Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Robust H_∞ Controller Design Using Frequency-Domain Data

Alireza Karimi [1] and Yuanming Zhu [2]

Automatic Control Laboratory, Ecole Polytechnique Fédérale de Lausanne (EPFL), Switzerland

Abstract: A new robust controller design method is developed for linear time-invariant single-input single-output systems presented by their frequency response data. The performance specifications are in terms of the upper bounds on the infinity norm of weighted closed-loop frequency responses. The designed controller is robust in terms of frequency-domain disk and polytopic uncertainty as well as multimodel uncertainty. The necessary and sufficient conditions for the existence of such controllers are presented by a set of convex constraints. The practical issues to compute fixed-order rational H_∞ controllers by convex optimization techniques are discussed. The experimental results on an electromechanical system illustrate the effectiveness of the proposed method.

1. INTRODUCTION

Data-driven controller design, in time- or in frequency-domain, is an attractive research field in control community (for a survey see Bazanella et al. [2012]). In this kind of methods, the controller is designed merely using measured data rather than parametric model of the plant. Therefore, the intermediate identification procedure or first principle modeling is not required. As a result, it is expected that these direct methods perform better than the model based approaches because of the absence of unmodelled dynamics and parametric errors (see Formentin et al. [2013]).

The majority of the data-driven methods use time-domain data for computing a controller that minimizes a model reference criterion or more generally an H_2 control criterion. Model Reference Adaptive Control (MRAC) in Landau et al. [2011], Iterative Feedback Tuning (IFT) in Hjalmarsson et al. [1998], Virtual Reference Feedback Tuning (VRFT) in Campi et al. [2002] and Iterative Correlation-based Tuning (ICbT) in Karimi et al. [2004] are among the well-known methods using time-domain data. Without a parametric model for the process, the stability and robustness of these methods are usually studied using frequency-domain data (see Kammer et al. [2000], van Heusden et al. [2011]).

There are a few methods that uses the frequency-domain data to compute robust controllers to meet some constraints on the stability margins or H_∞ norm of the sensitivity functions. A robust fixed-order controller design method using linear programming is proposed in Karimi et al. [2007]. In this method, the constraints on the gain margin, phase margin and crossover frequency are approximated with linear constraints for linearly parameterized controllers. The frequency response data are used in Hoogendijk et al. [2010] to compute the frequency response of a controller that achieves a desired closed-loop pole location. In Keel and Bhattacharyya [2008], a complete set of PID controllers is computed that guarantee a gain margin, phase margin and H_∞ performance specification using frequency-domain data. This method is extended to design of fixed-order linearly parameterized controllers in Parastvand and Khosrowjerdi [2014]. A data-driven synthesis methodology for fixed structure controller design problem with H_∞ performance is presented in Den Hamer et al. [2009]. This method uses the Q parameterization in the frequency domain and solves a non-convex optimization problem to find a local optimum. Another frequency-domain approach is presented in Khadraoui et al. [2013] to design reduced order controllers with guaranteed bounded error on the difference between the desired and achieved magnitude of closed-loop sensitivity functions. This approach also uses a non-convex optimization method.

Convex optimization is used in Karimi and Galdos [2010] to compute robust H_∞ controllers for SISO systems represented by their frequency response. The H_∞ robust performance constraints are convexified for linearly parameterized controllers with the help of a desired open loop transfer function. Based on this method, a public domain toolbox for Matlab is developed which is available in Karimi [2013]. This approach is extended to compute decoupling controllers for MIMO systems in Galdos et al. [2010].

In this paper, the necessary and sufficient conditions for the existence of robust controllers that guarantee bounded infinity norm on the closed-loop transfer functions are developed. It is shown that these conditions depends only on the frequency response of the plant model and can be represented by convex constraints with respect to the controller parameters. Therefore, fixed-order rational H_∞ controllers can be designed by convex optimization. The results are extended to systems with polytopic uncertainty in their frequency response, which are caused by mea-

[1] Corresponding author: `alireza.karimi@epfl.ch`
[2] The work of Yuanming Zhu is partially supported by Chinese Scholarship Council and the National Natural Science Foundation of China (61120106009).

surement noise or multimodel uncertainty. The developed conditions are necessary and sufficient for stable systems and only sufficient for unstable systems with polytopic uncertainty. The main advantage with respect to the work in Karimi and Galdos [2010] is that the whole conservatism of the approach is gathered in the controller structure and can be reduced by increasing its order.

