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Dynamic Output Feedback for Nonlinear Networked Control System with System Delays and Packet Dropout

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Abstract: Focusing on networked control systems with state delay and packet dropout, a state feedback control for a class of nonlinear networked control systems with system delays and packet dropout is proposed. Then an observer is designed to estimate the system state. Finally, we achieve stabilization of this class of systems through a dynamic output feedback without augmentation of the state space model. From an appropriate Lyapunov-Krasovskii function, sufficient conditions which guarantee the convergence of the state variables and state estimation errors to the origin are deduced and expressed in terms of simple LMIs. Usability and simplicity are the advantages of this approach. A numerical example is given to illustrate the effectiveness of the proposed approach.

Keywords: Networked control system, output feedback, bounded state feedback, observer design, system delay, packet dropout, nonlinear system.

1. INTRODUCTION

Networked control systems (NCS) are systems with the feedback loop closed through a real time communications network Zhao et al. (2011).

Low cost, system flexibility, simple installation, less cabling system, easier expansion and remote control are some of the main advantages of NCS over classical control systems. These advantages have given NCS great practical interest and allowed their use in various industrial environments such as unmanned aerial vehicles and automated highway systems (Seiler and Sengupta (2005, 2001)), networks with mobile sensors (Ogren et al. (2004)), haptic collaboration in Internet (Hespanh et al. (2000); Hikichi et al. (2002); Shirmohammadi and Woo (2004)) and remote surgical interventions (Meng et al. (2004)).

However, some factors as bandwidth constraints, packet delays, and packet dropping effects may often degrade the performances of a NCS or even cause instability of the feedback control loops. In order to prevent such problems, modelling, stability analysis, and control design of NCS have drawn considerable attention in recent years (*e.g.* Gao and Chen (2007, 2008); Mao and Jiang (2007); Mendez-Monroy and Benitez-Perez (2009); Zhang et al. (2001); W.A. Zhang (2007); Gupta and M.-Y. Chow (2010) and the references therein).

Li et al. derived sufficient conditions for stability based on linear matrix inequality (LMI) in Li et al. (2006), by choosing the proper Lyapunov-Krasovskii functionals and using a descriptor model transformation of the system. By considering all the possibilities of delays, an augmented state space model of the closed-loop system, which characterizes all the delay cases, was obtained in Tang and Ding (2012).

A control scheme which is constituted by a control prediction generator and a network delay compensator was proposed in Xia et al. (2006). In Xiong and Lam (2006), J. L. Xiong and J. Lam modeled the closed-loop system as new Markovian jump linear system with an extended state space, by considering the time varying state delay and the constant time delay in the mode signal.

A sufficient condition for exponential mean-square stability of the NCS was obtained in Li et al. (2011), by designing an observer and an augmented model for NCSs, based on Lyapunov stability theory with LMIs techniques.

The problem of the robust memoryless H_∞ controllers for uncertain NCSs with the effects of both networked-induced delay and data dropout was considered in Yue et al. (2005). A class of discrete-time networked nonlinear systems with mixed random delays and packet dropouts was introduced in Yang et al. (2011), and the filtering problem was investigated. Sufficient conditions for the existence of an admissible filter were established, which ensured the asymptotical stability as well as a prescribed H_∞ performance.

Most papers in the literature deal only with one of the two major problems in NCS, packet dropout or transmission (input/output) delays, while ignoring the other. The few papers that address this issue concern mainly linear NCS (in addition of (Yue et al. (2005); Yang et al. (2011)), we may refer the reader to Sun and Jiang (2013), Xia et al. (2006) and Yu et al. (2005)).

In this paper, first, we give a bounded state feedback controller for the stabilization of a class of nonlinear NCS with state delay and data packet dropout. Second, an observer is designed to estimate the system state. Third, stabilization of this class of systems through an output feedback is achieved without augmenting the state space model which could increase computational complexity, especially for large systems. From an appropriate Lyapunov-Krasovskii function, sufficient conditions that guarantee the convergence of the state variables and state estimation errors to the origin are deduced and expressed in terms of simple LMIs. Usability and simplicity are the advantages of this method.

The paper is organized as follows: the problem formulation and modeling are presented in section 2. Section 3 deals with state feedback stabilization. In section 4, we introduce the observer to estimate the system state. Section 5 is devoted to the dynamic output feedback stabilization. A numerical example is given to illustrate the effectiveness of the proposed design approach in Section 6 and concluding remarks are given in section 7.

