Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Robust Control of Dynamical Networks with Nonminimum Phase Agents[1]

Igor B. Furtat*

**Institute for Problems of Mechanical Engineering Russian Academy of Sciences, 61 Bolshoy ave V.O., St.-Petersburg, 199178, Russia (Tel: +7-812-321-47-66; e-mail: cainenash@mail.ru).*
**ITMO University, 49 Kronverkskiy ave, Saint Petersburg, 197101, Russia.*

Abstract: This paper describes the problem of a robust control for some class of network with nonminimum phase agents (subsystems). Only scalar input and output of each agents are available for measurement. Simulation results for network consisting of five agents with communication time delay are given.

1. INTRODUCTION

One of the fundamental assumptions for control of a parametric uncertainty single input single output (SISO) plant is an assumption of minimum phase plant model. Designing a stable controller is the main reason of this assumption (Fradkov et al., 1999).

Currently the problem for control of uncertainty SISO nonminimum phase plants has few solutions. The shunt method is used in Fradkov et al., 1999 and it is applicable only for linear plants. Moreover, this method is not effective for control of a plant with a disturbance because extended output is used, where extended output is equal to sum of the plant output and shunt output. The dynamic adaptive controller allowing get an extended model of the plant with vector input is designed in Hovakimyam and Calise, 2002. Such regulator compensates the positive zeros of the plant transfer function. Similar solution was designed in Tsykunov 2005. However, the methods of Hovakimyam and Calise, 2002, Tsykunov 2005 are effective only for stabilization of nonminimum phase plants without disturbances.

The control problem is more complicated if we control a lot of interconnected plants (network systems) (Das and Lewis, 2010, Fax and Murray, 2004, Li et al., 2009, Ren and Beard, 2005).

There are many examples of an important problem for control of the network systems (Das and Lewis, 2010, Fax and Murray, 2004, Li et al., 2009, Ren and Beard, 2005). There is a control of a multi-transmission and information processing, various transport networks, high-tech manufacturing networks, a complex crystal grid and nanostructured plants, etc. Among of control problems for networks with nonminimum phase agents we can note a coordinated motion aircraft control, control of underwater vehicles and mobile robots, distributed control systems and power grids control, etc. However, only dynamic networks with minimum phase agents are considered in Das and Lewis, 2010, Fax and Murray, 2004, Li et al., 2009, Ren and Beard, 2005.

The paper deals with the design of a robust control for some class of dynamic networks with nonminimum phase agents. We consider dynamic networks in conditions of parametric uncertainties and uncontrollable external disturbances. Only a scalar input and output of each local subsystem are available for measurement. We consider the digraph associated with the network, where each vertex of the digraph is associated with a corresponding node of the network and arcs of the digraph are associated with information connections between the network agents. Synthesis of the control law is based on the auxiliary loop method. First it is proposed for control of one plant in Tsykunov, 2007. This method is generalized for control of dynamic networks with minimum phase agents in Furtat et al., 2011. Designed control system guaranties the network synchronization with the required accuracy. Simulation results illustrating the algorithm efficiency are given.

2. PROBLEM STATEMENT

Consider a digraph $\Gamma = (V, E)$ associated with a network S, where $V = \{v_1, \ldots, v_k, v_L\}$ is a set of nodes, $E \subseteq V \times V$ is a set of arcs. Denote by $C = (c_{ij})$ and $S = (s_{iL})$ the adjacency matrixes of the digraph Γ such that $c_{ij} = 1$ and $s_{iL} = 1$ if $j \in N_{jL}$, else $c_{ij} = 0$ and $s_{iL} = 0$, where $N_{jL} = \{v_j \in V$: (v_j, v_i) , $(v_i, v_L) \in E\}$ is a set of neighbor nodes for the node v_i. Let each subsystem S_i of the network S be described by the following equation

$$Q_i(p)y_i(t) = k_i R_i(p)u_i(t) + f_i(t),$$
$$p^{l-1}y_i(0) = y_{i0}^l, \quad l = 1,\ldots,n, \quad i = 1,\ldots,k, \tag{1}$$

where $y_i(t) \in R$ is an output, $u_i(t) \in R$ is an input and $f_i(t) \in R$ is a smooth uncontrollable bounded disturbance, $Q_i(p)$, $R_i(p)$ are linear differential operators, $\deg Q_i(p) = n$, $\deg R_i(p) = m$, $n - m \geq 1$, $k_i > 0$, y_{i0}^l are unknown initial conditions.

[1] This work was supported by grants of Russian Foundation for Basic Researcher No. 13-08-01014, 12-01-00808, 14-08-01015 and the researcher program OEMMPU-14 Russian Academy of Sciences and was partially financially supported by Government of Russian Federation, Grant 074-U01 and Ministry of Education and Science of Russian Federation (Project 14.Z50.31.0031).

Let the leading subsystem be described by the following equation

$$Q_L(p)y_L(t)=k_L r(t). \tag{2}$$

Here $y_L(t) \in R$ is a reference model output, $r(t) \in R$ is a reference signal, $Q_L(p)$ is a known operator, $\deg Q_L(p) = n$, $k_L > 0$ is a known gain.