The outline of this paper is as follows. Section 2 presents the preliminaries and notation as well as the main results on the convex parameterization of robust controllers. The extension of the results to systems with polytopic uncertainty in the frequency domain are given in Section 3. The implementation issues are discussed and the experimental results are illustrated in Section 4. The paper ends with concluding remarks in Section 5.

2. CONVEX PARAMETERIZATION OF ROBUST CONTROLLERS

2.1 Preliminaries and notation

It is assumed that the frequency response of a causal LTI-SISO system is given by:

$$G(j\omega) = N(j\omega)M^{-1}(j\omega), \quad \omega \in \Omega \tag{1}$$

where $\Omega := \mathbb{R} \cup \{\infty\}$ and $N(j\omega), M(j\omega)$ are the frequency responses of bounded analytic functions in the right half plane. It is also assumed that $G(j\infty) = 0$, which leads to $N(j\infty) = 0$ and $M(j\infty) \neq 0$. This representation includes time-delay systems as well as unstable plant with unbounded infinity norm. For stable systems, a trivial choice is $N(j\omega) = G(j\omega)$ and $M(j\omega) = 1$.

Consider the controller structure, $K = XY^{-1}$, where X and Y are stable transfer functions with bounded infinity norm ($X, Y \in RH_\infty$). These transfer functions may be discrete- or continuous-time but for the ease of presentation we consider the continuous-time transfer functions. All results can be straightforwardly used for computing discrete-time controllers.

The aim is to design a controller that meets some constraints on the infinity norm of the weighted sensitivity functions. The four closed-loop sensitivity functions are given by:

$$S = (1 + GK)^{-1} = MY(NX + MY)^{-1} \tag{2}$$

$$T = GK(1 + GK)^{-1} = NX(NX + MY)^{-1} \tag{3}$$

$$U = K(1 + GK)^{-1} = MX(NX + MY)^{-1} \tag{4}$$

$$V = G(1 + GK)^{-1} = NY(NX + MY)^{-1} \tag{5}$$

In the following, we consider only an upper bound on the infinity-norm of $H(j\omega) = W_1(j\omega)S(j\omega)$, where $W_1(j\omega)$ is the frequency function of a stable system with bounded infinity norm. Therefore, the control objective is to find a stabilizing controller K such that

$$\sup_{\omega \in \Omega} |H(j\omega)| < \gamma \tag{6}$$

The result can be extended straightforwardly to the other weighted sensitivity functions. For the simplicity of notation, $j\omega$ will be dropped when there is no risk of confusion.

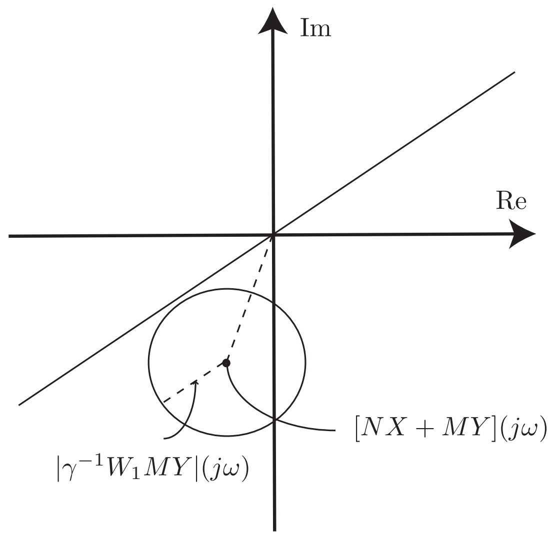

Fig. 1. Graphical illustration of nominal performance

2.2 Main Lemma

The main objective is to find a set of convex constraints (with respect to X and Y) to satisfy the control objective in (6). The following lemma will be used in the proof of the main results of this paper:

Lemma 1. Suppose that $H(j\omega) = W_1MY(NX + MY)^{-1}$ is the frequency response of a bounded analytic function in the right half plane. Then, (6) is met if and only if there exists a stable proper rational transfer function $F(s)$ that satisfies

$$Re\{(NX + MY - \gamma^{-1}|W_1MY|)F(j\omega)\} > 0, \qquad \forall\omega \in \Omega$$

