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Performance Evaluation of Preventive Maintenance Considering Transportation Delays

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Abstract: This paper considers a production system of a single product, with constant transportation delays of items, between the machine and the buffer and between the buffer and the customer. The demand arrives with a constant rate. The system is modeled by a continuous-flow model. The machine is subject to time-dependent failures and the piloting policy is of hedging point type. We employed a preventive maintenance policy that is realized at each certain period. A performance evaluation of the system is made by simulation.

Keywords: single-product manufacturing system, transfer delay, transportation delay, preventive maintenance, continuous flow model.

1. INTRODUCTION

This study deals with a continuous-flow model (CFM) of a failure-prone manufacturing system. Indeed, this model is very useful for continuous production systems when the number of parts produced in a manufacturing system is important and could be considered as a continuous flow (Mourani et al. (2008)). A lot of authors have performed this model to evaluate performances or to control the manufacturing system and the literature is extended. For example, Plambeck et al. (1996) investigated a failure-prone tandem production line. Machines are arranged in series and each one has different processing rates. Between each pair of machines there is a finite capacity buffer. Wardi and Melamed (2001) considered single-flow continuous flow models and their results were extended to multi-flow continuous flow models. A continuous-flow system, with two stages separated by a finite capacity buffer and different single-flow rates related to each state, was considered in Tan and Gershwin (2009).

However, in CFM or other models, the impact of possible delays in production systems is generally neglected or implicitly considered. To the best of our knowledge, few works consider explicitly delays such as transportation delays or transfer delays. Van Ryzin et al. (1991) showed for the first time the importance of considering delays in a job shop. We can also cite Mourani et al. (2008) who considered an assembly production system of a single-part-type, with machines subject to failures with a period of time that the materials flowing out from the machine must wait before arriving to its downstream buffer, and Turki et al. (2012) who studied a failure-prone manufacturing system composed of one machine with a constant delay between the buffer and the customer. The objective of this paper is to generalize these works by considering transfer and transportation delays in the proposed CFM model.

The considered control policy of the machine is the so called hedging point policy. Kimenia and Gershwin (1983) were the first to study the stochastic optimal control problem of a simple failure-prone manufacturing system with continuous production flow. They established a strategy known as hedging point, with a production surplus h that minimizes the cost, while satisfying the demand. Akella and Kumar (1986) considered a manufacturing system subject to failures, producing a single commodity. The system can be in two states: functional or breakdown state. It was modeled as a continuous time system. The aim of this model is to control the production rate at time t. For this purpose they obtained a critical number called the optimal inventory level which determines the production rate.

Boukas (1998) studied a failure-prone manufacturing system of one machine that produces one piece. This model takes into account the corrective maintenance. Afterwards, the model was widespread to a system formed by M machines and P different parts. This planning control problem showed to be more efficient than the one given by Akella and Kumar (1986) and it increases the system availability and improves the productivity. In Bielecki and Kumar (1988) a model of a simple failure-prone manufacturing system, that produces a single commodity was considered. It was proved that for some conditions, a zero-inventory policy seems to be efficient. A manufacturing system of multiple part-type and with a single machine that is subject to failures, was considerate by Perkins and Srikant (1995). They established optimality conditions of JIT (Just-In-Time) policies and furnished upper bounds on the buffer levels and upper and lower bounds on optimal hedging points.

A more recent work (Gharbi et al. 2010) considered a manufacturing system composed of one-machine that produces one piece and that is subject to failures, taking into account the interactions between periodic preventive maintenance and inventory control problem, with nonnegligible repair and maintenance durations. They considered a three-parameter joint control policy that yields an important cost reduction with respect to the use of a hedging point policy with a conventional periodic preventive maintenance policy.

This study is an extension of the work presented in Gomez Urrutia et al. (2011), taking into consideration the notion of delays. In this paper we present a manufacturing system composed of one machine that produces a single commodity, a finished product buffer and a customer. This system is modeled by a continuous-flow model. We consider constant delays between the machine and the buffer and between the buffer and the customer. The machine is subject to time-dependent failures with TBF (Time between failures) and TTR (Time to repair) following a distribution with an increasing failure rate. We also consider a block preventive maintenance policy to reduce the global costs or increase the availability of the system. In this paper our interest is to make the performance evaluation instead of the control/optimization problem, which will be studied in future works.

