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A decentralized fault tolerant control strategy for multi-robot systems

Filippo Arrichiello, * **Alessandro Marino,** ** **Francesco Pierri** ***

* *University of Cassino and Southern Lazio, Via G. Di Biasio 43, 03043 Cassino (FR), Italy* email: f.arrichiello@unicas.it
** *University of Salerno, Via Giovanni Paolo II, 132, 84084, Salerno (SA), Italy* email: almarino@unisa.it
*** *University of Basilicata, Via dell'Ateneo Lucano 10, 85100 Potenza (PZ), Italy* email: francesco.pierri@unibas.it.

Abstract: The paper presents a fault tolerance control strategy for distributed multi-robot systems. The proposed approach is based on a distributed controller-observer architecture that allows each robot to estimate the global system state using local communication. We derive residual dynamics that allows each robot to detect and isolate faults of other robots, even if they are not directly connected. Then, maximum time values to detect the faults are used to implement a fault tolerant control strategy that, in case of fault of one of the robots, allows to reconfigure the team. Numerical simulation results are provided to validate the approach.

1. INTRODUCTION

Networked robotic systems have been widely investigated in the last decade due to their flexibility, large application domain and their capability to accomplish complex missions not possible with a single unit. However, when considering more units, the probability of occurrence of a fault of one of the robots increases accordingly and, to accomplish the assigned mission even with a reduced number of agents, a proper strategy to make the system tolerant to faults is required. Such a strategy requires, at first, the adoption of a Fault Detection and Isolation (FDI) schema to detect the occurrence of a fault and identify the faulty robot; then, a Fault Tolerant Control (FTC) strategy is required to make the system able to accomplish the mission by handling the faults.

Despite several FDI and FTC approaches have been presented for single unit systems, very few approaches have been designed for the case of decentralized multi-robot systems. In Wang et al. [2009] a FDI scheme for networked systems is presented where a centralized station collects information about actuators and sensors of the robots, and detects and isolates faults over the network. A comparison between a centralized and decentralized architectures is presented in Meskin and Khorasani [2009], where the diagnosis problem is formulated in terms of isolability index for a given family of fault signatures. Works Ferrari et al. [2009] and Zhang and Zhang [2012] present a bank of local adaptive observers where each observer uses only measurements and information from neighboring subsystems and allows to detect and isolate faults in interconnected subsystems. Unknown Input Observers (UIOs) are proposed in Shames et al. [2011] for the FDI of networks of interconnected systems controlled with a decentralized control law. Most of the proposed approaches allow a healthy unit to detect and isolate the faulty ones only if the latter are directly connected to the former, i.e., they can communicate or sense each other. This limitation prevents these solutions from being used in case the control law of each robot depends on the state of all the others. In this case, it is required all robots are able to detect and isolate faulty units in order to re-arrange their control laws to accommodate the fault. Voulgaris and Jiang [2004] find conditions so that a controller, distributed under n nodes, guarantees pre-specified performance levels taking into account nodal failures and communication noise. In Xiao-Zheng and Guang-Hong [2009], a class of distributed state feedback controllers is constructed to automatically compensate the fault and the disturbance effects that are adaptively estimated. In Panagi and Polycarpou [2011], multiple faults occurring in local subsystem dynamics and the interconnections between the subsystems are compensated via adaptive laws. In Fonti et al. [2011], a FDI strategy is designed for fleet of underwater gliders where, in presence of a faulty vehicle, the formation automatically rearranges if the recovery is not possible.

The approach proposed in this paper originates from a distributed controller-observer architecture derived in Antonelli et al. [2011, 2013b,a] and that allows formation control of a multi-robot system by making each robot of the team able to estimate the global system state using local communication. In Arrichiello et al. [2013], we developed a FDI schema for the above mentioned controller-observer schema that allows each robot to detect and isolate faults on board of other robots. Here, we analytically derive the residual dynamics that allows each robot to detect and isolate faults on board of other robots, and we design a proper fault-tolerance strategy that consists in reducing the observer size by excluding the faulty vehicles from the team. Numerical simulation results are provided to validate the approach.

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2. DECENTRALIZED OBSERVER-CONTROLLER

Let us consider a system composed of N robots, where the ith robot's state is denoted by $\boldsymbol{x}_i \in \mathbb{R}^n$. Each robot is characterized by a single-integrator dynamics

$$\dot{\boldsymbol{x}}_i = \boldsymbol{u}_i + \boldsymbol{\phi}_i, \tag{1}$$

where $\boldsymbol{u}_i \in \mathbb{R}^n$ is the input vector and $\boldsymbol{\phi}_i \in \mathbb{R}^n$ is an additive fault term that is zero in normal operating conditions. The collective state is given by $\boldsymbol{x} = \begin{bmatrix}\boldsymbol{x}_1^{\mathrm{T}} \ \dots \ \boldsymbol{x}_N^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{Nn}$ and the collective dynamics is then expressed as

$$\dot{\boldsymbol{x}} = \boldsymbol{u} + \boldsymbol{\phi}, \tag{2}$$

where $\boldsymbol{u} = \begin{bmatrix}\boldsymbol{u}_1^{\mathrm{T}} \ \dots \ \boldsymbol{u}_N^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$ and $\boldsymbol{\phi} = \begin{bmatrix}\boldsymbol{\phi}_1^{\mathrm{T}} \ \dots \ \boldsymbol{\phi}_N^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$ are respectively the collective input and fault vectors.

It is supposed that each agent has access to a noisy measure $\boldsymbol{x}_{i,m}$ of its own state:

$$\boldsymbol{x}_{i,m} = \boldsymbol{x}_i + \boldsymbol{\eta}_i\,, \tag{3}$$

where $\boldsymbol{\eta}_i \in \mathbb{R}^n$ is the additive noise, assumed norm-bounded by a positive scalar, $\overline{\eta}$, i.e.,

$$\|\boldsymbol{\eta}_i\| \le \overline{\eta} \quad \forall i = 1, 2, \dots, N. \tag{4}$$

The collective noisy measure of the system state is

$$\boldsymbol{x}_m = \boldsymbol{x} + \boldsymbol{\eta}\,, \tag{5}$$

where $\boldsymbol{\eta} = [\boldsymbol{\eta}_1^{\mathrm{T}} \dots \boldsymbol{\eta}_N^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{Nn}$ is the collective noise.