Proof : The basic idea is similar to that of the proof of Theorem 1 in Rantzer and Megretski [1994]. From Fig. 1, it is clear that (6) is satisfied if and only if the disk of radius $\gamma^{-1}|W_1MY|$ centered at $NX + MY$ does not include the origin for all $\omega \in \Omega$. This is equivalent to the existence of a line passing through origin that does not intersect the disk. Therefore, at every given frequency, ω, there exists a complex number $f(j\omega)$ that can make rotate the disk such that it lays inside the right hand side of the imaginary axis. Hence, we have

$$Re\{(NX + MY - \gamma^{-1}|W_1MY|)f(j\omega)\} > 0 \tag{7}$$

for all $\omega \in \Omega$. In Rantzer and Megretski [1994], it is shown that $f(j\omega)$ can be approximated arbitrarily well by the frequency response of a rational stable transfer function $F(s)$, if and only if

$$Z = (NX + MY - \gamma_0^{-1}|W_1MY|)^{-1} \tag{8}$$

is analytic in the right half plane for all $\gamma_0 > \gamma$. However, $(NX + MY)^{-1}$ is stable because of the stability of H. On the other hand, by decreasing γ_0 from infinity to γ, the poles of Z move continuously with γ_0. Thus, Z is not analytic in the right half plane if and only if $Z^{-1}(j\omega) = 0$ for a given frequency, which is not the case because the disk shown in Fig. 1 does not include the origin. □

2.3 Nominal and robust performance

The set of all controllers that meet the nominal performance defined by the weighted norm of sensitivity functions are given in the following theorem.

Theorem 1. Given the frequency response model G in (1) and the frequency response of a bounded weighting filter W_1, the following statements are equivalent:

(a) There exists a controller K that stabilizes G and

$$\sup_{\omega\in\Omega} |W_1(1+GK)^{-1}| < \gamma \tag{9}$$

(b) There exist $X, Y \in RH_\infty$ with $K = XY^{-1}$, such that

$$\gamma^{-1}|W_1 MY|(j\omega) < Re\{[NX + MY](j\omega)\}, \quad \forall\omega \in \Omega$$

Proof : ($\mathbf{b} \Rightarrow \mathbf{a}$) Since $NX + MY$ is analytic in the right half plane and its real part is positive for all $\omega \in \Omega$, it will not turn around the origin when ω turns around the Nyquist contour, so its inverse is stable and therefore K stabilizes G. On the other hand, we have

$$|[NX + MY](j\omega)| \geq Re\{[NX + MY](j\omega)\}, \quad \forall\omega \in \Omega$$

which leads to

$$|W_1 MY|(j\omega) < \gamma|NX + MY|(j\omega) \quad \forall\omega \in \Omega$$

and consequently to (9) in Statement (**a**).

($\mathbf{a} \Rightarrow \mathbf{b}$) Assume that $K = X_0 Y_0^{-1}$ satisfies Statement (**a**) but not Statement (**b**). Then, according to Lemma 1 there exists a stable proper rational transfer function $F(s)$, such that

$$Re\{(NX_0 + MY_0 - \gamma^{-1}|W_1 MY_0|)F(j\omega)\} > 0 \quad \forall\omega \in \Omega$$

Therefore, there exist $X = X_0 F$ and $Y = Y_0 F$ with $K = XY^{-1} = X_0 Y_0^{-1}$, such that Statement (**b**) hold. □

This theorem can be applied straightforwardly to other sensitivity functions in (3)-(5).

The necessary and sufficient conditions for robust performance of closed-loop systems with frequency-domain uncertainty can be developed in the same way. Suppose that the frequency response of the plant model with some disk additive uncertainty is available as :

$$\tilde{N}(j\omega) = N(j\omega) + |W_n(j\omega)|\delta_n e^{j\theta_n} \tag{10}$$

$$\tilde{M}(j\omega) = M(j\omega) + |W_m(j\omega)|\delta_m e^{j\theta_m} \tag{11}$$

where $|\delta_n| \leq 1$, $|\delta_m| \leq 1$ and $\theta_n, \theta_m \in [0\,,\,2\pi]$. This type of models can be easily obtained by spectral analysis of measured data, where W_n and W_m are computed from the covariance of the estimates for a given confidence interval (see Ljung [1999]).