It is necessary to synthesis the continuous control law, is that the following condition

$$\left|y_i(t)-y_L(t)\right|<\varepsilon \text{ for } t>T, \tag{3}$$

holds, where $T > 0$, $\varepsilon > 0$ is small enough number.

Assumptions:

1. The unknown coefficients of $Q_i(p)$, $R_i(p)$ and coefficient $k_i > 0$ belong to a known bounded set Ξ.
2. Only $y_i(t)$ and $u_i(t)$ are available for measurement.
3. The digraph Γ has directed spanning tree (Ren and Beard, 2005). The root of the tree is associated with the leading subsystem (2).

3. MAIN RESULTS

According to Furtat (2010), Furtat (2011), Furtat, Fradkov, and Tsykunov (2011), Furtat, Fradkov, and Tsykunov (2013), Furtat (2013), represent the operator $R_i(p)$ of the form

$$R_i(p)=\left(R_{0i}(p)+\theta_i p\Delta R_{0i}(p)\right)R_i^-(p), \tag{4}$$

where $R_{0i}(\lambda)$ and $R_i^-(\lambda)$ are polynomials with negative real parts of the roots, $R_{0i}(\lambda)$ is some Hurwitz polynomial, $\deg R_{0i}(\lambda)=m_1$, $\deg R_i^-(\lambda)=m_2$, λ is complex variable, $\theta_i > 0$ is a small parameter, $\Delta R_{0i}(\lambda)$ is an unstable polynomial. Substitute (4) in (1), we get

$$\begin{gathered} Q_i(p)y_i(t)= \\ =k_i R_i^-(p)\left[R_{0i}(p)+\theta_i p\Delta R_{0i}(p)\right]u_i(t)+f_i(t). \end{gathered} \tag{5}$$

Transform equation (5) to the form

$$\begin{gathered} \dot{x}_i(t)=A_i x_i(t)+B_i\left(u_i(t)+\sigma_{\mathrm{i}}(t)\right)+C_i f_i(t), \\ y_i(t)=L_{1i}x_i(t), \end{gathered} \tag{6}$$

$$\dot{z}_i(t)=\theta_i^{-1}F_i z_i(t)+N_i\dot{u}_i(t), \quad \sigma_{\mathrm{i}}(t)=L_{2i}z_i(t), \tag{7}$$

where $x_i(t) \in R^n$, $z_i(t)\in R^{m_1}$ are vectors of the fast and slow components respectively, A_i, B_i, C_i, F_i, N_i, L_{1i}, L_{2i} are matrixes with respectively dimensions. Since A_i, B_i and L_{1i} depend on coefficients of the stable polynomials $Q_i(\lambda)$ and $k_i R_i^-(\lambda)R_{0i}(\lambda)$, it is seen that system (6) is minimum phase when $\sigma_i(t) = 0$. Since F_i, N_i and L_{2i} depend on the coefficients of the stable polynomial $R_{0i}(\lambda)$ and unstable polynomial $\Delta R_{0i}(\lambda)$, it follows that system (7) is nonminimum phase. Find the disturbance $\sigma_i(t)$ such that difference between solutions of system (6)-(7) when $\sigma_i(t) \neq 0$ and system (6)-(7) when $\sigma_i(t) = 0$ is small enough.

Rewrite (6)-(7) in the form

$$\begin{gathered} \dot{x}_i(t)=A_i x_i(t)+B_i\left(u_i(t)+\sigma_i(t)\right)+C_i f_i(t), \\ y_i(t)=L_{1i}x_i(t), \\ \theta_{1i}\dot{z}_i(t)=F_i z_i(t)+\theta_{2i}N_i\dot{u}_i(t), \quad \sigma_i(t)=L_{2i}z_i(t), \end{gathered} \tag{8}$$

where $\theta_{1i} = \theta_{2i} = \theta_i$. Use the following Lemma for investigation a solution of system (8).

Lemma 1 (Furtat, (2013)). Let the system be described by the following differential equation

$$\dot{x}=f(x,\mu_1,\mu_2,t), \tag{9}$$

where $x\in R^{S1}$, $\mu=\mathrm{col}(\mu_1,\mu_2)\in R^{S2}$, $f(x, \mu_1, \mu_2, t)$ is Lipchitz continuous function in x. Let (9) have a bounded closed set of attraction $\Omega = \{x \mid P(x) \le C\}$ for $\mu_2 = 0$, where $P(x)$ is piecewise-smooth, positive definite function in R^{S1}. In addition let there exist some numbers $C_1 > 0$ and $\bar{\mu}_1>0$ such that the following condition

$$\sup_{|\mu_1|\le\bar{\mu}_1}\left[\left\langle\left[\frac{\partial P(x)}{\partial x}\right]^{\mathrm{T}}, f(x,\mu_1,0,t)\right\rangle\middle| \quad P(x)=C\right]\le -C_1 \tag{10}$$

holds.

Then there exists $\mu_0 > 0$ such that the system (9) has the same set of attraction Ω for $\mu_2 \le \mu_0$.

Lemma 1 is an extension of Brusin's lemma (Brusin, 1995) to time-varying systems.

