375, 0.875\}$ like for the three others. Thus, three couples out of four share the same values of $F_{i_2}(x_{i_2})$. The representativity of the results could be improved by equally using the different possible values of $F_{i_2}(x_{i_2})$. In this paper, the initial points of the trajectories or the centers of the radial points are sampled by considering the quantiles hyperspace as a discrete Latin hypercube. As it is considered that $n = m$, the number of points in a discrete Latin hypercube with m values in each dimension is equal to the number n of trajectories or radial points. As presented at the beginning of this subsection, an initial discrete Latin hypercube sampling may still lead to representativity problems, including the calculation of the same elementary effect with different trajectories. With variations of 0.5, the representativity problems are avoided by varying the factors in a given order instead of a random order. Typically, the second point of a trajectory is obtained by varying the first factor, the third point is obtained by varying the second factor, so forth so on. Figure 3 shows all the projections of $n = 4$ trajectories generated for $k = 3$ factors.

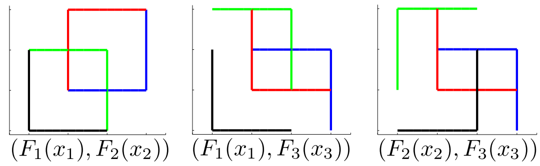

Fig. 3. Projections of improved trajectories

None of the problems previously presented appears. The application of this new method to radial points is direct. The radial points centers are generated like the trajectories initial points, and the elementary effects are then normally calculated from these centers. The generation of the n points used as initial points for the trajectories or centers for the radial points is fast. Let M be an $[n * k]$ matrix such that each row corresponds to a point and each column corresponds to a factor. The i-th column can be obtained with a random permutation of the vector $[X_i^1, .., X_i^m]$. Thus, the generation of the matrix M only requires k permutations.

3. APPLICATION

3.1 Calculation of the quality indicator

As a consequence of the limited number of calculations it requires, the Morris method is considered as qualitative. It is used to compare the relative importance of the factors. The results of the Morris method are also not normalized, contrary to the results of the Sobol's method. To develop a quality indicator, the way the results of the Morris method are analyzed must be clearly defined. Indeed, the indicator shows if the way the results are analyzed leads to wrong conclusions or not. The results are considered as perfect if the factors are in the right order, and the overestimation or underestimation of a factor is measured with both the number of factors involved and the relative difference of μ^*. The values of μ^* are first normalized. Let the linear regression of the μ^* be expressed as $a_1 S_t + a_0$, the normalized μ^* are such that $\mu_n^* = (100/a1)(\mu^* - a_0)$. The linear regression is $100 S_t$ for the μ_n^*. In Figure 4, the point P_7 should be higher than the points P_5 and P_6 to respect the order of the S_t. A measure of this problem is $R = r_{57}^2 + r_{67}^2$ where $r_{57} = (\mu_n^*(P_5) - \mu_n^*(P_7))$ and $r_{67} = (\mu_n^*(P_5) - \mu_n^*(P_7))$. More generally:

$$R = \sum_i \sum_j r_{ij}^2 \delta_{ij} \tag{6}$$

where $\delta_{ij} = [(S_t(P_j) - S_t(P_i)) > 0] \cap [(\mu_n^*(P_j) - \mu_n^*(P_i)) < 0]$.

3.2 Test functions and store separation

Five typical test functions of the literature of sensitivity analysis are first used. The results are then analyzed to improve the application of the Morris method to a separation between a space launcher and an aircraft. The five test functions and the store separation problem are briefly presented.

• The Morris function f_M depends on factors x_i uniformly distributed on $[0, 1]$ (Morris, 1991). The configuration

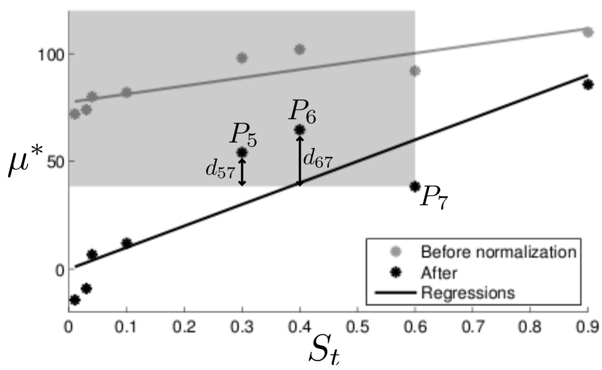

Fig. 4. Determination of the quality of the Morris method results

used, based on $k = 20$ factors, is also used in (Campolongo et al., 2011) and (Campolongo et al., 2007):