If we consider the nominal performance as defined in (6), the robust performance condition becomes:

$$\sup_{\omega\in\Omega} \frac{|W_1 MY| + |W_1 W_m|}{|NX + MY| - |W_n X| - |W_m Y|} < \gamma \tag{12}$$

Equivalently, at any $\omega \in \Omega$ a disk of radius

$$r(\omega) = \gamma^{-1}|W_1 MY| + \gamma^{-1}|W_1 W_m| + |W_n X| + |W_m Y| \tag{13}$$

centered at $[NX + MY](j\omega)$ should not include the origin. This can be presented as a set of convex constraints with respect to X and Y as follows:

$$r(\omega) < Re\{[NX + MY](j\omega)\}, \quad \forall\omega \in \Omega \tag{14}$$

3. POLYTOPIC UNCERTAINTY

Another frequency-domain uncertainty is the polytopic uncertainty that covers multimodel uncertainty and parametric uncertainty, as will be explained later, with some approximation. In this type of uncertainty, the frequency response of the model is given as:

$$G(\lambda, j\omega) = N(\lambda, j\omega)M^{-1}(\lambda, j\omega) \tag{15}$$

where

$$N(\lambda, j\omega) = \sum_{i=1}^{m} \lambda_i N_i(j\omega) \tag{16}$$

$$M(\lambda, j\omega) = \sum_{i=1}^{m} \lambda_i M_i(j\omega) \tag{17}$$

and $\lambda_i \geq 0$, $\sum_{i=1}^{q} \lambda_i = 1$ and λ is the convex hull of λ_is. It can be shown that the following constraints

$$\gamma^{-1}|W_1 M_i Y| < Re\{[N_i X + M_i Y](j\omega)\}, \quad \forall\omega \in \Omega \tag{18}$$

and for $i = 1, \ldots, m$ are sufficient conditions for robust performance of the closed-loop system with polytopic uncertainty. It suffices to compute the convex combination of the constraints in (18) as

$$\gamma^{-1}\sum_{i=1}^{m} \lambda_i |W_1 M_i Y| < Re\left\{\sum_{i=1}^{m} \lambda_i [N_i X + M_i Y]\right\}, \forall\omega \in \Omega$$

Noting that:

$$\left|\sum_{i=1}^{m} \lambda_i W_1 M_i Y\right| \leq \sum_{i=1}^{m} \lambda_i |W_1 M_i Y| \tag{19}$$

we obtain:

$$\gamma^{-1}|W_1 M(\lambda)Y| < Re\{N(\lambda)X + M(\lambda)Y](j\omega)\}, \quad \forall\omega \in \Omega$$

Then, according to Theorem 1 the upper bound for the weighted sensitivity function is satisfied for all λ.

Remark: In a data-driven approach, a parametric model of the plant can be identified together with its parametric uncertainty using the classical prediction error methods (see Ljung [1999]). The parametric uncertainty is characterized by an ellipsoid in the parameter space and can be computed using the asymptotic covariance matrix of the parameters for a given probability level. This parametric uncertainty can be transferred into the frequency domain by a linear transformation, which is accurate enough for large data length. In the complex plane, this parametric uncertainty is represented by an ellipse at each frequency and can be very well approximated with an m-side polygon ($m > 2$) of minimum area that circumscribes each ellipse. This way, the parametric uncertainty can be taken into account using the polytopic frequency-domain uncertainty with almost no conservatism.

Although the constraints for polytopic uncertainty are only sufficient, for some class of models and some sensitivity functions the necessary and sufficient conditions can be developed. The following theorem represents the results for systems that have polytopic uncertainty only in N.

Theorem 2. Consider the frequency response model given in (15) with $N(\lambda, j\omega)$ in (16) and $M(\lambda, j\omega) = M(j\omega)$. Then, the following statements are equivalent:

(a) Controller K stabilizes $G(\lambda) = N(\lambda)M^{-1}, \forall\lambda$ and

$$\sup_{\omega\in\Omega} |W_1(1 + G(\lambda)K)^{-1}| < \gamma$$

(b) There exist $X, Y \in RH_\infty$ such that $K = XY^{-1}$, and

$$\gamma^{-1}|W_1 MY(j\omega)| < Re\{[N_i X + MY](j\omega)\}, \forall\omega \in \Omega \tag{20}$$

and for $i = 1, \ldots, m$.