The information exchange between the robots is described by a graph $\mathcal{G}(\mathcal{E}, \mathcal{V})$ characterized by its topology, i.e., the set $\mathcal{V}$ of the indexes labeling the N vertices (nodes), the set of edges (arcs) $\mathcal{E} = \mathcal{V} \times \mathcal{V}$ connecting the nodes, and its $\boldsymbol{L}(N \times N)$ Laplacian matrix. We assume that the i th robot receives information from a reduced set of neighbors $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$, and it does not know the overall topology. Some properties and definitions about the communication graphs, used in the following, are listed in Antonelli et al. [2013b], while more details can be found in Mesbahi and Egerstedt [2010].

Each vehicle runs a local observer, that uses only local information and a suitable vector from neighbour vehicles, in order to estimate the overall team state [Arrichiello et al. 2013]. It is worth noting that the same observer is adopted both for control purposes and for the FDI strategy without increasing the information exchange burden.

Let $\boldsymbol{\Gamma}_i$ be a $(n \times Nn)$ selection matrix

$$\boldsymbol{\Gamma}_i = \{\boldsymbol{O}_n \ \cdots \ \underbrace{\boldsymbol{I}_n}_{i \text{ th node}} \ \cdots \ \boldsymbol{O}_n\}$$

that allows to extract the components of the i th robot from a collective vector and let $\boldsymbol{\Pi}_i$ be the $(Nn \times Nn)$ matrix $\boldsymbol{\Pi}_i = \boldsymbol{\Gamma}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i$. The estimate of the collective state $\boldsymbol{x}$ is computed by the i th robot by using the observer

$${}^i\dot{\hat{\boldsymbol{x}}} = k_o \left(\sum_{j \in \mathcal{N}_i} \left({}^j\hat{\boldsymbol{y}} - {}^i\hat{\boldsymbol{y}}\right) + \boldsymbol{\Pi}_i \left(\boldsymbol{y}_m - {}^i\hat{\boldsymbol{y}}\right) \right) + {}^i\hat{\boldsymbol{u}}, \tag{6}$$

where $k_o > 0$ is a scalar gain; $\boldsymbol{y}_m = \boldsymbol{x}_m - \int_{t_0}^{t} \boldsymbol{u}(\tau)d\tau$, and ${}^i\hat{\boldsymbol{y}} = {}^i\hat{\boldsymbol{x}} - \int_{t_0}^{t} {}^i\hat{\boldsymbol{u}}(\tau)d\tau$, where t_0 is the initial time instant; ${}^i\hat{\boldsymbol{u}}$ is the estimate of the collective input elaborated by the i th robot on the base of its estimate of the collective state and of the control law; clearly, it is $\boldsymbol{u}_i = \boldsymbol{\Gamma}_i{}^i\hat{\boldsymbol{u}}$.

It is worth noticing that (6) depends only on local information available to vehicle i, and that each observer is updated using only the estimates ${}^j\hat{\boldsymbol{y}}$ received from direct neighbors. Thus, ${}^j\hat{\boldsymbol{y}} \in \mathbb{R}^{Nn}$ is the only information that is required to be exchanged among neighbors.

The collective estimation dynamics, in the absence of faults ($\boldsymbol{\phi}_i = \boldsymbol{0}$, $i = 1, 2, \dots, N$), is

$$\dot{\hat{\boldsymbol{x}}}^{\star} = -k_o \boldsymbol{L}^{\star}\hat{\boldsymbol{y}}^{\star} + k_o \boldsymbol{\Pi}^{\star}\tilde{\boldsymbol{y}}^{\star} + k_o \boldsymbol{\Pi}^{\star}\boldsymbol{\eta}^{\star} + \hat{\boldsymbol{u}}^{\star}, \tag{7}$$

where $\boldsymbol{L}^{\star} = \boldsymbol{L} \otimes \boldsymbol{I}_{Nn}$, with $\otimes$ denoting the Kronecker product operator and

$$\boldsymbol{\Pi}^{\star} = \mathrm{diag}\left\{[\boldsymbol{\Pi}_1 \ \dots \ \boldsymbol{\Pi}_N]\right\}, \tag{8}$$

$\hat{\boldsymbol{x}}^{\star} = [{}^1\hat{\boldsymbol{x}}^{\mathrm{T}} \dots {}^N\hat{\boldsymbol{x}}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{N^2 n}$, $\hat{\boldsymbol{y}}^{\star} = [{}^1\hat{\boldsymbol{y}}^{\mathrm{T}} \dots {}^N\hat{\boldsymbol{y}}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{N^2 n}$, $\hat{\boldsymbol{u}}^{\star} = \left[{}^1\hat{\boldsymbol{u}}(t, {}^1\hat{\boldsymbol{x}}) \dots {}^N\hat{\boldsymbol{u}}(t, {}^N\hat{\boldsymbol{x}})\right] \in \mathbb{R}^{N^2 n}$, $\boldsymbol{\eta}^{\star} = \boldsymbol{1}_N \otimes \boldsymbol{\eta}$ and $\tilde{\boldsymbol{y}}^{\star} = \boldsymbol{1}_N \otimes \boldsymbol{y} - \hat{\boldsymbol{y}}^{\star} \in \mathbb{R}^{N^2 n}$ with $\boldsymbol{y} = \boldsymbol{x} - \int_{t_0}^{t} \boldsymbol{u}(\tau)d\tau$.

2.1 The control objective and the feedback control law

The control objective and feedback control law considered in this work are inherited from Antonelli et al. [2011, 2013a,b]; here, we recall their essential concepts to make this paper self-contained.