From Lemma 1, we consider system (8) for $\theta_2 = 0$

$$\begin{gathered} \dot{x}_i(t)=A_i x_i(t)+B_i\left(u_i(t)+\bar{\sigma}_i(t)\right)+C_i f_i(t), \\ y_i(t)=L_{1i}x_i(t), \end{gathered} \tag{11}$$

$$\theta_{1i}\dot{\bar{z}}_i(t)=F_i\bar{z}_i(t), \quad \bar{\sigma}_i(t)=L_{2i}\bar{z}_i(t). \tag{12}$$

Since matrix F_i is Hurwitz and $\theta_{1i} > 0$, we see that system (12) is asymptotically stable. Represent equations (11)-(12) of the form

$$Q_i(p)y_i(t)=k_i R_i^-(p)R_{0i}(p)\left[u_i(t)+\bar{\sigma}_i(t)\right]+f_i(t). \tag{13}$$

Represent $R_i^-(p)R_{0i}(p)$ and $Q_i(p)$ of the form

$$R_i^-(p)R_{0i}(p)=1+\Delta R_i(p), \; Q_i(p)=Q_L(p)+\Delta Q_i(p). \tag{14}$$

Here $\deg \Delta Q_i(p) < n$, $\deg \Delta R_i(p) = m$. Taking into account (2), (13) and (14), we obtain equation of the tracking error $e_i(t)=\sum_{j\in N_{jL}}c_{ij}\left(y_i(t)-y_j(t)\right)+s_{iL}\left(y_i(t)-y_L(t)\right)$ of the form

$$Q_L(p)e_i(t)=\sum_{j\in N_{jL}}c_{ij}\left(\varphi_i(t)-\varphi_j(t)\right)+s_{iL}\left(\varphi_i(t)-k_L r(t)\right), \tag{15}$$

where the function $\varphi_i(t)$ is presented by

$$\begin{aligned} \varphi_i(t) &= k_i u_i(t) + k_i \Delta R_i(p) u_i(t) + \\ &+ k_i \bar{\sigma}_i(t) + k_i \Delta R_i(p) \bar{\sigma}_i(t) - \Delta Q_i(p) y_i(t) + f_i(t). \end{aligned} \quad (16)$$

From (16) it follows that the function $\varphi_i(t)$ depends on uncertainty of ith subsystem and uncertainty of neighboring j-subsystem.

Introduce the auxiliary loop (Tsykunov, 2007, Furtat et al., 2011)

$$Q_L(p) e_{ai}(t) = \alpha u_i(t), \quad (17)$$

where $\alpha > 0$. Taking into account (15) and (17), form the function $\zeta_i(t) = e_i(t) - e_{ai}(t)$. Differentiating $\zeta_i(t)$ with respect to t, we get

$$Q_L(p)\zeta_i(t) = \phi_i(t).$$

Here $\phi_i = \sum_{j \in N_{jL}} c_{ij}(\varphi_i - \varphi_j) + s_{iL}(\varphi_i - k_L r) - \alpha u_i$.

Introduce the control law

$$u_i(t) = -\alpha^{-1} Q_L(p) \bar{\zeta}_i(t). \quad (18)$$

Here $\bar{\zeta}_i(t)$ is an estimate of the function $\zeta_i(t)$. Taking into account (18), rewrite equation (15) of the form

$$Q_L(p) e_i(t) = g^{\mathrm{T}} \delta_i(t) \quad (19)$$

where $g \in R^{n+1}$ is a vector such that coefficients of g are coefficients of the polynomial $Q_L(\lambda)$, $\delta_i^{\mathrm{T}} = \left[\zeta_i, \zeta_i', \ldots, \zeta_i^{(n)}\right]^{\mathrm{T}} - \left[\bar{\zeta}_i, \bar{\zeta}_i', \ldots, \bar{\zeta}_i^{(n)}\right]^{\mathrm{T}}$.

For implementation of the control law (18) use the observer

$$\dot{\xi}_i(t) = G_0 \xi_i(t) + D_0\left(\bar{\zeta}_i(t) - \zeta_i(t)\right), \quad \bar{\zeta}_i(t) = L\xi_i(t), \quad (20)$$

where $\xi_i(t) \in R^n$, parameters in (20) are chosen by Atassi and Khalil, 1999: $G_0 = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}$, I_{n-1} is an identity matrix of order $n-1$, $D_0 = -[d_1\mu^{-1}, d_2\mu^{-2}, \ldots, d_n\mu^{-n}]^{\mathrm{T}}$, coefficients $d_1, d_2, \ldots, d_n$ are chosen such that the matrix $G = G_0 - DL$ must be Hurwitz, $D = [d_1, d_2, \ldots, d_n]^{\mathrm{T}}$, $L = [1, 0, \ldots, 0]$, $\mu > 0$ is small enough number.