$$f_M(x_1,..,x_k) = \beta_0 + \sum_{i=1}^{20} \beta_i w_i + \sum_{i<j}^{20} \beta_{i,j} w_i w_j + \sum_{i<j<l}^{20} \beta_{i,j,l} w_i w_j w_l + \sum_{i<j<l<s}^{20} \beta_{i,j,l,s} w_i w_j w_l w_s \quad (7)$$

where $w_i = 2(x_i - 0.5)$ for all the values of i except for $i = 3, 5, 7$. In these cases, $w_i = 2[1.1x_i/(x_i + 0.1) - 0.5]$. Some of the coefficients β are set to relatively large values, increasing the importance of the corresponding factors:

$$\begin{aligned} \text{First order :} \quad & \beta_i = +20 \text{ for } i = 1, ..., 10 \\ \text{Second order :} \quad & \beta_{i,j} = -15 \text{ for } i, j = 1, ..., 6 \\ \text{Third order :} \quad & \beta_{i,j,l} = -10 \text{ for } i, j, l = 1, ..., 5 \\ \text{Fourth order :} \quad & \beta_{i,j,l,s} = +5 \text{ for } i, j, l, s = 1, ..., 4 \end{aligned} \quad (8)$$

The remaining first and second order coefficients are generated from a standard normal distribution, the remaining third and fourth order coefficients are set to zero.

• The Sobol function f_G (Archer et al., 2007) also depends on factors x_i uniformly distributed on $[0, 1]$. The number of factors is set to $k = 8$ with $a_i = \{0, 1, 4.5, 9, 99, 99, 99, 99\}$, a configuration for example directly implemented in the software Simlab (Saltelli et al., 2004). A larger coefficient a_i reduces the importance of the corresponding factor.

$$f_G(x_1,..,x_k) = \prod_{i=1}^{k} g_i \text{ where } g_i = \frac{|4x_i - 2| + a_i}{1 + a_i} \quad (9)$$

• The modified Sobol function f_G^* introduces a shift δ_i and a curvature α_i in f_G (Saltelli et al., 2010):

$$f_G^*(x_1,..,x_k) = \prod_{i=1}^{k} g_i^* \text{ where}$$
$$g_i^* = \frac{(1 + \alpha_i)|2(x_i + \delta_i - I[x_i + \delta_i]) - 1|^{\alpha_i} + a_i}{1 + a_1} \quad (10)$$

The number of factors is set to $k = 20$ with a configuration used in (Campolongo et al., 2011). It results in 10 important factors. The δ_i are generated in $U(0, 1)$ and:

$$\begin{aligned} a_i = \{&100, 0, 100, 100, 100, 100, 1, 10, 0, 0, \\ &9, 0, 100, 100, 4, 100, 100, 7, 100, 2\} \\ \alpha_i = \{&1, 4, 1, 1, 1, 1, 0.4, 3, 0.8, 0.7, \\ &2, 1.3, 1, 1, 0.3, 1, 1, 1.5, 1, 0.6\} \end{aligned} \quad (11)$$

• The Bratley function f_K (Bratley et al., 1992) also depends on factors x_i uniformly distributed on $[0, 1]$. As in (Campolongo et al., 2011), $k = 20$ factors are used:

$$f_K(x_1,..,x_k) = \sum_{i=1}^{k} \left((-1)^i \prod_{j=1}^{i} x_j \right) \quad (12)$$

• The Saltelli function f_B (Saltelli et al., 2008) depends on factors x_i following normal distributions $N(\overline{x_i}, \sigma_i)$.

$$f_B(x_1,..,x_k) = \sum_{i=1}^{k/2} x_i x_{k/2+i} \quad (13)$$

The number of factors is set to $k = 20$ and the configuration used in (Campolongo et al., 2011) is applied:

$$\begin{aligned} \overline{x_i} = \{&0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \\ &1, 2, 2, 2, 3, 3, 1.5, 3, 2, 2\} \\ \sigma_i = \{&0.5, 0.5, 1, 1, 2, 2, 1, 0.5, 1.5, 2, \\ &2, 2, 1, 1, 1, 3, 3, 3, 5, 5\} \end{aligned} \quad (14)$$

• The simulation of the air-launch-to-orbit separation is used to evaluate the risk of collision. The output is the minimum distance between the launcher and the aircraft over all the time steps. It depends on 49 factors uniformly distributed. 48 of them are classic scalar factors and represent for example a mass, an inertia, a distance, or a time. The last factor is multidimensional and represents the evolution of the white noise used with the Dryden model for the wind turbulences. Using multi-dimensional factors in the Morris method is complex and leads to a variability of the results. Thus, any improvement of the Morris method is particularly important.