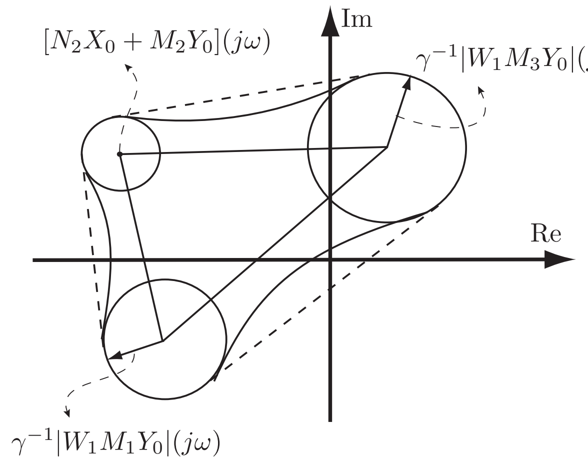

Fig. 2. Graphical illustration of the constraints for polytopic uncertainty with 3 vertices

Proof : ($\mathbf{b} \Rightarrow \mathbf{a}$) The convex combination of the constraints in (20) leads to

$$\gamma^{-1}|W_1MY(j\omega)| < Re\{[N(\lambda)X + MY](j\omega)\} \qquad (21)$$

for all $\omega \in \Omega$ and for all λ. So Statement (**a**) can be concluded using the result of Theorem 1.

($\mathbf{a} \Rightarrow \mathbf{b}$) Suppose that (**a**) is satisfied with the controller $K = X_0Y_0^{-1}$. Therefore, all disks of the same radius, $\gamma^{-1}|W_1MY_0|$, centered inside a polygon created by the m vertices, $N_iX_0 + MY_0$, do not include the origin. This represents a convex set, which is the convex hull of the m disks. Therefore, there exists always a line that passes through the origin and does not intersect this convex set. As a result, similar to the proof of Lemma 1, there exists a stable transfer function $F(s)$ such that:

$$Re\{[N_iX_0 + MY_0 - \gamma^{-1}|W_1MY_0|]F(j\omega)\} > 0, \quad \forall\omega \in \Omega$$

and for $i = 1, \ldots, m$. Hence $X = X_0F$ and $Y = Y_0F$ satisfies the inequalities in Statement (**b**). □

Remark: Theorem 2 considers only the plant model with polytopic uncertainty in N. This represents the class of stable systems that may have some fixed poles on the imaginary axis. It covers also the unstable systems with no uncertainty in M. A polytopic uncertainty in M will change the radius of the disks centered at $N_iX_0 + M_iY_0$, such that the whole set of the disks will not be necessarily convex. Figure 2 shows a case in which the set of the disks is not convex but is inside the convex hull of the disks. This is always true because of the constraint in (19). In the special case shown in Fig. 2, we observe that the set of disks does not include the origin but the convex hull does. Similarly, Statement (**b**) in Theorem 2 is a sufficient condition for satisfying an upper bound on the weighted sensitivity functions T or V, since the radius of the disks, at each frequency, will not be constant for the whole polygon. However, it will be still necessary and sufficient for an upper bound on the weighted sensitivity function U in (4).

4. FIXED-ORDER CONTROLLER DESIGN

In this section we show how an optimal fixed-order controller can be designed for frequency-domain models by convex optimization. For simplicity of presentation, we consider only the case without uncertainty. This problem is defined as:

$$\min_{X,Y} \quad \gamma$$

subject to (22)

$$\gamma^{-1}|W_1MY(j\omega)| < Re\{[NX + MY](j\omega)\}, \forall\omega \in \Omega$$

There are different practical and implementation issues in this optimization problem that will be discussed in this section. Note that the nonlinearity caused by the multiplication of γ^{-1} and one of the optimization variables, Y, can be easily solved by the iterative bisection algorithm.

4.1 Controller parameterization

A linear parameterization of X and Y keeps the constraints in (22) convex. As a result, $X(s)$ and $Y(s)$ are linearly parameterized as $X(s) = \rho_x^T\phi(s)$ and $Y(s) = \rho_y^T\phi(s)$, where $\rho_x^T = [\rho_{x_0} \cdots \rho_{x_n}]$ and $\rho_y^T = [1, \rho_{y_1} \cdots \rho_{y_n}]$ are the vectors of the controller parameters and

$$\phi^T(s) = [1, \phi_1(s) \cdots, \phi_n(s)] \qquad (23)$$

is a vector of stable orthogonal basis functions. A simple choice, is the Laguerre basis functions given by

$$\phi_i(s) = \frac{\sqrt{2\xi}(s-\xi)^{i-1}}{(s+\xi)^i}$$

with $\xi > 0$ and $i = 1, \cdots, n$. These basis functions have only one parameter, ξ, to be selected.