The rest of the paper is organized as follows. Section 2 introduces the studied system, the piloting policy, the maintenance policy and the cost function. Section 3 gives an analytical study of the paths of the inventory level and the inventory position. Section 4 contains numerical results of the performance evaluation gotten using simulation. Finally section 5 presents some conclusions and perspectives.

2. STUDIED SYSTEM

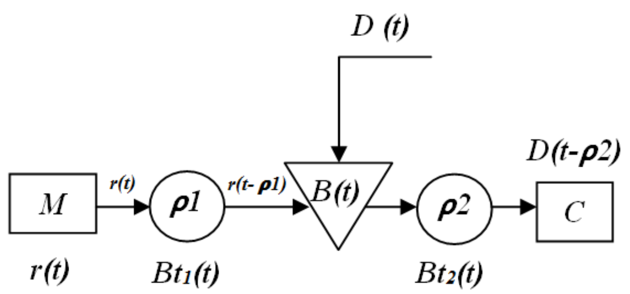

Fig. 1. Studied system

The system is consisting of a machine M, a buffer B with a finite capacity h and a customer C. Products are stored while a constant demand D arrives. Material flow is modeled as a continuous-flow model. This system considers transfer delay between the machine and the buffer, and transportation delay between the buffer and the customer. The number of transported parts at time t from M to B and from B to C is denoted by $Bt_1(t)$ and $Bt_2(t)$ respectively. The transfer and transportation delays $\rho 1$ and $\rho 2$ considered are strictly positive and constant. The machine is subject to dependant time random breakdowns, following a distribution with an increasing failure rate. If M fails an action of corrective maintenance is realized. The preventive maintenance policy consists of realizing a preventive maintenance action whenever the time reaches a certain period tp (block preventive maintenance policy).

Let $\phi(t)$ be a variable that represents the machine state at time t. $\phi(t) = 1$ if M is up, $\phi(t) = 0$ if M is down or repair and $\phi(t) = 2$ if M is in a preventive maintenance.

$$\phi(t) = \begin{cases} 0, \text{if the machine } M \text{ is down or repair} \\ 1, \text{if the machine } M \text{ is up} \\ 2, \text{if the machine } M \text{ is in preventive maintenance} \end{cases} \quad (1)$$

The optimal control policy has a hedging point structure (Kimemia and Gershwin, 1983). The control policy is described as follows: if the inventory level $B(t)$ is lower than the hedging point h^*, the machine produces at its maximum rate R. If $B(t)$ is upper than h^* or the machine is down or in a preventive maintenance, the machine does not produce at all, but if $B(t)$ is equal to h^* it produces at a rate equal to the demand.

Analytically, we can represent the production rate as follows:

$$r(t) = \begin{cases} D, \text{if } \phi(t) = 1 \text{ and } B(t) = h^* \\ R, \text{if } \phi(t) = 1 \text{ and } B(t) < h^* \\ 0, \text{if } \phi(t) = 0 \text{ or } \phi(t) = 2 \text{ or } B(t) > h^* \end{cases} \quad (2)$$

Assumption (1): The maximum production rate R is upper than the demand D in order to satisfy the demand ($R > D$).

Let $I(t)$ be the inventory position at time t.

The inventory position $I(t)$ at time t matches to the sum of the inventory level $B(t)$ at time t and the transported parts at time t. It is given by (3).

$$I(t) = Bt_1(t) + Bt_2(t) + B(t) \quad (3)$$

The system dynamics is given as follows:

$$\frac{dB(t)}{dt} = r(t - \rho 1) - D(t) \quad (4)$$

$$dI(t)/dt = r(t) - D(t - \rho 2), \text{with } D(t - \rho 2) = D \quad (5)$$

The total amount of products in transit between machine M and buffer B is given by (6), and the total amount of products in transit between the buffer B and the customer is given by (7).

$$Bt_1(t) = \int_{t-\rho 1}^{t} r(s).ds \quad (6)$$

$$Bt_2(t) = \int_{t-\rho 2}^{t} D(s).ds \quad (7)$$

Assumption (2). The maintenance policy, is subject to perfect repair, that means that the system returns to "as good as new" after a maintenance action.