The control objective is to make the team centroid and the relative formation follow desired time-varying references. The two tasks are represented via the task functions:

- the *centroid* of the system:

$$\boldsymbol{\sigma}_1(\boldsymbol{x}) = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{x}_i \ = \boldsymbol{J}_1\boldsymbol{x}, \tag{9}$$

  where $\boldsymbol{J}_1 \in \mathbb{R}^{n \times Nn}$ is the Jacobian of the task.
- the *formation* of the system, expressed as an assigned set of relative displacement between the robots:

$$\boldsymbol{\sigma}_2(\boldsymbol{x}) = \left[(\boldsymbol{x}_2 - \boldsymbol{x}_1)^{\mathrm{T}} \dots (\boldsymbol{x}_N - \boldsymbol{x}_{N-1})^{\mathrm{T}}\right]^{\mathrm{T}} = \boldsymbol{J}_2\boldsymbol{x}, \tag{10}$$

  where $\boldsymbol{J}_2 \in \mathbb{R}^{(N-1)n \times Nn}$ is the Jacobian of the task.

Let us combine both the tasks in a single vector $\boldsymbol{\sigma} \in \mathbb{R}^{Nn}$

$$\boldsymbol{\sigma} = \begin{bmatrix}\boldsymbol{\sigma}_1\\ \boldsymbol{\sigma}_2\end{bmatrix} = \begin{bmatrix}\boldsymbol{J}_1\\ \boldsymbol{J}_2\end{bmatrix}\boldsymbol{x} = \boldsymbol{J}\boldsymbol{x}, \ \dot{\boldsymbol{\sigma}} = \boldsymbol{J}\dot{\boldsymbol{x}}; \tag{11}$$

it can be easily shown that matrix $\boldsymbol{J} \in \mathbb{R}^{Nn \times Nn}$ is non singular and, then, invertible.

Each robot, using its estimate ${}^i\hat{\boldsymbol{x}}$, computes an estimate of the collective input via the feedback control law:

$${}^i\hat{\boldsymbol{u}} = {}^i\hat{\boldsymbol{u}}(t, {}^i\hat{\boldsymbol{x}}) = \boldsymbol{J}^{\dagger}\left[\dot{\boldsymbol{\sigma}}_d + k_c{}^i\tilde{\boldsymbol{\sigma}}({}^i\hat{\boldsymbol{x}})\right], \tag{12}$$

where

$${}^i\tilde{\boldsymbol{\sigma}}({}^i\hat{\boldsymbol{x}}) = \begin{bmatrix}{}^i\tilde{\boldsymbol{\sigma}}_1({}^i\hat{\boldsymbol{x}})\\ {}^i\tilde{\boldsymbol{\sigma}}_2({}^i\hat{\boldsymbol{x}})\end{bmatrix} = \begin{bmatrix}\boldsymbol{\sigma}_{1,d} - {}^i\boldsymbol{\sigma}_1({}^i\hat{\boldsymbol{x}})\\ \boldsymbol{\sigma}_{2,d} - {}^i\boldsymbol{\sigma}_2({}^i\hat{\boldsymbol{x}})\end{bmatrix},$$

is the estimate of the task error $\tilde{\boldsymbol{\sigma}} = [\tilde{\boldsymbol{\sigma}}_1(\boldsymbol{x})^{\mathrm{T}} \ \tilde{\boldsymbol{\sigma}}_2(\boldsymbol{x})^{\mathrm{T}}]^{\mathrm{T}}$. The input vector $\boldsymbol{u}_i$ to robot i is computed selecting the relative component from ${}^i\hat{\boldsymbol{u}}$, i.e., $\boldsymbol{u}_i = {}^i\hat{\boldsymbol{u}}_i = \boldsymbol{\Gamma}_i{}^i\hat{\boldsymbol{u}}$.

2.2 Convergence of the observer-controller schema

First of all, the exponentially convergence to the origin of $\tilde{\boldsymbol{y}}^{\star}$, in absence of error and measurement noise, is proven by the following theorem.

Theorem 1. In the presence of a directed strongly connected communication graph (or connected undirected graph) and in the absence of faults ($\boldsymbol{\phi}_i = \boldsymbol{0}$, $i = 1, 2, \dots, N$)

and measurement noise (i.e., $\boldsymbol{\eta}_i = \mathbf{0}, i = 1, 2, \ldots, N$, and $\boldsymbol{y} = \boldsymbol{y}_m$), with the update law in eq. (7), $\tilde{\boldsymbol{y}}^\star$ is exponentially convergent to the origin.

proof 1. The proof can be found in Arrichiello et al. [2013].

In the presence of bounded measurement noise, the term $k_o \boldsymbol{\Pi}^\star \boldsymbol{\eta}^\star$ in (7) can be viewed as a non-vanishing bounded perturbation whose upper bound is given by

$$\|k_o \boldsymbol{\Pi}^\star \boldsymbol{\eta}^\star\| \leq \|k_o \boldsymbol{\eta}\| \leq k_o \sqrt{N} \overline{\eta}. \tag{13}$$

The exponential stability of the origin of the nominal system ensures that the solutions of the perturbed system are globally uniformly ultimately bounded (see Lemma 9.2 p.347 in Khalil [2002]).

Theorem 2. In the presence of a directed strongly connected communication graph (or connected undirected graph) and in the absence of faults and measurement noise (i.e., $\boldsymbol{\phi}_i = \mathbf{0}, \boldsymbol{\eta}_i = \mathbf{0}, i = 1, 2, \ldots, N$), with the update law in eq. (7), the stacked vector of the collective state estimation errors, $\tilde{\boldsymbol{x}}^\star$, is exponentially convergent to the origin.

proof 2. The proof can be found in Arrichiello et al. [2013].

Again, in the presence of bounded measurement noise, by resorting to the Lemma 9.2 in Khalil [2002], the exponential stability of the origin of the nominal system ensures that the solutions of the perturbed system are globally uniformly ultimately bounded.