4.2 Frequency response models

Finding the coprime factors of a given plant is a standard problem in control when the model of the plant is available (Zhou [1998]). For stable systems, a trivial choice is $N = G$ and $M = 1$. For unstable systems, in a data-driven setting, a stabilizing controller is needed for data acquisition purpose. In this case, $N(j\omega)$ is the frequency function between the reference signal and the measured output, while $M(j\omega)$ is the frequency function between the reference signal and the plant input. It is evident that in this case $N(j\omega)$ and $M(j\omega)$ are both stable and $N(j\omega)/M(j\omega)$ represents the frequency response of the plant model. Although, the coprime factors are not unique for a given system, their choice has only an effect for low order controller design and this effect will be reduced by increasing the controller order.

4.3 Finite number of constraints

The constraints in (22) should be satisfied for all $\omega \in \Omega$, which is an infinite set. This problem is known as semi-infinite programming or robust optimization and there exist different methods to solve it. A very simple and practical solution to this problem is to choose a finite set of frequencies

$$\Omega_p = \{\omega_1, \omega_2, \cdots, \omega_p\}$$

and satisfy the constraints for this set. This way, the problem is converted to a semi-definite programming that can be solved efficiently with the existing solvers.

Another solution is to use a randomized approach, in which the constraints are satisfied for a finite set of randomly chosen frequencies. In this approach a bound on the violation probability of the constraints can be derived which goes to zero when the number of samples goes to infinity (see Calafiore and Campi [2006] and Alamo et al. [2010]). It should be mentioned that in a data-driven framework, the frequency domain uncertainties are given by some stochastic bounds. Therefore, even if the constraints are met for all ω, the stability, robustness and performance are guaranteed with a probability level. As a result, the use of randomized method to solve the robust optimization problem in (22) is fully compatible with the uncertainty description of the frequency-domain model of the proposed approach.

4.4 Solution by linear programming

The convex constraints in (22) are equivalent to the following linear constraints:

$$Re\{[NX + MY](j\omega) - \gamma^{-1}e^{j\theta}W_1MY(j\omega)\} > 0, \quad (24)$$

$\forall\omega \in \Omega$ and $\forall\theta \in [0\,, 2\pi[$. In fact, $\gamma^{-1}e^{j\theta}W_1MY(j\omega)$ represents the circle in Fig. 1. Note that $e^{j\theta}$ can be very well approximated by a polygon of $q > 2$ vertices with least area that circumscribes it. By gridding ω and over bounding the circle $e^{j\theta}$, a finite set of linear constraints will be obtained as follows:

$$Re\left\{[NX + MY](j\omega_i) - \gamma^{-1}\frac{e^{j2\pi k/q}}{\cos(\pi/q)}W_1MY(j\omega_i)\right\} > 0$$
$$\text{for } i = 1, \dots, p \text{ and } k = 1, \dots, q \quad (25)$$

Therefore, the convex constraints in (22) can be replaced by the above $p \times q$ linear constraints and then γ can be minimized by an iterative bisection algorithm. At each iteration a linear feasibility problem can be solved efficiently even if the number of constraints are large.

4.5 Experimental results

In this example, the experimental data are used to compute a robust controller with respect to frequency-domain uncertainty. An electro-mechanical flexible transmission system which consists of three disks connected by elastic belts is considered. The first disk is coupled to a servo motor which is derived by a current amplifier. The position of the third disk is measured with an incremental encoder and controlled by a proportional controller. The input of the system is the reference position for the third disk (see Fig. 3). This system is excited by a PRBS signal with a sampling period of $Ts = 40$ms and the data length is 765. Figure 4 shows the experimental data that are used to identify a frequency domain model using `spa` command in Identification toolbox of Matlab. The Nyquist diagram of this spectral model together with the uncertainty disks of 0.95 probability are given in Fig. 5. The uncertainty disks are approximated by a polygon of $m = 20$ vertices and the goal is to design a stabilizing controller that minimizes γ where $\|W_1S\|_\infty < \gamma$, with $W_1(z) = \frac{z-0.96}{z-1}$.