The maintenance cost $MC(T)$ based on a type-block strategy is given by (8) (Chelbi and Ait-Kaidi 2004), where T is the preventive maintenance period, ccm matches to the cost of the corrective maintenance action, cpm is the cost of the preventive action and $N_{cm}(T)$ is the mean number of failures:

$$MC(T) = \frac{ccm.N_{cm}(T) + cpm}{T} \quad (8)$$

The total cost $C(t)$ matches to the sum of the storage, the transportation, the maintenance, and the shortage costs. We consider in this paper, that the transportation cost ct_i is fixed. ct_1 is the cost of the transported parts from the machine to the buffer and ct_2 is the cost of the transported pieces between the buffer and the customer.

$$C(t) = g\big(B(t)\big) + Bt_1(t).ct_1 + Bt_2(t).ct_2 + MC(t) \quad (9)$$

with:

$$g\big(B(t)\big) = \begin{cases} B(t).c^+, if\ B(t) \geq 0 \\ B(t).c^-, if\ B(t) < 0 \end{cases} \quad (10)$$

$g\big(B(t)\big)$ denotes the inventory cost at time *t,* c^+ is the unit inventory holding cost and c^- is the shortage cost per unit.
The total expected average cost over an infinite horizon $J(h)$ is then given by:

$$J(h) = \left[\lim_{T\to\infty} \frac{1}{T} \int_{t=0}^{t=T} E\Big(g\big(B(t,h)\big) + Bt_1(t).ct_1 + Bt_2(t).ct_2\Big).dt + MC(t)\right] \quad (11)$$

In the following part, we will study the obtained paths analytically.

3. ANALITICAL STUDY OF THE PATHS

The following theorem shows the link between the inventory level and the inventory position.

Theorem. $B(t) = I(t-\rho1) - D.\rho1 - D.\rho2,\ \forall\, t \geq \min(\rho1, \rho2)$.

Proof. We suppose that at $t = 0$, we have:

$$B(0) = B_0\ ,\ I(0) = I_0 \quad (12)$$

by (4) and (12) we can express the inventory level as:

$$B(t) = B_0 + \int_0^t \big(r(s-\rho1) - D(s)\big).ds \quad (13)$$

since $0 \leq \rho1 \leq t$, we can decompose (13) as follows:

$$B(t) = B_0 + \int_0^{\rho1} \big(r(s-\rho1) - D\big).ds + \int_{\rho1}^{t} \big(r(s-\rho1) - D\big).ds$$

we do $s = s - \rho1$ and we get:

$$B(t) = B_0 + \int_{-\rho1}^{0} \big(r(s)\big).ds + \int_0^{t-\rho1} \big(r(s) - D\big).ds - D.\rho1 \quad (14)$$

by (3) and initial conditions we have:

$$I_0 = B_0 + \int_{-\rho1}^{0} \big(r(s)\big).ds + \int_{-\rho2}^{0} D.ds$$

$$I_0 = B_0 + \int_{-\rho1}^{0} \big(r(s)\big).ds + D.\rho2 \quad (15)$$

So:

$$B_0 = I_0 - \int_{-\rho1}^{0} \big(r(s)\big).ds - D.\rho2 \quad (16)$$

By replacing (16) in (14), we obtain:

$$B(t) = I_0 + \int_0^{t-\rho1} \big(r(s) - D\big).ds - D.\rho1 - D.\rho2 \quad (17)$$

by (5) and (12) and assuming a constant demand we obtain:

$$\frac{dI(t)}{dt} = r(t) - D$$

$$I(t) = I_0 + \int_0^t \big(r(s) - D\big).ds$$

we do $t = t - \rho1$ and we get :

$$I(t-\rho1) = I_0 + \int_0^{t-\rho1} \big(r(s) - D\big).ds \quad (18)$$

Then:

$$I_0 = I(t-\rho1) - \int_0^{t-\rho1} \big(r(s) - D\big).ds \quad (19)$$

By replacing (19) in (17), we finally obtain:

$$B(t) = I(t-\rho1) - D.\rho1 - D.\rho2 \quad (20)$$

Q.E.D.

This result allows highlighting that the fact of using *I* for the calculation of the cost function has not much impact because *B* and *I* are dependant each other and the difference is:

$$I - B = D.\rho1 + D.\rho2,\ \forall\, t \geq \rho1$$

Assumption (3). The time between failures (TBF) and the time to repair (TTR) are i.i.d.