Remark 2.1. In Antonelli et al. [2011] has been proven that the exponential stability of the observer leads also to the exponential stability of the task errors $\tilde{\boldsymbol{\sigma}}_l$ $(l = 1, 2)$ with the control law in eq. (12).

3. FAULT DETECTION AND ISOLATION

In order to detect the occurrence of a fault, let us define for the robot ith $(i = 1, \ldots, N)$, the following residual vector

$$^{i}\boldsymbol{r} = \sum_{j \in \mathcal{N}_i} \left({}^{j}\hat{\boldsymbol{y}} - {}^{i}\hat{\boldsymbol{y}} \right) + \boldsymbol{\Pi}_i (\boldsymbol{y}_m - \hat{\boldsymbol{y}}_i); \tag{14}$$

the above quantity does not require additional information exchange since it makes use of the same quantity used in the local state observer. The vector ${}^{i}\boldsymbol{r}$ can be seen as a stacked vector, i.e., ${}^{i}\boldsymbol{r} = \left[{}^{i}\boldsymbol{r}_1^{\mathrm{T}}, {}^{i}\boldsymbol{r}_2^{\mathrm{T}}, \ldots, {}^{i}\boldsymbol{r}_N^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{R}^{Nn}$, where each component ${}^{i}\boldsymbol{r}_k \in \mathbb{R}^n$ represents the residual computed by robot i relative to robot k, and it can be expressed as

$$^{i}\boldsymbol{r}_k = (d_i + {}^{i}\delta_k) {}^{i}\tilde{\boldsymbol{y}}_k + \sum_{j \in \mathcal{N}_i} {}^{j}\tilde{\boldsymbol{y}}_k + {}^{i}\delta_k \boldsymbol{\eta}_k, \tag{15}$$

where ${}^{i}\delta_k$ is 1 if $i = k$, 0 otherwise, and d_i is the dimension of $\mathcal{N}_i$ (i.e., the indegree of node i).

The collective residual vector $\boldsymbol{r}^\star = \left[{}^{1}\boldsymbol{r}^{\mathrm{T}} \ \ldots \ {}^{N}\boldsymbol{r}^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{R}^{N^2 n}$ can be expressed as

$$\boldsymbol{r}^\star = \tilde{\boldsymbol{L}}^\star \tilde{\boldsymbol{y}}^\star + \boldsymbol{\Pi}^\star \boldsymbol{\eta}^\star. \tag{16}$$

From Theorem 1 and (16), it is straightforward to derive that, in the absence of faults and measurement noise, $\boldsymbol{r}^\star$ converges exponentially to zero, while, in presence of bounded noise, the collective residual is bounded as well.

3.1 Adaptive thresholds

In the presence of nonzero initial observer estimation errors and measurement noise, the residuals in eq. (14) can be different from zero even in the absence of faults. To avoid the occurrence of false alarms, adaptive thresholds can be defined and, then, the decision about the occurrence of a fault is made when a residual exceeds such thresholds.

In order to choose the thresholds, let us consider the vector $\tilde{\boldsymbol{y}}_k^\star = \left[{}^{1}\tilde{\boldsymbol{y}}_k^{\mathrm{T}}, {}^{2}\tilde{\boldsymbol{y}}_k^{\mathrm{T}}, \ldots, {}^{N}\tilde{\boldsymbol{y}}_k^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{R}^{Nn}$, collecting the estimation errors of $\boldsymbol{y}_k$ computed by the observers of each robot. It can be easily shown that it holds:

$$\tilde{\boldsymbol{y}}_k^\star = \mathrm{diag}\{\boldsymbol{\Gamma}_k, \boldsymbol{\Gamma}_k, \ldots, \boldsymbol{\Gamma}_k\} \tilde{\boldsymbol{y}}^\star = \boldsymbol{\Gamma}_k^\star \tilde{\boldsymbol{y}}^\star.$$

From Arrichiello et al. [2013], the dynamics of $\tilde{\boldsymbol{y}}_k^\star$ is

$$\begin{aligned} \dot{\tilde{\boldsymbol{y}}}_k^\star &= \boldsymbol{\Gamma}_k^\star \dot{\tilde{\boldsymbol{y}}}^\star = -k_o \boldsymbol{\Gamma}_k^\star \tilde{\boldsymbol{L}}^\star \tilde{\boldsymbol{y}}^\star - k_o \boldsymbol{\Gamma}_k^\star \boldsymbol{\Pi}^\star \boldsymbol{\eta}^\star + \boldsymbol{\Gamma}_k^\star \boldsymbol{\phi}^\star \\ &= -k_o \tilde{\boldsymbol{L}}_k^\star \tilde{\boldsymbol{y}}_k^\star - k_o \boldsymbol{\Pi}_k \boldsymbol{\eta} + \mathbf{1}_N \otimes \boldsymbol{\phi}_k. \end{aligned} \tag{17}$$

where $\tilde{\boldsymbol{L}}_k^\star = \boldsymbol{L} \otimes \boldsymbol{I}_n + \boldsymbol{\Pi}_k$ has all its eigenvalues in the right half-plane when the communication graph is strongly connected (this can be proven analogously to the case of $\tilde{\boldsymbol{L}}^\star$). Thus, system (17) is asymptotically stable and its solution is

$$\begin{aligned} \tilde{\boldsymbol{y}}_k^\star(t) &= e^{-\tilde{\boldsymbol{L}}_k^\star t} \tilde{\boldsymbol{y}}_k^\star(0) + \\ &+ \int_0^t e^{-\tilde{\boldsymbol{L}}_k^\star (t-\tau)} \left(\mathbf{1}_N \otimes \boldsymbol{\phi}_k(\tau) - k_o \boldsymbol{\Pi}_k \boldsymbol{\eta}(\tau) \right) d\tau. \end{aligned} \tag{18}$$