3.3 Results of the test functions

The classic random generation of trajectories and radial points is called "Normal", while the new improvement presented in Section 2.2 is called "Distinct". The methods based on the verification of the Euclidian and Manhattan distance between trajectories are called "Euclidian" and "Manhattan". These four methods are tested with $n = 4$, 6, and 8 trajectories, and the Normal and Distinct methods are also tested with the same numbers of radial points. In the Euclidian and Manhattan methods, the number of initial trajectories is set to $N = 25$ for $n = 4$, and $N = 15$ for $n = 6$ or $n = 8$. For each test, 1000 applications are used for the Normal and Distinct methods, while only 100 are used for the Euclidian and Manhattan methods because of the time they need. The quality indicator R requires the Sobol's total indexes. They are obtained with the Sobol's first order indexes by running 86016 times the functions based on $k = 20$ factors, and 36864 times the function based on $k = 8$ factors.

Table 1. Average and maximum values of R for trajectories - Minima underlined

Function		**Normal (1000app.)**			**Distinct (1000app.)**			**Euclidian (100app.)**			**Manhattan (100app.)**		
		4	**6**	**8**	**4**	**6**	**8**	**4**	**6**	**8**	**4**	**6**	**8**
Morris	avg	1240	733	522	1170	585	386	1010	702	493	969	643	450
	(max)	(17400)	(9730)	(4300)	(32900)	(5040)	(4580)	(4780)	(6460)	(3380)	(5850)	(2710)	(3460)
Sobol	avg	0.553	9490	4.77	2.61e-5	0.205	0.0617	0.329	95.8	0.737	0.240	388	0.774
	(max)	(36)	(8.28e6)	(1830)	(6.16e-4)	(9.87)	(0.977)	(5.69)	(3710)	(24.5)	(2.58)	(37600)	(16.2)
Modified Sobol	avg	7400	4950	4038	2270	2290	1640	6120	6850	3520	6890	4710	3050
	(max)	(1.81e5)	(1.65e5)	(1.44e5)	(38700)	(34000)	(29800)	(1.31e5)	(1.27e5)	(64100)	(1.01e5)	(34700)	(24600)
Bratley	avg	31.3	9.63	5.26	2.00	0.991	0.585	25.4	7.14	5.61	14.7	12.9	5.41
	(max)	(4310)	(709)	(578)	(109)	(63.6)	(20.7)	(362)	(95.0)	(110)	(369)	(211)	(282)
Satelli	avg	1170	607	424	6.94	3.64	2.85	909	480	418	770	362	325
	(max)	(19600)	(6830)	(4760)	(6.94)	(11.5)	(11.4)	(79400)	(3070)	(2600)	(4980)	(1470)	(1470)

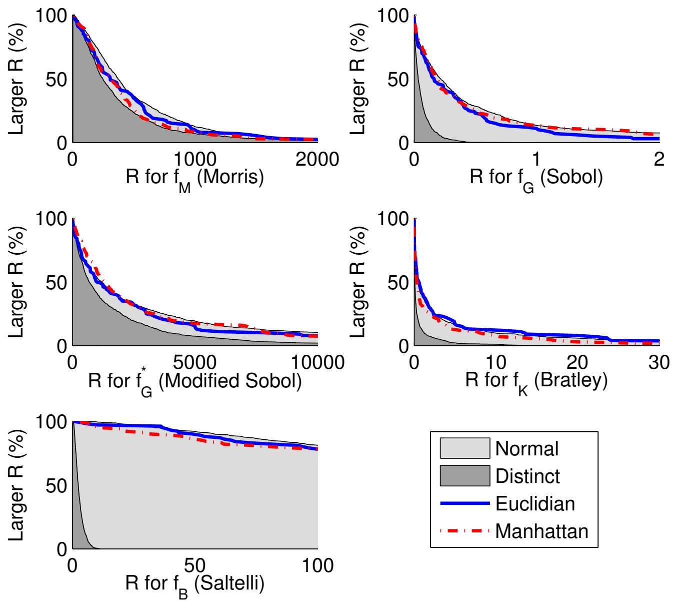

Fig. 5. Results with 8 trajectories

In Figure 5, the x-axis represents the estimator R and the y-axis represents the percentage of applications resulting in a larger R. The faster the curves decrease, the more efficient the corresponding method is. In Figure 5, the curve "Distinct" quickly decreases for the five test functions. The lowest benefit of the new improvement is obtained with the Morris function as the curve "Distinct" is close to the others, and its highest benefit is obtained with the Saltelli function as the curve "Distinct" decreases much faster than any other.