4. NUMERICAL RESULTS

To simulate this problem, we used a discrete event simulation, where possible events are: the failure of the machine M, machine repair, inventory saturation ($B(t)$ reaches h), inventory depletion ($B(t)$ becomes 0), the change in the production rate $r(t)$ of the machine M and the preventive maintenance. We develop a program in Dev-C++ IDE and it is tested on a 2.3 GHz Intel ® Core ™ i7-3610QM with Windows 7. The parameters of the simulation are defined in table 1.

Table 1. Simulation parameters

R	D	c^+	c^-	$\rho 1$	$\rho 2$	TBF		TTR	
						α	β	α	β
4	1	20	200	1	2	8	2	1.5	2

We realized the performance evaluation by employing integer values for the preventive maintenance periods and buffer level units. We calculate the cost function making use of a discrete event simulation. To calculate it, we studied the possible cases for the storage cost between two successive events. These cases were considered in Mourani et al. (2008).

Fig. 2 shows different curves of the cost function with respect to the inventory level for different preventive maintenance periods.

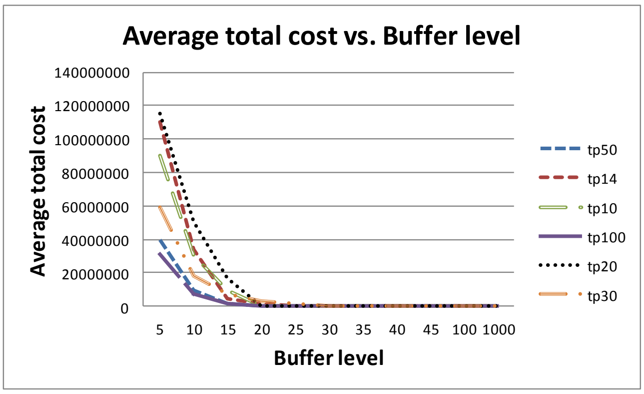

Fig. 2. Average total cost vs. buffer level.

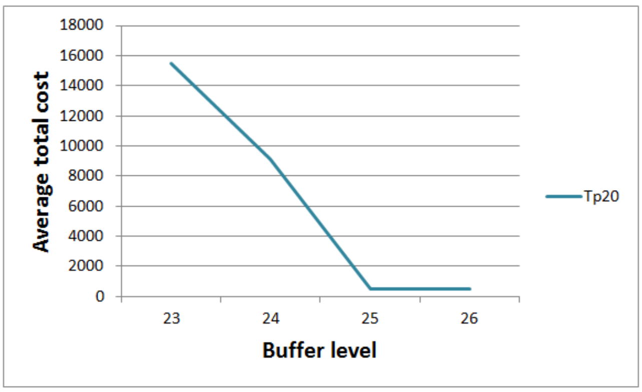

Fig. 3. Enlarged view of the cost function curve.

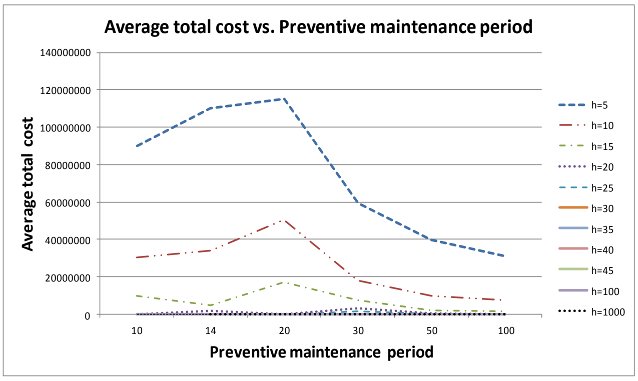

Fig 4. Average total cost vs. preventive maintenance period.

From Fig. 3 and Fig. 4, we can see that the minimum cost value is obtained with a preventive maintenance period equal to 20 units of time and a buffer level h^{*}= 25 units. The associated cost is 478.26 monetary units.