Introduce the vector $\bar{\eta}_i(t) = \bar{D}^{-1}\delta_i(t)$, where $\bar{D} = \mathrm{diag}\,\{\mu^{n-1}, \mu^{n-2}, \ldots, \mu, 1\}$. Taking into account (20), take the derivative in time of $\bar{\eta}_i(t)$. Therefore, we have

$$\dot{\bar{\eta}}_i(t) = \mu^{-1} G \bar{\eta}_i(t) + \bar{b}\zeta_i^{(n)}(t), \quad \bar{\Delta}_i(t) = \mu^{n-1} L \bar{\eta}_i(t),$$

where $\bar{b} = [0, \ldots, 0, 1]^{\mathrm{T}}$. Transform last equation to the form

$$\dot{\eta}_i(t) = \mu^{-1} G \eta_i(t) + b\dot{\zeta}_i(t), \quad \bar{\Delta}_i(t) = \mu^{n-1} L \eta_i(t). \quad (21)$$

Here $\eta_i(t) \in R^n$, $\eta_i^1(t) = \bar{\eta}_i^1(t)$ are the first components of the vectors $\eta_i(t)$ and $\bar{\eta}_i(t)$ respectively, $b = [1, 0, \ldots, 0]^{\mathrm{T}}$.

Taking into account (21), transform (20) to the form

$$\dot{x}_i(t) = A x_i(t) + \mu^{n-1} b g^{\mathrm{T}} \Delta_i(t), \quad e_i(t) = L x_i(t), \quad (22)$$

where $x_i(t) \in R^n$, A is Hurwitz matrix in Frobenius form with the characteristic polynomial $Q_L(\lambda)$, $\Delta_i(t) = \left[\eta_i^1(t), \dot{\eta}_i^1(t), \ldots, \left(\eta_i^1(t)\right)^{(n)}\right]^{\mathrm{T}}$.

Theorem 1. Let assumptions 1A-3A hold. If there exist $\alpha > 0$, $\mu_0 > 0$ and

$$\mu_0 \le \min\left\{\|Q_2\|\|Hb\|^{-2}, \sqrt[n-1]{0{,}5\|Q_1\|\|P\bar{b}g^{\mathrm{T}}\|^{-2}}\right\},$$

where P and H are solutions of the matrix equations $A^{\mathrm{T}}P + PA = -Q_1$, $G^{\mathrm{T}}H + HG = -Q_2$, $Q_1 = Q_1^{\mathrm{T}} > 0$, $Q_2 = Q_2^{\mathrm{T}} > 0$, then the control system (17), (18), (20) for $\mu \le \mu_0$ provides the goal (3) for dynamic network with subsystems (11)-(12) and leading subsystem (2).

Since $\bar{\sigma}_i(t)$ is exponentially decaying function, then Theorem 1 can be proved as in (Furtat, 2011).

However, it follows from the problem statement that subsystems (1) are nonminimum phase.

Consider the vector $\Delta z_i(t) = z_i(t) - \bar{z}_i(t)$. Taking into account third equation of (8), equation (12) and differentiating $\Delta z_i(t)$ with respect to t, we get

$$\Delta\dot{z}_i(t) = \theta_i^{-1} F_i \Delta z_i(t) + N_i \dot{u}_i(t), \Delta\sigma_i(t) = L_{2i}\Delta z_i(t). \quad (23)$$

Then the function $\varphi_i(t)$ in equation (23) is defined by

$$\begin{aligned} \varphi_i(t) &= k_i u_i(t) + k_i \Delta R_i(p) u_i(t) + \\ &+ k_i \sigma_i(t) + k_i \Delta R_i(p) \sigma_i(t) - \Delta Q_i(p) y_i(t) + f_i(t). \end{aligned} \quad (24)$$

Theorem 2. Let the assumptions 1A-3A hold. There exist $\mu > 0$ and $\theta_0 > 0$ such that the solutions of the matrix inequalities

$$\begin{aligned} &0{,}25(k-1)^{-1}\lambda_2(L(\Gamma))\left(A^{\mathrm{T}}H_1 + H_1 A\right) - \\ &\quad - 2\mu^{n-1} H_1 b g^{\mathrm{T}}\left(H_1 b g^{\mathrm{T}}\right)^{\mathrm{T}} \le -Q_1, \\ &0{,}25\theta_{0i}^{-1}(k-1)^{-1}\lambda_2(L(\bar{\Gamma}))\left(F_i^{\mathrm{T}} H_{2i} + H_{2i} F_i\right) - \\ &\quad - 2\alpha^{-2}\mu^{-1} H_{2i} N_i g^{\mathrm{T}} G_0 \left(H_{2i} N_i g^{\mathrm{T}} G_0\right)^{\mathrm{T}} - \\ &\quad - 2\alpha^{-2}\mu^{n-1} H_{2i} N_i g^{\mathrm{T}} D_0 L \left(H_{2i} N_i g^{\mathrm{T}} D_0 L\right)^{\mathrm{T}} \le -Q_2, \\ &G^T H_3 + H_3 G - 2H_3 b b^{\mathrm{T}} H_3 - 2\mu^n I \le -Q_3, \end{aligned} \quad (25)$$

are positive define matrixes H_1, H_2 and H_3, $Q_1 = Q_1^{\mathrm{T}} > 0$, $Q_2 = Q_2^{\mathrm{T}} > 0$, $Q_3 = Q_3^{\mathrm{T}} > 0$, $k \le \bar{k}$, $L(\Gamma)$ is symmetrized Laplacian of the digraph Γ, $\lambda_2(L(\Gamma))$ is the second (nonzero) eigenvalue of the $L(\Gamma)$ (Godsil and Royle, 2001). Then for $\theta < \theta_0$ algorithm (17), (18), (20) provides the goal (3) for the network with subsystem (1) and the -leading subsystem (2).