2. PROBLEM FORMULATION

The dynamic output feedback networked control system is considered as shown in Fig.1.

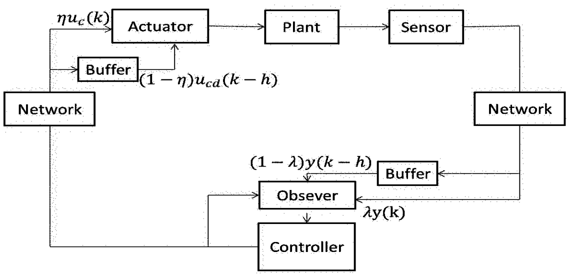

Fig. 1. Structure of dynamic output feedback NCS

Consider a discrete nonlinear time-invariant delay system in the following state space form:

$$x_{k+1} = Ax_k + A_d x_{k-h} + g(x_k)u_k, \tag{1}$$

$$y_k = Cx_k, \tag{2}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$ and $y_k \in \mathbb{R}^q$ denote the state, input and output vectors respectively at time instant k. A, A_d and C_d are constant matrices of appropriate dimensions. $g(x_k)$ is a nonlinear map of appropriate dimension and h is a constant positive number representing the delay. For simplicity of notations, we replace $g(x_k)$ by g_k in the rest of the paper.

The control of a networked control system means that communication will occur through the network from the sensor to the controller and from the controller to the actuator. So delay may occur in both communications; the controller signal (h_1) and the measurements output (h_2). Suppose that $h_1 < h$ and $h_2 < h$.

A buffer is added into the acceptance port of the actuator and another one in the acceptance port of the observer so that the output delay and the control delay are changed into a constant delay. Without loss of generality we consider that the value of this constant delay is equal to the state delay h.

The measurment data packet dropout from the sensor to the controller is modelled as Bernoulli process λ_k with the probability distribution as follow:

$$\begin{aligned} &Prob\{\lambda_k = 1\} = E\{\lambda_k\} = \lambda, \\ &Prob\{\lambda_k = 0\} = 1 - E\{\lambda_k\} = 1 - \lambda, \\ &Var\{\lambda_k\} = E\{(\lambda_k - \lambda)^2\} = \lambda(1-\lambda) = \bar{\lambda}, \end{aligned} \tag{3}$$

where $\lambda_k = 1$ means that the packet transmission will be successful, $\lambda_k = 0$ means that the packet will be lost, the positive constant $0 < \lambda < 1$ is the probability of successful packet transmission, and $\bar{\lambda}$ is the variance of λ_k.

Since the system state is not measurable, we will use an observer to estimate these state variables through the measured system output. If the transmission of system output to the observer through network is successful, then the data $y(k)$ will be used by the observer. Or, if the output data is lost then the most recent delayed data $y(k-h)$ will be used. Thus, the system output can be rewritten

$$y_c(k) = \lambda y(k) + (1-\lambda)y(k-h). \tag{4}$$

Similarly as for the output data, the controller signal can also be delayed or lost through network, and then we have

$$u(k) = u_k(x_k, x_{k-h}) = \eta u_c(k) + (1-\eta)u_{cd}(k-h), \tag{5}$$

$u_c(k)$ and u_{cd} will be detailed later (in Theorem 1). So, the control data transfer from the controller to the actuator is also modelled as Bernoulli process η_k with the probability distribution as follow:

$$\begin{aligned} &Prob\{\eta_k = 1\} = E\{\eta_k\} = \eta, \\ &Prob\{\eta_k = 0\} = 1 - E\{\eta_k\} = 1 - \eta, \\ &Var\{\eta_k\} = E\{(\eta_k - \eta)^2\} = \eta(1-\eta) = \bar{\eta}, \end{aligned} \tag{6}$$

where $\eta_k = 1$ when the packet is transferred successfully (in real time), $\eta_k = 0$ when the packet is lost, the known positive constant $0 < \eta < 1$ is the probability of packet successful transmission, and $\bar{\eta}$ is the variance of η_k.