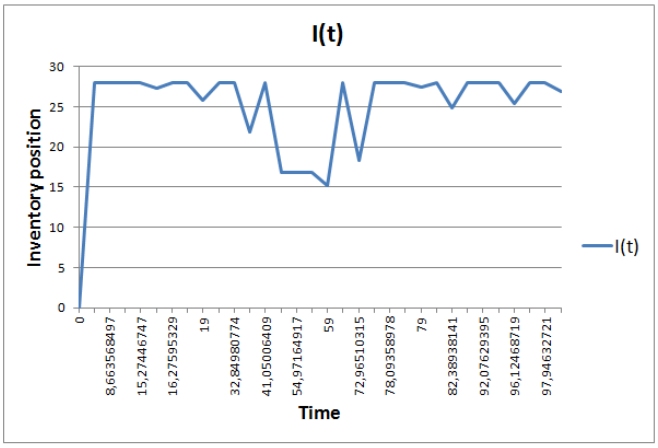

Fig. 5. Inventory position vs. time

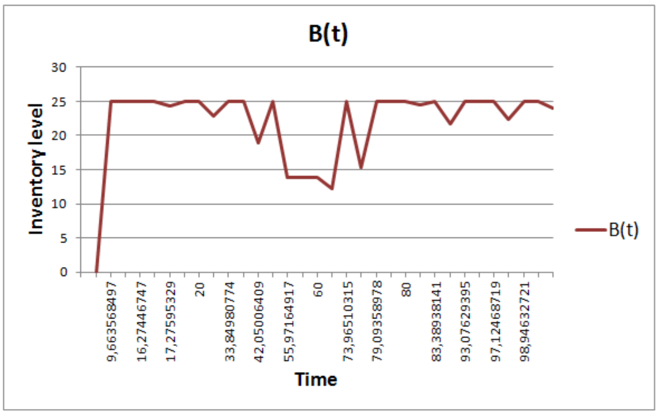

Fig. 6. Inventory level vs. time.

Fig. 5 and Fig. 6 show the link (see Theorem) between the inventory level and the inventory position. The both curves show the effect of the delays and of the constant demand.

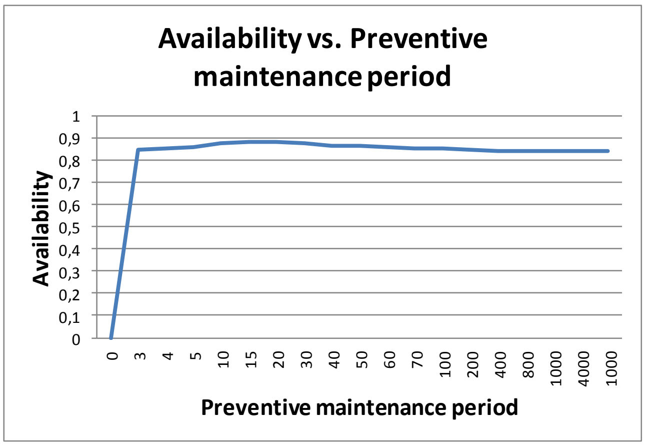

Fig. 7. Availability vs. preventive maintenance period

We calculated the availability of the system depending on different preventive maintenance periods for h^*=25 units (see Fig. 7).
The preventive maintenance period that maximizes the availability is equivalent to 20 units of time. The maximum value of the availability is 88.27%. Relative error and confidence interval are given in Table 2. The confidence intervals are stated at the 95% confidence level.

Table 2. Simulation results for the availability $\rho 1 = 1$ TU (Time units) and $\rho 2 = 2$ TU.

tp	Average availability	Relative error	Confidence interval
10	0,87534589	5,0621E-06	4,43109E-06
14	0,8812919	4,67466E-05	4,11974E-05
20	0,88273238	1,47422E-05	1,30135E-05
30	0,87751846	2,05024E-05	1,79912E-05
50	0,86181165	1,96105E-05	1,69006E-05
100	0,85124923	7,60789E-06	6,47621E-06

We simulated the problem with $\rho 1$ equal to 10 TU and $\rho 2$ equal to 20 TU. We obtained a buffer level h^*= 26 units with an associated cost of 524,34796 monetary units (see Fig. 8).