According to Furtat, (2014) control system (17), (18), (20) is available for network with communication time delay.

4. EXAMPLE

Consider the network S consisting of four subsystems S_i, $i = 1, \ldots, 4$ and the leading subsystem S_L. The digraph Γ, associated with the network S, is presented in Fig. 1.

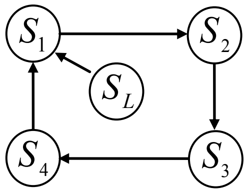

Fig. 1. The digraph Γ of the network S

Let each subsystem S_i be described by

$$\left(p^3 + q_{2i}p^2 + q_{1i}p + q_{0i}\right)y(t) = k_i\left(1 - \theta_i p\right)u_i(t) + f_i(t)\,. \quad (26)$$

The set Ξ is given by the following inequalities: $-5 \le q_{ji} \le 5$, $j = 0, 1, 2, , 1 \le k_i \le 2$, $|f_i(t)| \le 10$. The set of values $\theta_i > 0$ will be determined.

Consider the leading subsystem

$$\left(p^3 + 3p^2 + 3p + 1\right)y_L(t) = r(t)\,,\ r(t) = 1 + 2\sin t\,.$$

The goal is fulfillment of (3).

Choose $\alpha = 1$. According to (17), introduce the auxiliary loop of the form

$$\left(p^3 + 3p^2 + 3p + 1\right)e_{ai}(t) = u_i(t)\,.$$

Let $D_i = [3\ \ 3\ \ 1]^{\mathrm{T}}$ and $\mu = 0.01$ in (20). Then the observer equations (20) are described by

$$\begin{gathered}\dot{\xi}_i^1(t) = \xi_i^2(t) - 3\cdot 10^2\left(\xi_i^1(t) - \zeta_i(t)\right),\\ \dot{\xi}_i^2(t) = \xi_i^3(t) - 3\cdot 10^4\left(\xi_i^1(t) - \zeta_i(t)\right),\\ \dot{\xi}_i^3(t) = -10^6\left(\xi_i^1(t) - \zeta_i(t)\right),\ \ \xi_i(0) = \begin{bmatrix}0 & 0 & 0\end{bmatrix}^{\mathrm{T}}.\end{gathered}$$

Define the control law (18) as

$$u_i(t) = -\dot{\xi}_i^3(t) - 3\dot{\xi}_i^3(t) - 3\xi_i^2(t) - \xi_i^1(t)\,.$$

Let $R_{0i}(p) = \theta_i p + 1$, $\Delta R_{0i}(p) = -2\theta_i p$ in (4). Than in (7): $F_i = -1$, $N_i = -2$.

Let $Q_1 = 10^{-5}I_3$, $Q_2 = \theta_0$ and $Q_3 = I_3$ in (25). According to the digraph (Fig. 1), we get $\lambda_2(L(\Gamma)) = 0.382$. Change matrix inequalities to the matrix equations. Then those equations have solutions when $\theta_{0i} \in (0; 0.0009]$.

Let parameters in (26) be equal to

S_1: $q_{21} = -5$, $q_{11} = -5$, $q_{01} = -5$, $k_1 = 1$, $f_1(t) = 2 + \sin t$, $y_1(0) = 1$, $\dot{y}_1(0) = 0$, $\ddot{y}_1(0) = 1$;

S_2: $q_{22} = 1$, $q_{12} = -3$, $q_{02} = -3$, $k_2 = 2$, $f_2(t) = 1 + 8\sin 0{,}5t$, $y_2(0) = -1$, $\dot{y}_2(0) = 1$, $\ddot{y}_2(0) = 1$;

S_3: $q_{23} = 5$, $q_{13} = -5$, $q_{03} = 0$, $k_3 = 1$, $f_3(t) = 1 - 2\sin 1{,}5t$, $y_3(0) = 0$, $\dot{y}_3(0) = 2$, $\ddot{y}_3(0) = 0$;

S_4: $q_{24} = 1$, $q_{14} = 5$, $q_{04} = -5$, $k_4 = 1$, $f_4(t) = 2 + 4\sin 2t$, $y_4(0) = 2$, $\dot{y}_4(0) = -1$, $\ddot{y}_4(0) = -1$;

From simulations it follows that for $\theta_{01} \in (0; 0.001]$, $\theta_{02} \in (0; 0.004]$, $\theta_{03} \in (0; 0.001]$ и $\theta_{04} \in (0; 0.007]$ the closed-loop system is stable. Let information get from j-th subsystem to i-th subsystem after time delay $\tau_{ji}(t) = 1(i + j) - 0.5e^{-0.5t}$.

Let $\theta_1 = 0.001$, $\theta_2 = 0.004$, $\theta_3 = 0.001$ and $\theta_4 = 0.007$ in (26). In Fig. 2-5 the transients of $\tilde{y}_1(t) = y_1(t) - y_L(t)$, $\tilde{y}_2(t) = y_2(t) - y_L(t)$, $\tilde{y}_3(t) = y_3(t) - y_L(t)$ and $\tilde{y}_4(t) = y_4(t) - y_L(t)$ are presented respectively.