Since $\tilde{\boldsymbol{L}}_k^\star$ is Hurwitz, there exists a constant $\lambda > 0$ such as

$$\|e^{-\tilde{\boldsymbol{L}}_k^\star t}\| \leq e^{-\lambda t}, \tag{19}$$

therefore, in the absence of faults, the following bound for $\tilde{\boldsymbol{y}}_k^\star(t)$ can be derived

$$\begin{aligned} \|\tilde{\boldsymbol{y}}_k^\star(t)\| &\leq \left\| e^{-\tilde{\boldsymbol{L}}_k^\star t} \tilde{\boldsymbol{y}}_k^\star(0) \right\| + \int_0^t \left\| e^{-\tilde{\boldsymbol{L}}_k^\star (t-\tau)} k_o \boldsymbol{\Pi}_k \boldsymbol{\eta}(\tau) \right\| d\tau \\ &\leq \|\tilde{\boldsymbol{y}}_k^\star(0)\| e^{-\lambda t} + \frac{\sqrt{N} k_o \overline{\eta}}{\lambda} \left(1 - e^{-\lambda t} \right). \end{aligned} \tag{20}$$

By virtue of (15) the following chain of inequalities holds

$$\begin{aligned} \left\| {}^{i}\boldsymbol{r}_k \right\| &\leq (d_i + {}^{i}\delta_k) \left\| {}^{i}\tilde{\boldsymbol{y}}_k \right\| + d_i \|\tilde{\boldsymbol{y}}_k^\star\| + {}^{i}\delta_k \|\boldsymbol{\eta}_k\| \\ &\leq (2d_i + {}^{i}\delta_k) \|\tilde{\boldsymbol{y}}_k^\star\| + {}^{i}\delta_k \overline{\eta}. \end{aligned} \tag{21}$$

Thus, on the basis of (20) and (21), the following time-varying threshold ${}^{i}\mu_k$ can be defined for the residual ${}^{i}\boldsymbol{r}_k$

$$\begin{aligned} {}^{i}\mu_k(t) &= (2d_i + {}^{i}\delta_k) \|\tilde{\boldsymbol{y}}_k^\star(0)\| e^{-\lambda t} + \\ &+ \left[\frac{(2d_i + {}^{i}\delta_k)\sqrt{N} k_o}{\lambda} \left(1 - e^{-\lambda t} \right) + {}^{i}\delta_k \right] \overline{\eta}. \end{aligned} \tag{22}$$

The threshold calculation requires a reliable estimate of $\|\tilde{\boldsymbol{y}}_k^\star(0)\|$ and λ; the first one can be estimated on the basis of approximate information about the initial conditions of the system (e.g., the vehicles start from a known bounded area), while the latter can be estimated as the minimum eigenvalue of the matrix $\tilde{\boldsymbol{L}}_k^\star$ computed by considering the worst case for the Laplacian matrix.

3.2 Residuals in the presence of faults

Let us consider a fault occurring on the lth robot at time $t_f > 0$, namely $\boldsymbol{\phi} = \left[\mathbf{0}^{\mathrm{T}} \ldots \boldsymbol{\phi}_l^{\mathrm{T}} \ldots \mathbf{0}^{\mathrm{T}} \right]^{\mathrm{T}}$.

In order to analyze the influence of the fault on the residuals, it is worth deriving the dynamics of the vector ${}^i\tilde{\boldsymbol{y}}_k$ ($\forall i, k \in \{1, \ldots, N\}$) in (15). From (17), it is

$$ {}^i\dot{\tilde{\boldsymbol{y}}}_k = -{}^i\varrho_k \boldsymbol{I}_n {}^i\tilde{\boldsymbol{y}}_k + k_o \sum_{h\in\mathcal{N}_i} {}^h\tilde{\boldsymbol{y}}_k - k_o {}^i\delta_k \boldsymbol{\eta}_k + {}^l\delta_k \boldsymbol{\phi}_l. \quad (23) $$

where ${}^i\varrho_k = k_o(d_i + {}^i\delta_k)$.

Therefore, the expression of ${}^i\tilde{\boldsymbol{y}}_k$ is

$$ {}^i\tilde{\boldsymbol{y}}_k(t) = {}^i\boldsymbol{\chi}_k(t) {}^i\tilde{\boldsymbol{y}}_k(0) + k_o \int_0^t {}^i\boldsymbol{\chi}_k(t-\tau) \sum_{h\in\mathcal{N}_i} {}^h\tilde{\boldsymbol{y}}_k d\tau + \quad (24) $$

$$ -k_o \int_0^t {}^i\boldsymbol{\chi}_k(t-\tau) {}^i\delta_k \boldsymbol{\eta}_k d\tau + \int_{t_f}^t {}^i\boldsymbol{\chi}_k(t-\tau) {}^l\delta_k \boldsymbol{\phi}_l d\tau, $$

where ${}^i\boldsymbol{\chi}_k(t) = \exp(-{}^i\varrho_k \boldsymbol{I}_n t)$.

By folding (24) in (15), it can be argued that all the residuals ${}^i\boldsymbol{r}_k$ for all $i = 1, \ldots, N$, and for all $k \neq l$ (i.e., all the residuals referred to a robot different to the faulty one) are insensitive to the fault since the last term in (24) is null (${}^l\delta_k = 0$). Therefore the fault $\boldsymbol{\phi}_l$, affecting the lth robot, can be detected and isolated by the robot i if

$$ \begin{cases} \exists t > t_f : & \|{}^i\boldsymbol{r}_l(t)\| > {}^i\mu_l(t) \\ \forall k \in (1, \ldots, N), k \neq l, \forall t > 0, & \|{}^i\boldsymbol{r}_k(t)\| \leq {}^i\mu_k(t). \end{cases} \quad (25) $$