Table 1 shows the average and the maximum values of R for the tests based on trajectories. The best results, which have been underlined, are mostly obtained with the new Distinct improvement. It reduces all the average and maximum values of more than 50%, except for the Morris function. In several cases, the benefit of the Distinct method is considerable. For example, one of the average values for the Saltelli function changes from 1170 to 6.94. Using more applications for the Euclidian and Manhattan methods would simply be likely to increase the corresponding maxima. Table 1 also shows that using 6 trajectories instead of 4 is a major advantage for most of the functions, both for the classic and the Distinct methods. The Sobol function is an exception because of its special shape leading to very good results with 4 discrete values per factor and 4 trajectories.

Table 2. Average and maximum values of R for radial points - Minima underlined

Function		**Normal (1000app.)**			**Distinct (1000app.)**		
		4	**6**	**8**	**4**	**6**	**8**
Morris	avg	1250	676	505	1080	571	397
	(max)	(17900)	(6530)	(3490)	(12100)	(6400)	(4150)
Sobol	avg	3.17e-5	370	2.89	1.12e-5	0.210	0.0536
	(max)	(2.79e-4)	(56600)	(954)	(1.26e-4)	(15.1)	(0.420)
Modified Sobol	avg	4680	2770	2460	2500	2290	1430
	(max)	(1.66e5)	(49800)	(52100)	(64100)	(33300)	(35100)
Braltey	avg	13.2	5.68	2.57	2.15	0.794	0.891
	(max)	(2050)	(965)	(955)	(93.4)	(48.2)	(119)
Saltelli	avg	1160	609	424	6.94	3.67	2.79
	(max)	(21100)	(7120)	(3900)	(6.94)	(12.6)	(13.0)

Table 2 shows the results for the radial points. The best results are underlined. 29 out of 30 are obtained with the Distinct method. It also appears that 22 of the 30 results obtained with the Normal radial points are lower than with the Normal trajectories of Table 1. Using 6 radial points instead of 4 appears again as a good recommendation. The results of the Distinct radial points and the Distinct trajectories are equivalent. The Distinct radial points and trajectories both have lower results in half of the cases.

The non-parametric Kolmogorov-Smirnov test is used to compare the distributions of R obtained with the different methods. The null hypothesis is that the distributions are equivalent, and it is rejected at the 5% significance level. When the hypothesis is rejected, the differences which appear in the figures and tables cannot be considered as random. The null hypothesis is rejected when the Distinct method is compared to the Normal method for the different numbers of trajectories and radial points in 29 out of 30 cases. The case which is not rejected corresponds to the analysis of the Modified Sobol function with 6 Distinct or Normal radial points. The null hypothesis is also rejected in most of the cases when Distinct trajectories are compared to Euclidian or Manhattan trajectories. The Morris function is a first exception. The difference between the Distinct trajectories and the Euclidian or Manhattan trajectories is not clear for any number of trajectory with this function. Another exception reveals that the difference between 6 Distinct trajectories and 6 Manhattan trajectories is also not clear for the Modified Sobol function. The null hypothesis is conserved in 26 out of 30 cases when the Euclidian and Manhattan trajectories are compared to the Normal trajectories, and always conserved when the Euclidian and Manhattan trajectories are compared to each other.

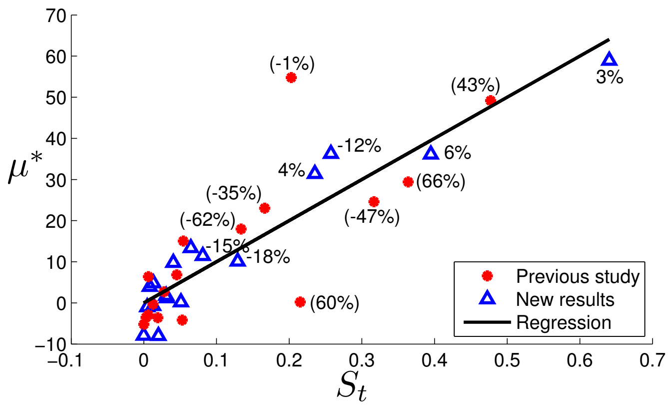

Fig. 6. Results for the store separation

3.4 Results of the store separation

A sensitivity analysis of the store separation problem had been carried out before the work presented in this paper. The Morris method was applied with 4 classic trajectories. It is now applied with 6 Distinct radial points. Indeed, the Distinct method showed particularly good results with the test functions, and the use of a 6 radial points or trajectories appeared as particularly effective. The radial points and the trajectories gave similar results with the Distinct method, but the radial points gave better results with the Normal method, which could be an argument for final selection.