As a result, we obtain the following networked control system:

$$x_{k+1} = Ax_k + A_d x_{k-h} + g(x_k)\eta u_c(k) + g(x_k)(1-\eta)u_{cd}(k-h), \tag{7}$$

$$y_c(k) = \lambda C x_k + (1-\lambda)C x_{k-h}. \tag{8}$$

This approach takes account of both of the main problems in NCS, namely, data packet dropout and system delays. Our approach considers that the data signal may arrive in real time, or, if it is lost or delayed, the last signal that arrived will be placed in the acceptance buffer in order to have a signal with constant delay for all measurements and control signals.

3. STATE FEEDBACK STABILIZATION

Before proceeding, let us define the sets

$$\Omega = \{x_k \in \mathbb{R}^n : x_k^T \left(A^T PA - P + Q + A^T P \times A_d M^{-1} A_d^T PA\right) x_k = 0,\ k = 0,1,\ldots\},$$

$$S1 = \{x_k \in \mathbb{R}^n : g_k^T PAx_k = 0,\ k = 0,1,\ldots\},$$

$$S2 = \{x_k \in \mathbb{R}^n : g_k^T PA_d x_{k-h} = 0,\ k = h, h+1,\ldots\},$$

$$H = \{x_k \in \mathbb{R}^n : A_d^T PAx_k - (Q - A_d^T PA_d)x_{k-h} = 0,\ k = h, h+1,\ldots\}.$$

Theorem 1. Suppose that there exists an $n \times n$ positive-definite matrix P and an $n\times n$ nonnegative-definite matrix Q, such that

$$\text{H1)} \begin{bmatrix} P-A^TPA-Q & A^TPA_d \\ A_d^TPA & M \end{bmatrix} \geq 0,$$

where

$$M = Q - A_d^TPA_d > 0.$$

If $\Omega \cap S1 \cap S2 \cap H = \{0\}$, then the nonlinear discrete-time delay system (7) is globally asymptotically stabilized by the bounded state feedback (5), where

$$u_c(k) = -\alpha_1 \left[I+g_k^TPg_k\right]^{-1} \frac{g_k^TPAx_k}{1+\|g_k^TPAx_k\|},$$

$$u_{cd}(k-h) = -\alpha_2 \left[I+g_k^TPg_k\right]^{-1} \frac{g_k^TPA_dx_{k-h}}{1+\|g_k^TPA_dx_{k-h}\|},$$

(for any $0<\alpha_1<1$ and $0<\alpha_2<1$).

Proof.

Set

$$\gamma_1 = \frac{\alpha_1 * \eta}{1+\|g_k^TPAx_k\|},$$

$$\gamma_2 = \frac{\alpha_2 * (1-\eta)}{1+\|g_k^TPA_dx_{k-h}\|},$$

then, the control bounded state feedback can also be written

$$u_k = u(k) = -\gamma_1 K_1 x_k - \gamma_2 K_2 x_{k-h}. \tag{9}$$

To show the stability of the closed-loop system (7)-(9), we consider the following Lyapunov-Krasovskii function

$$V_k = x_k^TPx_k + \sum_{i=k-h}^{k-1} x_i^TQx_i. \tag{10}$$

Notice that, since P is positive definite and Q nonnegative definite, V_k is then positive definite.

The difference of this Lyapunov function along the trajectory of the closed-loop (7)-(9) is given by

$$\Delta V_k = x_{k+1}^TPx_{k+1} + x_k^TQx_k - x_k^TPx_k - x_{k-h}^TQx_{k-h}. \tag{11}$$

Using (7) and (9) and after some matrix manipulations, we get

$$\begin{aligned}\Delta V_k &= x_k^T[A^TPA-P+Q]x_k + 2x_k^TA^TP[A_d-\gamma_2 g_kK_2]x_{k-h} \\ &\quad - x_{k-h}Mx_{k-h} - 2\gamma_1x_k^TA^TPg_kK_1x_k + \gamma_1^2x_k^TAPg_kK_1x_k \\ &\quad - 2\gamma_1x_{k-h}^TA_d^TPg_kK_1x_k + \gamma_1\gamma_2x_{k-h}^TA_d^TPg_kK_1x_k \\ &\quad - 2\gamma_2x_{k-h}^TA_d^TPg_kK_2x_{k-h} + \gamma_2^2x_{k-h}^TA_dPg_kK_2x_{k-h} - u_k^Tu_k,\end{aligned} \tag{12}$$

with $M = Q - A_d^TPA_d$.