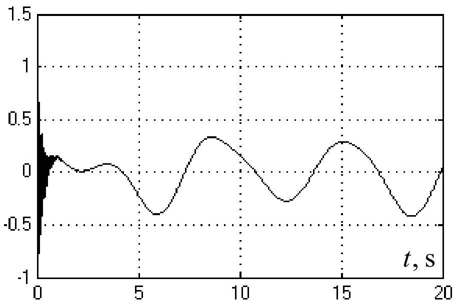

Fig. 2. Transient of $\tilde{y}_1(t)$

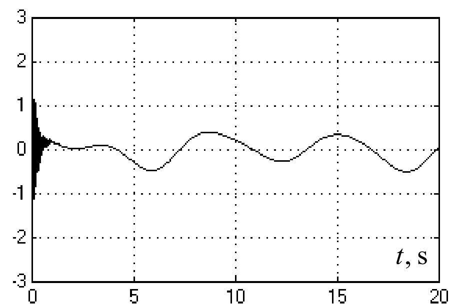

Fig. 3. Transient of $\tilde{y}_2(t)$

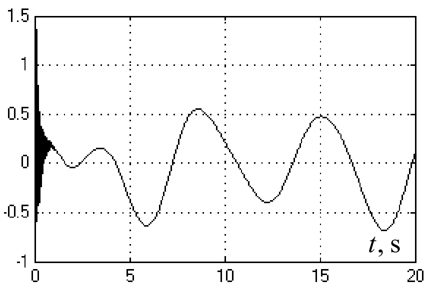

Fig. 4. Transient of $\tilde{y}_3(t)$

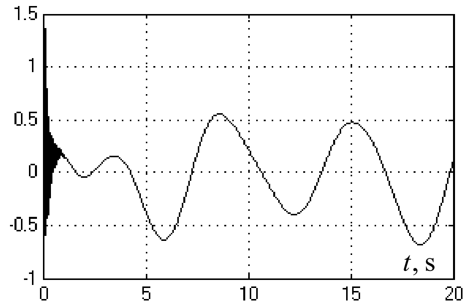

Fig. 5. Transient of $\tilde{y}_4(t)$

5. CONCLUSIONS

This paper considers the robust algorithm for some class of dynamic networks with nonminimum phase agents. It is supposed that each subsystem of the network can be represented as the main loop described by minimum-phase system and disturbance loop described by nonminimum phase system. Further subsystems are decomposed to systems of singularly perturbed differential equations. In these equations small parameter depends on positive zeros of the transfer functions of the subsystems. Found sets of small parameters such that the control algorithm designed for dynamic networks with minimum-phase agents holds for dynamic networks with nonminimum phase agents. The boundaries of these sets depend on subsystem parameters, coefficients in the control algorithm and the network topology. Simulations are confirmed by analytical results.

REFERENCES

Atassi, A.N. and H.K. Khalil (1999). A separation principle for the stabilization of class of nonlinear systems, *IEEE Transaction on Automatic Control*, 44, 1672-1687.

Brusin, V.A. (1995). A class of singularly perturbed adaptive systems. 1, *Automation and Remote Control*, 54(4), 119-127.

Das, A., and F.L. Lewis (2010). Distributed adaptive control for synchronization of unknown nonlinear networked systems, *Automatica*, 46(12), 2014-2021.

Fax, J.A., and R.M. Murray (2004). Information flow and cooperative control of vehicle formations, *IEEE Trans. on Automatic Control*, 49(9), 1465-1476.

Fradkov, A.L., I.V. Miroshnik, and V.O. Nikiforov (1999). *Nonlinear and Adaptive Control of Complex Systems*. Dordrecht: Kluwer.

Furtat, I.B., and A.M. Tsykunov (2010). Adaptive control of plants of unknown relative degree, *Automation and Remote Control*, 71(6), 1076-1084.

Furtat, I.B. (2011). Robust synchronization of dynamical networks with compensation of disturbances, *Automation and Remote Control*, 72(12), 2516–2526.

Furtat, I.B., A.L. Fradkov, and A.M. Tsykunov (2011). Robust synchronization of linear networks with compensation of disturbances, *Proc. of the 18th IFAC World Congress on Automatic Control*, Milan, 1255-1260.

Furtat I., A. Fradkov, and A. Tsykunov (2013). Robust synchronization of linear dynamical networks with compensation of disturbances, *International Journal of Robust and Nonlinear Control,* published online Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/rnc.3024.

Furtat, I.B. (2014). Robust Control for a Specific Class of Non-Minimum Phase Dynamical Networks, *Journal of Computer and Systems Sciences International*, 53(1), 1, 33-46.

Godsil, C., and G. Royle (2001). *Algebraic graph theory*. New York: Springer-Verlag.

Hovakimyam, N., B.-J. Yang, and A. J. Calise (2002). An adaptive output feedback control methodology for non-minimum phase systems, *Proc. of the 41st IEEE Conference on Decision and Control, Las Vegas*, 949-954.

Li, Z., Z. Duan, and L. Huang (2009). Leader-follower consensus of multi-agent systems, *Proc. of the American control conference, St. Louis*, 3256-3261.

Ren, W., and R.W. Beard (2005). Consensus seeking in multiagent systems under dynamically changing interaction topologies, *IEEE Trans. on Automatic Control*, 50(5), 655-661.