Moreover, from (24) and (15) it can be easily shown that the residuals ${}^i\boldsymbol{r}_l$ ($\forall i \in \{1, \ldots, N\}$) are affected by the fault $\boldsymbol{\phi}_l$ via the following term

$$ {}^i\boldsymbol{f}_l = (d_i + {}^i\delta_l) \int_{t_f}^t {}^i\boldsymbol{\chi}_l(t-\tau)\boldsymbol{\phi}_l d\tau + \sum_{j\in\mathcal{N}_i} \int_{t_f}^t {}^j\boldsymbol{\chi}_l(t-\tau)\boldsymbol{\phi}_l d\tau $$

$$ = \int_{t_f}^t \left[(d_i + {}^i\delta_l) {}^i\boldsymbol{\chi}_l(t-\tau) + \sum_{j\in\mathcal{N}_i} {}^j\boldsymbol{\chi}_l(t-\tau) \right] \boldsymbol{\phi}_l d\tau. \quad (26) $$

By straightforward calculations, omitted for the sake of brevity, the following sufficient detectability conditions can be derived

$$ \exists t > t_f : \left\|{}^i\boldsymbol{f}_l\right\| \geq 2\ {}^i\mu_l(t). \quad (27) $$

Remark 3.1. Since the residuals are decoupled in such a way that the fault $\boldsymbol{\phi}_l$ affects only the residuals ${}^i\boldsymbol{r}_l$, the proposed scheme is effective also in the presence of multiple faults affecting different robots.

4. FAULT RECOVERY STRATEGY

The presence of faulty robots might cause the mission failure. The developed FDI strategy allows to recognize these abnormal situations and to elaborate a proper recovery scheme. Different approaches could be adopted. In a first solution, the faulty vehicle locally modifies the control input in order to compensate the fault ($\boldsymbol{\phi}_i$ in (1)), of course it requires that the vehicle mobility is not completely compromised. In a more conservative solution, the faulty vehicle is removed from the team, and the remaining robots reorganize themselves in order to accomplish the mission. A combination of the two strategies is possible as well: once the fault has been detected and isolated, the faulty vehicle has a certain amount of time to recover itself and make the residuals return below the thresholds, after this time, it is excluded from the team. In this paper, the second solution is addressed.

4.1 Exclusion of the faulty vehicle

The exclusion of a faulty vehicle from the team requires to resize the dimension of all the involved variables: in particular, each healthy robot must resize the estimate of the collective state (${}^i\hat{\boldsymbol{x}}$) and input (${}^i\hat{\boldsymbol{u}}$) as well as the exchanged variable ${}^i\hat{\boldsymbol{y}}$, whose dimensions become $(N-1)n$ instead of Nn.

The detection time instant t in (25) is, in general, different for each robot, since each robot detects the fault asynchronously from the others. Thus, let us suppose that the ith vehicle has detected a fault on robot lth (i.e., $\left\|{}^i\boldsymbol{r}_l\right\| > {}^i\mu_l$) at time t_i. Then, the ith vehicle resizes the vector ${}^i\hat{\boldsymbol{y}}$ to be exchanged with its neighbors. If one or more of these neighbors has not yet detected the fault, they would receive a reduced size vector from the robot i but they have not any information about the faulty teammate, so they do not know which components have to be removed from their collective state estimate. This issue does not arise if the neighbors already know which one is the faulty vehicle, therefore the idea is to allow a vehicle to exchange a reduced size vector only when all the other healthy teammates have detected and isolated the fault. To this purpose, let us define the time ${}^it_d$ as the first instant at which $\left\|{}^i\boldsymbol{r}_l\right\| > {}^i\mu_l$ in (25), i.e.,

$$ {}^it_d = \min_t \left\|{}^i\boldsymbol{r}_l(t)\right\| > {}^i\mu_l(t). $$

and let be ${}^i\Delta t_d = {}^it_d - t_f$ as the time occurring from the fault occurrence and its detection/isolation by vehicle ith. Moreover, we define

$$ \Delta_{d,max} = \max_{i=1,\ldots,N} {}^i\Delta t_d \quad (28) $$

as the maximum detection delay. It is obvious that if robot i sends a reduced size vector starting from ${}^it_d + \Delta_{d,max}$, all the neighbors have complete knowledge about the faulty vehicle as well, and can resize their state estimate accordingly. In sum, the resize of the team occurs after

$$ t_r = \min_i {}^it_d + \Delta_{d,max}. \quad (29) $$

Thus, the problem is how to get a reliable estimate of $\Delta_{d,max}$. The solution of this problem is described in detail in the following section.

4.2 Estimate of the maximum detection delay

The estimate of $\Delta_{d,max}$ in (28) can depend on the particular fault $\boldsymbol{\phi}(t)$ occurred. In the following, the case of instantaneous and constant fault is considered, then a consideration is made for a more general class of faults. To this aim, given the fault instant t_f and a constant fault $\boldsymbol{\phi}_l(t) = \overline{\boldsymbol{\phi}}_l$ affecting the lth robot, we are looking for the time needed for condition (27) to be satisfied (in the particular case, $\left\|{}^i\boldsymbol{f}_l\right\| \geq {}^i\mu_l$ ($i = 1, 2, \ldots, N$)).

In this case, the right-hand side of (26) can be written as

$$ {}^i\boldsymbol{f}_l = \frac{(d_i + {}^i\delta_l)}{{}^i\varrho_l} \left(\boldsymbol{I}_n - {}^i\boldsymbol{\chi}_l(t-t_f)\right) \overline{\boldsymbol{\phi}}_l + \sum_{j\in\mathcal{N}_i} \frac{\boldsymbol{I}_n - {}^j\boldsymbol{\chi}_l(t-t_f)}{{}^j\varrho_l} \overline{\boldsymbol{\phi}}_l. \quad (30) $$