Adding and subtracting $x_k^TA^TP\hat{A}M^{-1}\hat{A}^TPAx_k$ to and from the inequality (12), we have

$$\begin{aligned}\Delta V_k &= x_k^T[A^TPA-P+Q+A^TP\hat{A}M^{-1}\hat{A}^TPA]x_k - 2\gamma_1x_k^TA^TPg_kK_1x_k \\ &\quad + \gamma_1^2x_k^TAPg_kK_1x_k - 2\gamma_1x_{k-h}^TA_d^TPg_kK_1x_k + \gamma_1\gamma_2x_{k-h}^TA_d^TPg_kK_1x_k \\ &\quad - 2\gamma_2x_{k-h}^TA_d^TPg_kK_2x_{k-h} + \gamma_2^2x_{k-h}^TA_dPg_kK_2x_{k-h} - u_k^Tu_k \\ &\quad + 2x_k^TA^TP\hat{A}x_{k-h} - x_{k-h}Mx_{k-h} - x_k^TA^TP\hat{A}M^{-1}\hat{A}^TPAx_k,\end{aligned} \tag{13}$$

where $\hat{A} = A_d - \gamma_2 g_kK_2$. Furthermore

$$\begin{aligned}\Delta V_k &= x_k^T[A^TPA-P+Q+A^TP\hat{A}M^{-1}\hat{A}^TPA]x_k - 2\gamma_1x_k^TA^TPg_kK_1x_k \\ &\quad + \gamma_1^2x_k^TAPg_kK_1x_k - 2\gamma_1x_{k-h}^TA_d^TPg_kK_1x_k + \gamma_1\gamma_2x_{k-h}^TA_d^TPg_kK_1x_k \\ &\quad - 2\gamma_2x_{k-h}^TA_d^TPg_kK_2x_{k-h} + \gamma_2^2x_{k-h}^TA_dPg_kK_2x_{k-h} - u_k^Tu_k \\ &\quad - [M^{-\frac{1}{2}}\hat{A}^TPAx_k - M^{\frac{1}{2}}x_{k-h}]^T[M^{-\frac{1}{2}}\hat{A}^TPAx_k - M^{\frac{1}{2}}x_{k-h}].\end{aligned} \tag{14}$$

Since $0<\eta<1$, $0<\alpha_1<1$ and $0<\alpha_2<1$, we have

$$2\gamma_1 > \gamma_1^2,\ \ 2\gamma_1 > \gamma_1\gamma_2 \text{ and } 2\gamma_2 > \gamma_2^2.$$

Then, from equation (14), we obtain the following inequality

$$\Delta V_k \leq x_k^T[A^TPA-P+Q+A^TP\hat{A}M^{-1}\hat{A}^TPA]x_k. \tag{15}$$

A sufficient condition to have $\Delta V_k \leq 0$ is

$$A^TPA-P+Q+A^TP\hat{A}M^{-1}\hat{A}^TPA \leq 0. \tag{16}$$

Let us compute $\hat{A}$.

$$\begin{aligned}\hat{A} &= A_d - \gamma_2 g_kK_2, \\ \hat{A} &= A_d - \gamma_2 g_k(I+g_k^TPg_k)^{-1}g_k^TPA_d, \\ \hat{A} &= \left(I - \gamma_2 g_k(I+g_k^TPg_k)^{-1}g_k^TP\right)A_d, \\ \hat{A} &= P^{-1}\left(P - P\gamma_2 g_k(I+g_k^TPg_k)^{-1}g_k^TP\right)A_d.\end{aligned}$$

Since $P - P\gamma_2 g_k(I+g_k^TPg_k)^{-1}g_k^TP \leq P$ we conclude that

$$\begin{aligned}&A^TPA-P+Q+A^TP\hat{A}M^{-1}\hat{A}^TPA \\ &\qquad \leq A^TPA-P+Q+A^TPA_dM^{-1}A_d^TPA.\end{aligned} \tag{17}$$