Tsykunov A.M. (2005). The application of dynamic adaptive controller for an output feedback control of the plant, *Proc. of the 4th International Conference "System Identification and Control Problems" SICPRO '05, Moscow*, 1349-1357.

Tsykunov, A.M. (2007). Robust control algorithms with compensation of bounded perturbations, *Automation and Remote Control*, 71(7), 1213-1224.

APPENDIX A.

Proof of Theorem 2. Consider Lemma 2 (Furtat, 2011).

Lemma 2. Let the digraph Γ has a directed spanning tree. Consider the quadratic form

$$W = \sum_{i=1}^{k} \sum_{j \in N_{iL}} \big(x_i(t) - x_j(t)\big)^T K \big(x_i(t) - x_j(t)\big),$$

where $K = K^{\mathrm{T}} > 0$. Then for $k > 1$ we have

$$W \geq 0{,}25(k-1)^{-1} \lambda_2(L(\Gamma))$$
$$\sum_{i=1}^{k} \sum_{j=1}^{k} \big(x_i(t) - x_j(t)\big)^{\mathrm{T}} K \big(x_i(t) - x_j(t)\big),$$
$$W \leq \sum_{i=1}^{k} \sum_{j=1}^{k} \big(x_i(t) - x_j(t)\big)^{\mathrm{T}} K \big(x_i(t) - x_j(t)\big).$$

Taking into account (18) and (20), rewrite (23) as

$$\Delta \dot z_i(t) = \theta_i^{-1} F_i \Delta z_i(t) + bT\left(G_0 \xi_i(t) + D_0 \mu^{n-1} L \eta_i(t)\right), \quad \Delta\sigma_i(t) = L_{1i}\Delta z_i(t), \tag{27}$$

Rewrite equations (21), (22) and (27) of the form

$$\begin{gathered} \dot x_i(t) = Ax_i(t) + \mu^{n-1} bg^{\mathrm T} \Delta_i(t) + \psi_i(t), \\ \theta_{1i}\Delta\dot z(t) = F_i \Delta z_i(t) - \\ -\alpha^{-1}\theta_{2i} N_i g^{\mathrm T}\left(G_0\xi_i(t) + D_0\mu^{n-1} L\eta_i(t)\right), \\ \dot\eta_i(t) = \mu^{-1} G\eta_i(t) + b\dot\zeta_i(t). \end{gathered} \tag{28}$$

Under the condition of Lemma 1, we consider Lyapunov function $V(t) = V(x_1(t), \ldots, x_k(t), \Delta z_1(t), \ldots, \Delta z_k(t), \eta_1(t), \ldots, \eta_k(t))$ of the form

$$V(t) = \sum_{i=1}^{k}\left[x_i^{\mathrm T}(t) H_1 x_i(t) + \right. \\ \left. + \Delta z_i^{\mathrm T}(t) H_{2i} \Delta z_i(t) + \eta_i^{\mathrm T}(t) H_3 \eta_i(t)\right] \tag{29}$$

According to Lemma 1, consider (28) when $\theta_{2i} = 0$. Since matrix F_i is Hurwitz, it follows that the second equation of (28) is asymptotically stable. Therefore, the function $\sigma_i(t)$ is bounded. According to Theorem 1, the system (28) is dissipative.

Now we must define the value θ_{0i} such that the system (28) is dissipative for $\theta_{2i} > 0$. Let $\theta_{1i} = \theta_{2i} = \theta_{0i}$. Take derivative in time of function (29) along the trajectories of (28), we get

$$\begin{gathered} \dot V(t) = \sum_{i=1}^{k}\left[x_i^{\mathrm T}(t)\left(A^{\mathrm T} H_1 + H_1 A\right)x_i(t) + \right. \\ + 2x_i^{\mathrm T}(t) H_1 \mu^{n-1} bg^{\mathrm T}\Delta_i(t) + \\ + \theta_{0i}^{-1}\Delta z_i^{\mathrm T}(t)\left(F_i^{\mathrm T} H_{2i} + H_{2i} F_i\right)\Delta z_i(t) + \\ - 2\Delta z_i^{\mathrm T}(t) H_{2i}\alpha^{-1} N_i g^{\mathrm T}\left(G_0\xi_i(t) + D_0\mu^{n-1} L\eta_i(t)\right) + \\ \left. + \mu^{-1}\eta_i^{\mathrm T}(t)\left(G^T H_3 + H_3 G\right)\eta_i(t) + 2\eta_i^{\mathrm T}(t) H_3 b\dot\zeta_i(t)\right]. \end{gathered} \tag{30}$$