By taking into account that

$$ {}^i\boldsymbol{\chi}_l(t-t_f) = e^{(-{}^i\varrho_k \boldsymbol{I}_n (t-t_f))} = e^{(-{}^i\varrho_k (t-t_f))} \boldsymbol{I}_n, $$

the following holds

$$\begin{aligned}\|{}^i\boldsymbol{f}_l\| &= \left(\frac{(d_i+{}^i\delta_l)\left(1-e^{-{}^i\varrho_k(t-t_f)}\right)}{{}^i\varrho_l} + \sum_{j\in\mathcal{N}_i}\frac{1-e^{-{}^j\varrho_k(t-t_f)}}{{}^j\varrho_l}\right)\|\overline{\boldsymbol{\phi}}_l\| \\ &\geq \left(\frac{\left(1-e^{-k_o(t-t_f)}\right)}{k_o} + \frac{d_i\left(1-e^{-k_o(t-t_f)}\right)}{k_oN}\right)\|\overline{\boldsymbol{\phi}}_l\| \\ &= \frac{N+d_i}{k_oN}\|\overline{\boldsymbol{\phi}}_l\| - \left(\frac{N+d_i}{k_oN}e^{k_ot_f}\|\overline{\boldsymbol{\phi}}_l\|\right)e^{-k_ot}. \quad (31)\end{aligned}$$

Let us rearrange, now, the threshold in (22) as

$$ {}^i\mu_l(t) = \alpha e^{-\lambda t} + \beta\,, \quad (32)$$

where

$$\alpha = (2d_i + {}^i\delta_k)\,\|\tilde{\boldsymbol{y}}_k^\star(0)\| - \frac{(2d_i+{}^i\delta_k)\sqrt{N}k_o\overline{\eta}}{\lambda},$$

$$\beta = \left(\frac{(2d_i+{}^i\delta_k)\sqrt{N}k_o}{\lambda} + {}^i\delta_k\right)\overline{\eta}.$$

Based on results in (31), condition (27) becomes

$$2\alpha e^{-\lambda t} + \left(\frac{N+d_i}{k_oN}e^{k_ot_f}\|\overline{\boldsymbol{\phi}}_l\|\right)e^{-k_ot} \leq \frac{N+d_i}{k_oN}\|\overline{\boldsymbol{\phi}}_l\| - 2\beta; \quad (33)$$

by defining $\gamma = \min\{k_o, \lambda\}$ it is

$$\begin{aligned}&2\alpha e^{-\lambda t} + \left(\frac{N+d_i}{k_oN}e^{k_ot_f}\|\overline{\boldsymbol{\phi}}_l\|\right)e^{-k_ot} \leq \left(2\alpha + \frac{N+d_i}{k_oN}e^{k_ot_f}\|\overline{\boldsymbol{\phi}}_l\|\right)e^{-\gamma t} \\ &\leq \frac{N+d_i}{k_oN}\|\overline{\boldsymbol{\phi}}_l\| - 2\beta. \quad (34)\end{aligned}$$

Finally, the instant t in (27) is

$$ {}^it_d \geq -\frac{1}{\gamma}\ln\frac{\frac{N+d_i}{k_oN}\|\overline{\boldsymbol{\phi}}_l\| - 2\beta}{\left(2\alpha + \frac{N+d_i}{k_oN}e^{k_ot_f}\|\overline{\boldsymbol{\phi}}_l\|\right)}, \ \forall i \quad (35)$$

and

$$\Delta_{d,i} \geq -\frac{1}{\gamma}\ln\frac{\frac{N+d_i}{k_oN}\|\overline{\boldsymbol{\phi}}_l\| - 2\beta}{\left(2\alpha + \frac{N+d_i}{k_oN}e^{k_ot_f}\|\overline{\boldsymbol{\phi}}_l\|\right)} - t_f.$$

It can be shown that $\Delta_{d,i}$ monotonically decreases with respect to d_i; thus, the detection time decreases with respect to the in-degree (d_i). This agrees with the intuition that the less the network is connected the less the information flows through the network and more difficult is the detection of a fault. Moreover, concerning t_f, $\Delta_{d,i}$ reaches its maximum for $t_f = 0$ (the initial instant); this is in accordance with the fact that a fault in the initial phase requires a larger time to be detected, because of the larger value of the adaptive thresholds ${}^i\mu_l$ in this phase.
In our analysis, the worst case is considered ($d_i = 1$, $t_f = 0$) for the computation of $\Delta_{d,max}$, i.e.,

$$\Delta_{d,max} = -\frac{1}{\gamma}\ln\frac{\frac{N+1}{k_oN}\|\overline{\boldsymbol{\phi}}_l\| - 2\beta}{\left(2\alpha + \frac{N+1}{k_oN}\|\overline{\boldsymbol{\phi}}_l\|\right)}. \quad (36)$$

Remark 4.1. Equation (34) provides also a sufficient condition for a fault to be detected; in particular it is required:

$$\|\overline{\boldsymbol{\phi}}_l\| \geq 2\frac{k_oN}{N+d_i}\beta; \quad (37)$$

since β linearly depends on the noise bound, $\overline{\eta}$, the larger the value of $\overline{\eta}$, the larger $\|\overline{\boldsymbol{\phi}}_l\|$ is required.

Remark 4.2. In the case of non constant faults, condition (36) can still be used, since it still holds for any fault whose absolute value is greater than $\overline{\phi}_l$ for a time greater or equal to the time needed for the fault to be detected ($\Delta_{d,max}$).