So, if H1) is verified, then

$$\Delta V_k = V_{k+1} - V_k \leq 0.$$

This prove that the closed loop (7)-(5) is Lyapunov stable. To show the asymptotic stability of the origin, it suffices to show that the largest subset of $\Delta V_k = 0$ invariant under closed-loop dynamics is $\{0\}$.
Setting $\Delta V_k = 0$, it follows from (14) that

$$x_k^T[A^TPA-P+Q+\hat{A}^TPA_dM^{-1}A_d^TP\hat{A}]x_k = 0, \tag{18}$$

$$g_k^TPAx_k = 0, \tag{19}$$

$$g_k^TPA_dx_{k-h} = 0, \tag{20}$$

$$M^{-1/2}A_d^TP\hat{A}x_k - M^{1/2}x_{k-h} = 0, \tag{21}$$

$$u(k) = 0. \tag{22}$$

Using (22), equations (18), (19), (20) and (21) becomes

$$x_k^T[A^TPA-P+Q+A^TPA_dM^{-1}A_d^TPA]x_k = 0, \tag{23}$$

$$g_k^TPAx_k = 0, \tag{24}$$

$$g_k^TPA_dx_{k-h} = 0, \tag{25}$$

$$A_d^T PAx_k - (Q - A_d^T PA_d)x_{k-h} = 0. \tag{26}$$

Thus, we can conclude from the assumption

$$\Omega \cap S1 \cap S2 \cap H = \{0\}$$

that

$$\Delta V(x_k) = 0,$$

for

$$k = 0, 1, \dots$$

implies

$$x_k \equiv 0.$$

The asymptotic stability is, then, proved because all the conditions of LaSalle's invariance principle are verified.

Therefore, the origin is an asymptotically stable equilibrium of the closed-loop system (7)-(9) since $V(x_k) \to \infty$ as $\|x_k\| \to \infty$.

4. OBSERVER DESIGN

In this section a simple and a useful observer design, without state augmentation, for a nonlinear discrete-time networked control delay system will be given.

Theorem 2. Suppose that the function g_k is globally Lipschitz on $\mathbb{R}^{n\times n}$ with a Lipschitz constant β, i. e.

$$\|g(x_k^1) - g(x_k^2)\| \le \beta \|x_k^1 - x_k^2\|.$$

If there exists an $n\times n$ positive-definite matrix S, and an $n\times n$ nonnegative-definite matrix F, such the following LMI holds

$$\text{H1)} \begin{bmatrix} \Pi_1 & \Pi_2 & -\lambda C^T L^T S & 0 \\ * & \Pi_3 & (1-\lambda)C^T L^T S & (1-\lambda)C^T L^T S \\ * & * & S & 0 \\ * & * & * & S \end{bmatrix} \ge 0,$$

where

$$\Pi_1 = S - A^T SA - F + I + \lambda A^T SLC + \lambda C^T L^T SA,$$
$$\Pi_2 = A^T SA_d - (1-\lambda)A^T SLC - \lambda C^T L^T SA_d,$$
$$\Pi_3 = F - 2A_d^T SA_d + 2(1-\lambda)A_d^T SLC + 2(1-\lambda)C^T L^T SA_d.$$

then, the following observer

$$\xi_{k+1} = A\xi_k + A_d\xi_{k-h} + g(\xi_k)u(k) + L[y_c(k) - \lambda C\xi_k - (1-\lambda)C\xi_{k-h}] \tag{27}$$

is an asymptotic observer for the system (7)-(8).

Proof. Let

$$e_k = x_k - \xi_k, \tag{28}$$

then,

$$e_{k+1} = (A - \lambda LC)e_k + (A_d - (1-\lambda)LC)e_{k-h} + [g(x_k) - g(\xi_k)]u(k). \tag{29}$$

Let $\hat{A} = A - \lambda LC$, $A = A_d - (1-\lambda)LC$ and $\phi = g(x_k) - g(\xi_k)$.