Taking into account Theorem 2 and Lemma 2, we can obtain the following estimates in (30):

$$\sum_{i=1}^{k} x_i^{\mathrm T}\left(A^{\mathrm T} H_1 + H_1 A\right)x_i \le$$

$$\le 0{,}25(k-1)^{-1}\lambda_2(L(\Gamma))\sum_{i=1}^{k}\tilde x_i^{\mathrm T}\left(A^{\mathrm T} H_1 + H_1 A\right)\tilde x_i \;,$$

where $\tilde x_i = \sum_{j=1}^{k}\left(x_i - x_j\right) + x_i - x_L$;

$$\sum_{i=1}^{k} 2x_i^{\mathrm T} H_1 \mu^{n-1} bg^{\mathrm T}\Delta_i \le$$

$$\le 2\mu^{n-1}\sum_{i=1}^{k}\tilde x_i^{\mathrm T} H_1 bg^{\mathrm T}\left(H_1 bg^{\mathrm T}\right)^{\mathrm T}\tilde x_i + 2\mu^{n-1}\sum_{i=1}^{k}\left|\Delta_i\right|^2;$$

$$\sum_{i=1}^{k}\theta_{0i}^{-1}\Delta z_i^{\mathrm T}\left(F_i^{\mathrm T} H_{2i} + H_{2i} F_i\right)\Delta z_i \le$$

$$\le 0{,}25(k-1)^{-1}\lambda_2(L(\overline\Gamma))$$

$\sum_{i=1}^{k}\sum_{j=1}^{k}\theta_{0i}^{-1}\Delta z_{ij}^{\mathrm T}\left(F_i^{\mathrm T} H_{2i} + H_{2i} F_i\right)\Delta z_{ij}$, where $\Delta z_{ij} = \Delta z_i - \Delta z_j$;

$$-2\sum_{i=1}^{k}\Delta z_i^{\mathrm T} H_{2i}\alpha^{-1} N_i g^{\mathrm T} G_0 \xi_i \le$$

$$\le 2\alpha^{-2}\mu^{-1}\sum_{i=1}^{k}\sum_{j=1}^{k}\Delta z_{ij}^{\mathrm T} H_{2i} N_i g^{\mathrm T} G_0$$

$$\left(H_{2i} N_i g^{\mathrm T} G_0\right)^{\mathrm T}\Delta z_{ij} + 2\mu\sum_{i=1}^{k}\left|\xi_i\right|^2;$$

$$-2\sum_{i=1}^{k}\Delta z_i^{\mathrm T} H_{2i}\alpha^{-1} N_i g^{\mathrm T} D_0 \mu^{n-1} L\eta_i \le$$

$$\le 2\alpha^{-2}\mu^{n-1}\sum_{i=1}^{k}\sum_{j=1}^{k}\Delta z_{ij}^{\mathrm T} H_{2i} N_i g^{\mathrm T} D_0 L$$

$$\left(H_{2i} N_i g^{\mathrm T} D_0 L\right)^{\mathrm T}\Delta z_{ij} + 2\mu^{n-1}\sum_{i=1}^{k}\eta_i^{\mathrm T}\eta_i;$$

$$\sum_{i=1}^{k} 2\eta_i^{\mathrm T} H_3 b\dot\zeta_i \le 2\mu^{-1}\sum_{i=1}^{k}\eta_i^{\mathrm T} H_3 bb^{\mathrm T} H_3 \eta_i + 2\mu\sum_{i=1}^{k}\dot\zeta_i^2 \;.$$

Substitute these estimates in (30). Taking into account (25), rewrite (31) of the form

$$\dot V(t) \le -\sum_{i=1}^{k}\tilde x_i^{\mathrm T}(t) Q_1 \tilde x_i(t) - \sum_{i=1}^{k}\sum_{j=1}^{k}\Delta z_{ij}^{\mathrm T}(t) Q_2 - \\ -\mu^{-1}\sum_{i=1}^{k}\eta_i^{\mathrm T}(t) Q_3 \eta_i(t) + \mu\delta, \tag{31}$$

where $\delta = 2\sup_t\left(\sum_{i=1}^{k}\left(\left|\xi_i(t)\right|^2 + \dot\zeta_i^2(t)\right)\right)$.

From Lemma 2 we can estimate (29) as

$$V(t) \le \sum_{i=1}^{k}\sum_{j=1}^{k}\left[\tilde x_i^{\mathrm T}(t) H_1 \tilde x_i(t) + \Delta z_{ij}^{\mathrm T}(t) H_{2i}\Delta z_{ij}(t)\right] + \\ + \sum_{i=1}^{k}\eta_i^{\mathrm T}(t) H_3 \eta_i(t). \tag{32}$$

Let

$$\chi = \min\left\{\frac{\lambda_{\min}(R_1)}{\lambda_{\max}(H_1)}, \frac{\lambda_{\min}(R_2)}{\theta_0\lambda_{\max}(H_{2i})}, \frac{\lambda_{\min}(R_3)}{\mu\lambda_{\max}(H_3)}\right\}.$$

Taking into account (32), rewrite (31) as

$$\dot V(t) \le -\chi V(t) + \mu\delta, \tag{33}$$

Solve (33) for variable $V(t)$

$$V(t) \le V(0)e^{-\chi t} + \mu\chi^{-1}\left(1 - e^{-\chi t}\right)\delta \;.$$

From the last inequality and (25) it follows that in the goal (3) ε is defined by

$$\varepsilon \le \sqrt{\lambda_{\min}^{-1}(H_1)\left(V(0)e^{-\chi T} + \mu\chi^{-1}\left(1 - e^{-\chi T}\right)\delta\right)}. \tag{34}$$

for $t = T$.

From (25) and (34) there exist the values θ_i such that an algorithm designed for networks with minimum phase agents holds for some class of networks with nonminimum phase agents. Moreover, from the last inequality it follows that increase of the value μ the value ε can be reduced in (3).