5. NUMERICAL SIMULATIONS

A team of 5 robots ($N = 5$) moving in the plane ($n = 2$) is commanded to track a time-varying centroid while keeping a circular formation. Each vehicle implements the observer-controller scheme presented in Section 2. In particular, gains k_o, k_c in (6) and (12) were set to 5 and 3, respectively. The measurement noise $\boldsymbol{\eta}_i$ in (3) is assumed to be a normally distributed random vector with null mean, uncorrelated components and standard deviation of 0.03 m. The network topology is fixed directed and strongly connected. The desired trajectories of the centroid, $\boldsymbol{\sigma}_{1,d}(t)$ is time-varying, while the desired formation $\boldsymbol{\sigma}_{2,d}(t)$ is constant and it corresponds to a regular circular formation around the centroid with radius 0.3 m.
From instant $t_f = 60$ s, one of the vehicles (the second vehicle in our case) presents a constant fault given as::

$$\boldsymbol{\phi}_2(t) = \begin{cases}\boldsymbol{0} & \text{if } t < t_f \\ [0.5\ 0.5]^{\mathrm{T}} & \text{if } t \geq t_f.\end{cases} \quad (38)$$

Time $\Delta_{d,max}$ in Section 4.1 was set to 6 s. After the healthy vehicles have identified the faulty one, the latter is excluded from the team. The remaining vehicles keep tracking the same centroid while, for the formation, they are required to assume again a regular circular formation around the centroid with the same radius (0.3 m).
Figure 1 shows the residual norms $\|{}^i\boldsymbol{r}_2\|$ ($i = 1, 2, \ldots, N$) and the corresponding thresholds calculated by all the vehicles of the team and relative to the faulty robot, moreover, in each plot, it is also highlighted the instant t_f in which the fault occurs, the instant ${}^it_d$ in which the fault is detected and identified by vehicle i (red vertical line), and the time t_r in (29) in which the team is effectively resized. It is worth noticing that, after the team resize, all the residuals except $\|{}^2\boldsymbol{r}_2\|$, return below their thresholds since the data from the faulty robot are not fed to the resized observers. The residuals relative to an healthy robot are not shown for the sake of brevity; they remain always below the thresholds, therefore the fault is correctly isolated on the second vehicle.

In Figure 2 the team trajectories are plotted. In particular, the faulty vehicle (before and after the fault) is represented by a red marker. Moreover, the configurations at the fault instant t_f and at instant t_r are represented. As it can be seen, because of the fault, the second vehicle drives away form the team, while the remaining ones reconfigure themselves from instant t_r in order to reach again a regular polygon formation.

Finally, Figure 3 (top) shows the observer error $\|\tilde{\boldsymbol{x}}^\star\|$. It can be noticed that, in healthy condition ($t < t_f = 60$ s), the error exponentially approaches a neighborhood of the origin (depending on the bound $\overline{\eta}$ in (4)), then, in the time interval $[t_f,\, t_r]$, namely in interval between the fault occurrence and the team resize, the error grows up and finally it decreases again after the removal of the faulty vehicle from the team ($t > t_r$). The same behavior is exhibited by the centroid task error $\tilde{\boldsymbol{\sigma}}_1$ (Figure 3 (center)) and by the formation task error $\tilde{\boldsymbol{\sigma}}_2$ (Figure 3 (bottom)).

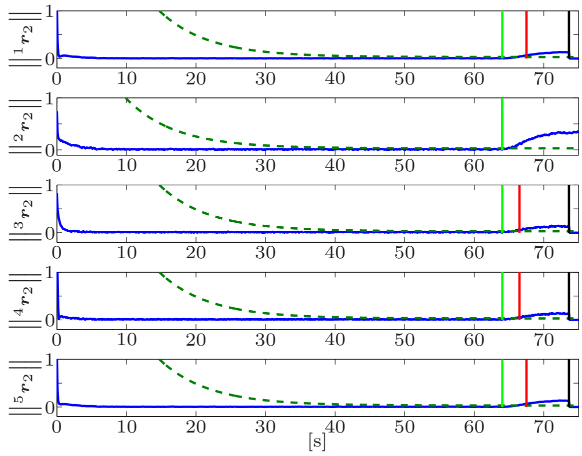

Fig. 1. Residuals $\|{}^i\boldsymbol{r}_2\|$ (solid blue lines) as calculated by vehicle i and relative to vehicle 2 (the faulty vehicle). Dashed green lines represent the thresholds, vertical green lines the fault instant t_d, vertical red lines the detection instants ${}^it_d$, vertical black lines the team resize time t_r as in (29).

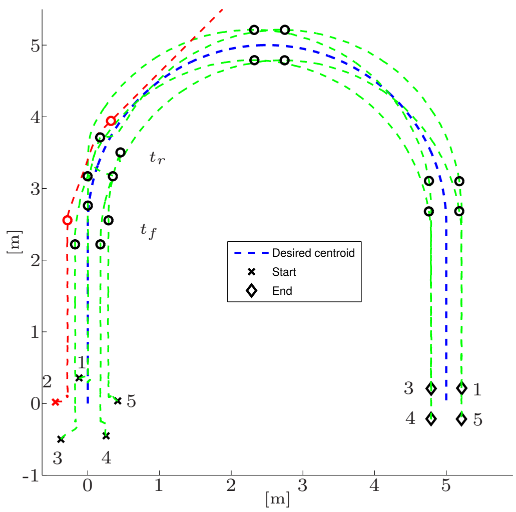

Fig. 2. Vehicle trajectories. The dashed blue line is the desired centroid path, while dashed green lines are the vehicles' paths. Vehicles' positions at instants t_f and t_r are shown. In red, the faulty vehicle.

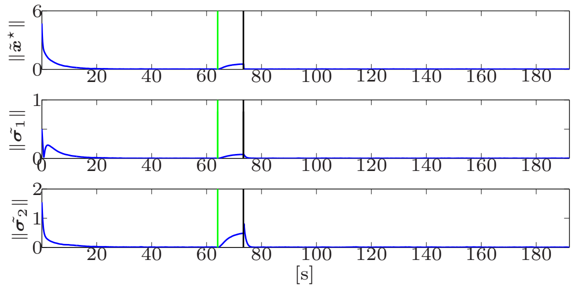

Fig. 3. Observer and task errors. Top. Observer error $\|\tilde{\boldsymbol{x}}^{\star}\|$. Center. Centroid task error. Bottom. Formation error. Vertical green lines are in correspondence of the fault instant t_f, vertical black lines are in correspondence of the team resize instant t_r as in (29).

6. CONCLUSIONS

A distributed fault tolerant scheme for multi-robot system was presented which is based on a properly designed observer-controller scheme. After the detection and isolation phase, a recovery technique was developed consisting in resizing the state vector by eliminating the faulty components. Experiments will be run as future work.

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