The Lyapunov-Krasovskii function is given by

$$W_k = e_k^T Se_k + \sum_{i=k-h}^{k-1} e_i^T Fe_i, \tag{30}$$

then,

$$\Delta W_k = W_{k+1} - W_k, \quad = e_{k+1}^T Se_{k+1} + e_k^T Fe_k - e_k^T Se_k - e_{k-h}^T Fe_{k-h}, \tag{31}$$

or, equivalently

$$\Delta W_k = e_k^T[\hat{A}^T S\hat{A} - S + F]\, e_k + e_{k-h}^T[\tilde{A}^T S\tilde{A} - F]e_{k-h} + u(k)^T\phi^T S\phi u(k) + e_k^T\hat{A}^T S\phi u(k) + u(k)^T\phi^T S\hat{A}e_k + e_k^T\hat{A}^T S\tilde{A}e_{k-h} + e_{k-h}^T\tilde{A}^T S\hat{A}e_k + e_{k-h}^T\tilde{A}^T S\phi u(k) + u(k)^T\phi S^T\tilde{A}e_{k-h}, \tag{32}$$

then,

$$\Delta W_k = e_k^T\,[\hat{A}^T S\hat{A} - S + F]\,e_k + u(k)^T\phi^T S\phi u(k) + 2e_k^T\hat{A}^T S\phi u(k) - e_{k-h}^T M1 e_{k-h} + 2e_k^T\hat{A}^T S\tilde{A}e_{k-h} + 2u(k)^T\phi^T S\tilde{A}e_{k-h}, \tag{33}$$

with $M1 = F - \tilde{A}^T S\tilde{A}$.

Since $2z^T Dy \le z^T Dz + y^T Dy$, we have

$$\Delta W_k \le e_k^T\,[\hat{A}^T S\hat{A} - S + F]\,e_k + u(k)^T\phi^T S\phi u(k) + 2e_k^T\hat{A}^T S\phi u(k) - e_{k-h}^T M1 e_{k-h} + 2e_k^T\hat{A}^T S\tilde{A}e_{k-h} + u(k)^T\phi^T S\phi u(k) + e_{k-h}^T\tilde{A}^T S\tilde{A}e_{k-h}, \tag{34}$$

then,

$$\Delta W_k \le e_k^T\,[\hat{A}^T S\hat{A} - S + F]\,e_k + u(k)^T(\phi^T S\phi + \phi^T S\phi)u(k) + 2e_k^T\hat{A}^T S\phi u(k) - e_{k-h}^T(M1 - \tilde{A}^T S\tilde{A})e_{k-h} + 2e_k^T\hat{A}^T S\tilde{A}e_{k-h}, \tag{35}$$

or,

$$\Delta W_k \le e_k^T\,[\hat{A}^T S\hat{A} - S + F]\,e_k + 2u(k)^T\phi^T S\phi u(k) + 2e_k^T\hat{A}^T S\phi u(k) - e_{k-h}^T Ne_{k-h} + 2e_k^T\hat{A}^T S\tilde{A}e_{k-h}, \tag{36}$$

with $N = M1 - \tilde{A}^T S\tilde{A} = F - 2\tilde{A}^T S\tilde{A}$.

Adding and subtracting $e_k^T\hat{A}^T S\tilde{A}N^{-1}\tilde{A}^T S\hat{A}e_k$, we have

$$\Delta W_k \le e_k^T\,[\hat{A}^T S\hat{A} - S + F + \hat{A}^T S\tilde{A}N^{-1}\tilde{A}^T S\hat{A}]\,e_k + 2u(k)^T\phi^T S\phi u(k) + 2e_k^T\hat{A}^T S\phi u(k) - [N^{-1/2}\tilde{A}^T S\hat{A}e_k - N^{1/2}e_{k-h}]^T \times [N^{-1/2}\tilde{A}^T S\hat{A}e_k - N^{1/2}e_{k-h}]. \tag{37}$$

This, in turn, implies

$$\Delta W_k \le e_k^T\,[\hat{A}^T S\hat{A} - S + F + \hat{A}^T S\tilde{A}N^{-1}\tilde{A}^T S\hat{A}]\,e_k + 2u(k)^T\phi^T S\phi u(k) + 2e_k^T\hat{A}^T S\phi u(k). \tag{38}$$

Using the hypothesis H2), we have

$$\Delta W_k \le -e_k^T e_k + 2u(k)^T\phi^T S\phi u(k) + 2e_k^T\hat{A}^T S\phi u(k), \tag{39}$$

or,

$$\Delta W_k < -e_k^T e_k + 2u(k)^T\,[g(x_k) - g(\xi_k)]^T S[g(x_k) - g(\xi_k)]\,u(k) + 2e_k^T\hat{A}^T S\,[g(x_k) - g(\xi_k)]\,u(k. \tag{40}$$

From the Lipschitz condition of $g(.)$ and the boundness of the state feedback $u(k)$ ($u(k) < \alpha = \alpha_1 + \alpha_2$), we deduce that

$$\Delta W_k < -\|e_k\|^2(1 - 2\alpha^2\beta^2\|S\| - 2\alpha\beta\|(A - \lambda LC)S\|), \tag{41}$$

where β is the Lipschitz constant associated with $g(.)$.