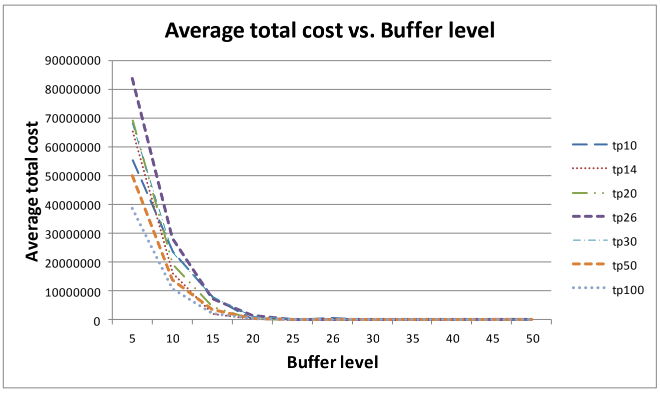

Fig. 8. Average total cost vs. buffer level $\rho 1$=10 TU and $\rho 2$=20 TU.

The preventive maintenance period that maximizes the availability is equal to 26 units of time with a value of 88.28% (see Fig.9). Relative error and confidence interval are given in Table 4.

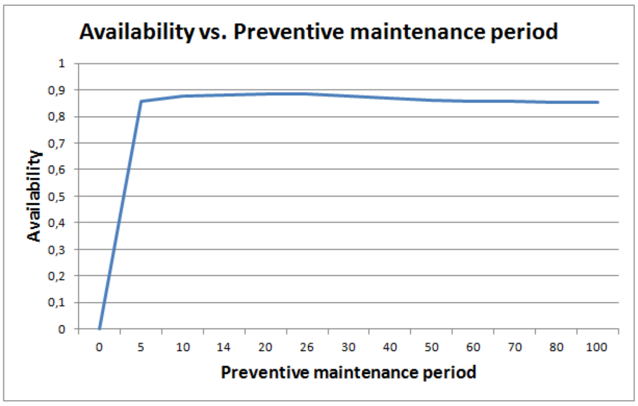

Fig. 9. Availability vs. preventive maintenance period $\rho 1$=10 TU and $\rho 2$=20 TU.

The average total cost is given in Table 3 in both cases. It is specified for the optimal preventive maintenance period.

Table 3. Simulation results for the average total cost

$\rho 1$=1 TU, $\rho 2$=2 TU and *tp*=20 TU		$\rho 1$=10 TU, $\rho 2$=20 TU and *tp*=26 TU	
h	Average total cost	*h*	Average total cost
5	115165704	5	83748291,6
10	50665175,2	10	28198691,3
15	17041543,2	15	7297396,85
20	62557,95	20	1583674,15
25	478,26	25	252303,82
26	498,47	26	524,35
30	627,73	30	604,14
35	677,58	35	704,15
40	777,52	40	804,53
45	877,51	45	904,13
50	977,56	50	1004,06

Table 4. Simulation results for the availability $\rho 1$=10 TU and $\rho 2$=20 TU.

tp	Average availability	Relative error	Confidence interval
10	0,875041111	4,1024E-05	3,58977E-05
14	0,881127778	5,55821E-06	4,89749E-06
20	0,882756	8,71511E-17	7,69331E-17
30	0,877513	1,75344E-16	1,53866E-16
50	0,86182	8,92682E-17	7,69331E-17
100	0,851244	9,03773E-17	7,69331E-17

5. CONCLUSION

In this paper, we study a single-product failure-prone manufacturing system with constant demand and delays. The machine is subject to time-dependent failures, which allows us the use of a preventive maintenance policy. We focus on the performance evaluation instead of optimization problem. We test several values for the preventive maintenance period and for the inventory level. We note that this problem is very sensitive to the chosen parameter values. We have seen that the constant demand as well as the delays are reflected on the path of the inventory position and of the inventory level. The results show that the preventive maintenance policy increases the operation time and the optimal preventive maintenance period maximizes the availability. We also proved by simulation and theoretical study, the link between the inventory level and the inventory position.

For future works, we will use a random demand as well as random delays. We will also apply an optimization method to find the optimal buffer level and the optimal preventive maintenance period. In the same way, we could employ another piloting policy different to the hedging point policy. These results could be extended to assembly/disassembly lines or to transfer lines such as in Fu and Xie (2002). They solve the problem by defining sub-systems composed of two machines separated by a buffer, studying these sub-systems and generalizing the obtained results to the all system.

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