In the proposed method, discrete-time Laguerre basis functions of order 4 with $a = 0$ (FIR filter) are considered for X and Y. The resulting controller is

$$K(z) = \frac{20.3(z^2 - 1.88z + 0.92)(z^2 - 1.278z + 0.6057)}{(z + 0.72)(z - 1)(z^2 + 0.209z + 0.563)},$$

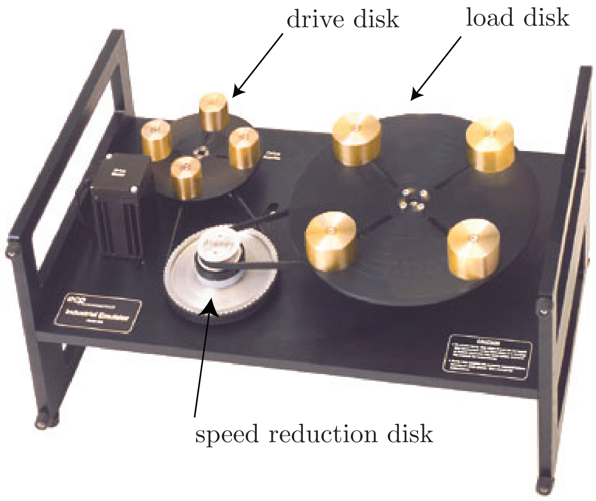

Fig. 3. Flexible transmission system

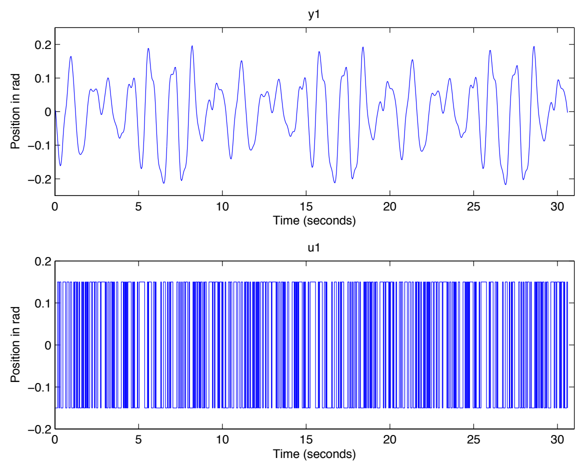

Fig. 4. Experimental identification data.

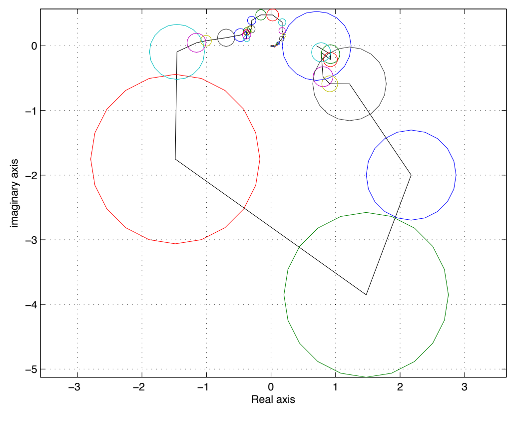

Fig. 5. Nyquist diagram of the spectral model together with uncertainty disks

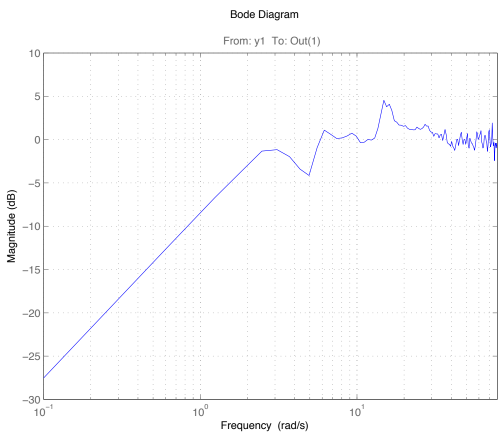

Fig. 6. Magnitude Bode diagram of the sensitivity function

which achieves an optimal performance of $\gamma = 2.12$. Figure 6 shows the magnitude of the Bode diagram of the sensitivity function for the nominal model. It can be observed that the sensitivity function is small at low frequencies and its maximum value is less than 5db which guarantees a good stability margin.

5. CONCLUSIONS

In comparison with the classical model-based H_∞ controller design the following features can be highlighted:

- Only frequency response of the plant is used for controller design and no parametric model is required.
- Pure input/output time delay can be considered with no approximation.
- Frequency-domain uncertainty is taken into account with almost no conservatism.
- Parametric uncertainty in identified models with noisy data can be considered in a stochastic sense with almost no conservatism.
- Fixed-order controllers can be designed with direct optimization (no need for model or controller order reduction).