Obviously it is possible to choose $\alpha > 0$ sufficient small so that for some θ, $0 < \theta < 1$

$$\Delta W_k = W_{k+1} - W_k < -\theta e_k^T Se_k, \tag{42}$$

then, we ensure the global asymptotic stability of the system (29). We can conclude also that

$$\|e_k\| \leq \sigma, \text{ for } k = 1,2,3,\ldots \text{ and } \sigma > 0. \tag{43}$$

5. DYNAMIC OUTPUT FEEDBACK

Theorem 3. Under Assumptions H1) and $\Omega \cap S \cap H = \{0\}$, a discrete-time MIMO nonlinear system (7)-(8) can be globally asymptotically stabilized by the dynamic compensator (27) with the control input defined as in (5)

$$u(k) = u_k(\xi_k, \xi_{k-h}), \tag{44}$$

for a sufficient small $\alpha > 0$ ($\alpha = \alpha_1 + \alpha_2$). $g(\xi)$ is globally Lipschitz and L is such that H2) is verified.

Proof. By theorem 1, for a given definite positive matrix B, the following inequality

$$\|A\xi_k + A_d\xi_{k-h} + g(\xi_k)u(k)\|_B^2 \leq \|\xi_k\|_B^2, \ \text{k=1,2,3,...} \tag{45}$$

is satisfied.

Without loss of generality, let B = I in (45). We deduce from (27), (29) and (45) that

$$\begin{aligned} \|\xi_{k+1}\| &\leq \|A\xi_k + A_d\xi_{k-h} + g(\xi_k)u(k)\| + \|LC_d e_{k-h}\| \\ &\leq |\xi_k\| + \sigma \quad \leq \ldots \\ &\leq |\xi_0\| + \sigma. \end{aligned} \tag{46}$$

Inequalities (43) and (46) allow us to conclude that all trajectories of the closed loop system (27)-(29) are bounded.

Now, we consider that (e_k, ξ_k) is a trajectory of system (27)-(29) with the initial value (e_0, ξ_0).

Let *mo* denotes its ω-limit set. It is clear that *mo* is nonempty, compact, and invariant because (e_k, ξ_k) is bounded for $k = 1,2,3,\ldots$. In addition, we conclude from theorem 2 that $\lim_{k\to\infty} e(k) = 0$.

Then, any point in *mo* must be of the form $(0, \xi_k)$. Let $(0, \bar{\xi}) \in$ *mo* and $(0, \bar{\xi}_k)$ be the corresponding trajectory.This trajectory is described by the equation

$$\bar{\xi}_{k+1} = A\bar{\xi}_k + A_d\bar{\xi}_{k-h} + g(\bar{\xi}_k)u_k. \tag{47}$$

We already proved that this trajectory is globally asymptotically stable at $\xi = 0$. This means that the global asymptotic behavior of the closed-loop system (27)-(29) at (e, ξ) = (0,0) is determined by the flow on the invariant manifold governed by system (47) (Carr (1981)). Since this last system is globally asymptotically stable, so is the closed-loop system (27)-(29)

Remark 1. Compared with other approaches in the literature:
1- The approach proposed in this paper has the advantage of taking into account the two major problems in the NCS, namely data packet dropout as well as system delays, from both the sensor-to-controller and the controller-to-actuator.
2- We did not augment the state space to obtain an augmented delay-free system. This will increase computational complexity, especially for large systems .

6. NUMERICAL EXAMPLE

In this section, a numerical example is presented, to illustrate how the methods developed so far can be used to solve the problems of stabilization, observer design and dynamic output feedback stabilization of nonlinear NCS.