The Morris method is always applied twice with the same radial points or trajectories but with different samples of the multi-dimensional factor. The results of the two applications are averaged and the relative difference $2(\mu^{*1} - \mu^{*2})/(\mu^{*1} + \mu^{*2})$ is shown as a percentage in Figure 6. The Sobol method is used as a reference, but it is only applied to the 17 main factors in order to limit the number of simulations required to 2304. Fixing the other factors has limited consequences as the μ^* of the 18^{th} factor is more than 10 times smaller than the μ^* of the 1^{st} factor in both studies. The new study first improved the identification of the 17 main factors. For example, the 7^{th} most important factor of the new study is not amongst the 17 main factors of the past study. As a consequence, the new study required a new application of the Sobol's method. Figure 6 shows that the points of the new study are closer to the regression line, revealing a better quality of the results. The new study avoids two errors from the past study, one overestimation and one underestimation of a factor. The indicator R equals 6180 for the past study and only 610 for the new study. Figure 6 finally shows that using a larger number of radial points also reduces the variability due to the multi-dimensional factors, shown in brackets.

The most important factor appears to be the interaction force on the y-axis of the launcher with a μ^* of 0.0199. A group of three factors, all related to the aerodynamics, also appears as particularly important with different μ^* in $[0.0125, 0.0138]$: the uncertainty on the center of pressure of the launcher, the modification of this center of pressure by the interactions, and the wind turbulences. Finally, a group of six factor also has a non-negligible μ^* in $[0.0066, 0.0076]$: two of them represent the initial orientation of the launcher under the aircraft, one represents the interaction force on the z-axis of the aircraft, and the last one represents the hooks' opening time. It would be necessary to design a guidance law robust to variations of these parameters.

4. CONCLUSION

This paper presents a method based on the Morris method improving the estimation of the main sensitivity factors of a system at no additional computational cost. Classic radial points also appear to be more effective than the more common classic trajectories, but the difference fades out when the Distinct method is applied. Six radial points or trajectories are much more efficient than four, in particular when multi-dimensional factors are involved. It raises the question of the convergence, that is how much the results can be improved with more calculations. The identification of the main factors of an air-launch-to-orbit separation has been improved, helping to the development of new robust controllers.

REFERENCES

Archer, G.E.B., Saltelli, A., and Sobol, I.M. (2007). Sensitivity measures,anova-like techniques and the use of bootstrap. *Journal of Statistical Computation and Simulation*, 58, 99–120.

Bratley, P., Fox, B.L., and Niederreiter, H. (1992). Implementation and tests of low-discrepancy sequences. *ACM Transactions on Modeling and Computer Simulation*, 2, 195–213.

Campolongo, F., Cariboni, J., and Saltelli, A. (2007). An effective screening design for sensitivity analysis of large models. *Environmental Modelling & Software*, 22, 1509–1518.

Campolongo, F., Saltelli, A., and Cariboni, J. (2011). From screening to quantitative sensitivity analysis. a unified approach. *Computer Physics Communications*, 182, 978–988.

Morris, M.D. (1991). Factorial sampling plans for preliminary computational experiments. *Technometrics*, 33, 161–174.

Ratto, M., Pagano, A., and Young, P. (2007). State dependent parameter metamodelling and sensitivity analysis. *Computer Physics Communications*, 177, 863–876.

Saltelli, A., Annoni, P., Azzini, I., Campolongo, F., Ratto, M., and Tarantola, S. (2010). Variance based sensitivity analysis of model output. design and estimator for the total sensitivity index. *Computer Physics Communications*, 181, 259–270.

Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M., and Tarantola, S. (2008). *Global Sensitivity Analysis: The Primer*. Wiley.

Saltelli, A., Tarantola, S., Campolongo, F., and Ratto, M. (2004). *Sensitivity Analysis in Practice - A Guide to Assessing Scientific Models*. Wiley.

Sobol, I.M. (2001). Global sensitivity indices for nonlinear mathematical models and their monte carlo estimates. *Mathematics and Computers in Simulation*, 55, 271–280.

van Griensven, A., Meixner, T., Grunwald, S., Bishop, T., Diluzio, M., and Srinivasan, R. (2006). A global sensitivity tool for the pparameter of multi-variable catchment models. *Journal of Hydrology*, 324, 10–23.