Clearly, the choice of basis functions affects the optimization results for low-order controllers. Their optimal choice and the extension of the results to multivariable systems are considered for future research works.

REFERENCES

T. Alamo, R. Tempo, and A. Luque. On the sample complexity of probabilistic analysis and design methods. In *Perspectives in Mathematical System Theory, Control, and Signal Processing*, pages 39–55. Springer, 2010.

A. S. Bazanella, L. Campestrini, and D. Eckhard. *Data-driven Controller Design: The H_2 Approach*. Springer, 2012.

G. Calafiore and M. C. Campi. The scenario approach to robust control design. *IEEE Trans. on Automatic Control*, 51(5):742–753, May 2006.

M. C. Campi, A. Lecchini, and S. M. Savaresi. Virtual reference feedback tuning: A direct method for the design of feedback controllers. *Automatica*, 38:1337–1346, 2002.

A. J. Den Hamer, S. Weiland, and M. Steinbuch. Model-free norm-based fixed structure controller synthesis. In *48th IEEE Conference on Decision and Control*, pages 4030–4035, Shanghai, China, 2009.

S. Formentin, K. Van Heusden, and A. Karimi. A comparison of model-based and data-driven controller tuning. *Int. Journal of Adaptive Control and Signal Processing*, 2013. doi: 10.1002/acs.2415.

G. Galdos, A. Karimi, and R. Longchamp. H_∞ controller design for spectral MIMO models by convex optimization. *Journal of Process Control*, 20(10):1175 – 1182, 2010.

H. Hjalmarsson, M. Gevers, S. Gunnarsson, and O. Lequin. Iterative feedback tuning: Theory and application. *IEEE Control Systems Magazine*, pages 26–41, 1998.

R. Hoogendijk, A. J. Den Hamer, G. Angelis, R. van de Molengraft, and M. Steinbuch. Frequency response data based optimal control using the data based symmetric root locus. In *IEEE Int. Conference on Control Applications*, pages 257–262, Yokohama, Japan, 2010.

L. C. Kammer, R. R. Bitmead, and P. L. Bartlett. Direct iterative tuning via spectral analysis. *Automatica*, 36 (9):1301–1307, 2000.

A. Karimi. Frequency-domain robust control toolbox. In *52nd IEEE Conference in Decision and Control*, pages 3744 – 3749, 2013.

A. Karimi and G. Galdos. Fixed-order H_∞ controller design for nonparametric models by convex optimization. *Automatica*, 46(8):1388–1394, 2010.

A. Karimi, L. Mišković, and D. Bonvin. Iterative correlation-based controller tuning. *Int. Journal of Adaptive Control and Signal Processing*, 18(8):645–664, 2004.

A. Karimi, M. Kunze, and R. Longchamp. Robust controller design by linear programming with application to a double-axis positioning system. *Control Engineering Practice*, 15(2):197–208, February 2007.

L. H. Keel and S. P. Bhattacharyya. Controller synthesis free of analytical models: Three term controllers. *IEEE Trans. on Automatic Control*, 53(6):1353–1369, July 2008.

S Khadraoui, HN Nounou, MN Nounou, A Datta, and SP Bhattacharyya. A measurement-based approach for designing reduced-order controllers with guaranteed bounded error. *Int. Journal of Control*, 86(9):1586–1596, 2013.

I. D. Landau, R. Lozano, M. M'Saad, and A. Karimi. *Adaptive Control: Algorithms, Analysis and Applications*. Springer-Verlag, London, 2011.

L. Ljung. *System Identification - Theory for the User*. Prentice Hall, NJ, USA, second edition, 1999.

H. Parastvand and M. J. Khosrowjerdi. Controller synthesis free of analytical model: fixed-order controllers. *Int. Journal of Systems Science*, (ahead-of-print), 2014.

A. Rantzer and A. Megretski. Convex parameterization of robustly stabilizing controllers. *IEEE Trans. on Automatic Control*, 39(9):1802–1808, September 1994.

K. van Heusden, A. Karimi, and D. Bonvin. Data-driven model reference control with asymptotically guaranteed stability. *Int. Journal of Adaptive Control and Signal Processing*, 25(4):331–351, 2011.

K. Zhou. *Essentials of Robust Control*. Prentice Hall, New Jersey, 1998.