Consider the system (7)-(8) with the following matrices

$$A = \begin{bmatrix} 0.73624 & 0.0452 \\ 0.0915 & 0.4462 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.2456 & 0.0151 \\ 0.0305 & 0.1487 \end{bmatrix},$$

$$g(x_k) = \begin{pmatrix} g_1(x_k) \\ g_2(x_k) \end{pmatrix},$$

where

$$g_1(x_k) = \frac{x_1(k)}{1+x_1^2(k)+x_2^2(k)}, \quad g_2(x_k) = \frac{x_2(k)}{1+x_1^2(k)+x_2^2(k)},$$

and

$$C = [1 \quad 0],$$

with

$$x_0 = \begin{pmatrix} -3.5 \\ 2.8 \end{pmatrix}, \quad \xi_0 = \begin{pmatrix} 2.5 \\ -2.3 \end{pmatrix} \text{ and } h = 4.$$

The unforced dynamics of this system is not asymptotically stable but only Lyapunov stable. So, the whole system need to be asymptotically stabilized.

Resolution of the LMI H1) gives:

$$P = \begin{bmatrix} 614.0062 & -48.2905 \\ -48.2905 & 456.6416 \end{bmatrix}, Q = \begin{bmatrix} 162.4833 & -41.6186 \\ -41.6186 & 212.3097 \end{bmatrix}.$$

And resolution of H2) gives the following results:

$$S = \begin{bmatrix} 2.2751 & -0.0006 \\ -0.0006 & 2.2766 \end{bmatrix}, F = \begin{bmatrix} 1.8578 & -0.0200 \\ -0.0200 & 1.6645 \end{bmatrix},$$

which allows to compute L :

$$L = \begin{pmatrix} 0.0248 \\ 0.0186 \end{pmatrix}.$$

Applying the control law (44) with the observer (27), with $\lambda = 0.5$ and $\eta = 0.5$, we ensure, as shown in Figures 2-5, the decrease of the Lyapunov-Krasovskii functions (10) and (30), and that (x, e) = (0,0) is a global asymptotically stable equilibrium of (7)-(8)-(29) .

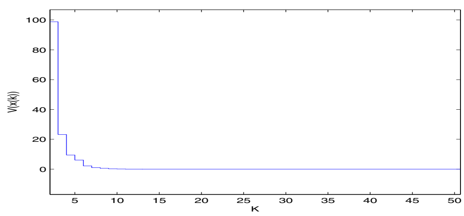

Fig. 2. $V(x_k)$ with respect to sampling time k.

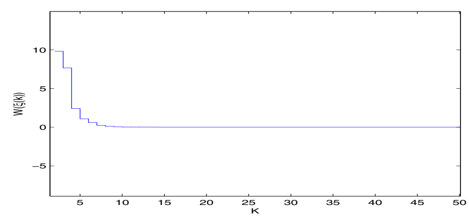

Fig. 3. $W(e_k)$ with respect to sampling time k.

Numerical example illustrate how the developed approach is simple to implement and the practical applicability of the LMI conditions H1) and H2) in the stabilization of a class of a NCS.

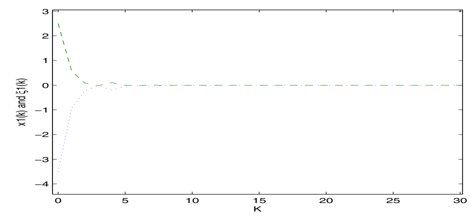

Fig. 4. $x_1(k)$ and $\xi_1(k)$ with respect to sampling time k.

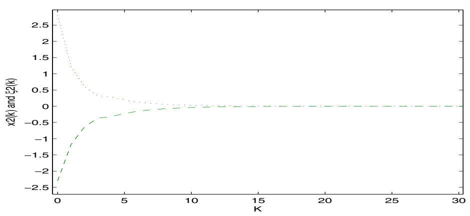

Fig. 5. $x_2(k)$ and $\xi_2(k)$ with respect to sampling time k.

7. CONCLUSION

In this paper, we studied the stabilization of a class of nonlinear NCS with state delay and packet dropout. A state feedback that guarantees the convergence of the state variables to the origin was presented. Then, we introduced an observer that estimates the state variables of this class of systems. Finally, a dynamic output feedback which stabilizes this class of nonlinear NCS was achieved. LMI sufficient conditions to characterize the state feedback controller, the observer and dynamic output feedback have been developed. Finally, we illustrated the approach developed so far using a numerical example. Our approach consider the two main problems in NCS, namely, data dropout and system delays. The advantages of this method are simplicity and usability.

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