Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Finite Element Model Reduction and Model Updating of structures for Control

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Abstract:
Estimation and control of unmeasurable performance variables in complex large-scale systems is an important issue in systems and control. The preferred solution to this problem is to have a relatively low-order and accurate standard plant model which can be used for control purposes. For this purpose, a two-step procedure is proposed. The first step is to generate a reduced Finite Element (FE) model based on the selection of desired Degrees Of Freedom (DOFs), resulting in reduced-order mass, damping, and stiffness matrices. The second step is updating of the reduced-order FE model that is carried out to minimize the differences between the model and the measurements from the structure with the focus on input-output behavior. The presented approach helps to create sufficiently accurate reduced-order dynamic models which can be used for control purposes. The approach will be examined on a planar plate FE model.

Keywords: Model updating, model reduction, finite element model, identification, model-based control.

1. INTRODUCTION

A wafer scanner lithography machine is the state-of-the-art equipment for the automated production of ICs. During the production, a wafer stage must track a predefined reference trajectory in six motion DOFs with a very high accuracy. Due to the developments in lithographic production processes, next-generation wafer stages are expected to be lightweight. Reasons for this are: (a) market viability requires a high throughput of the wafers demanding high accelerations in all six motion DOFs; high acceleration requires a reduction of the mass, motivating a lightweight system design, and (b) the wafer diameter is expected to increase from 300 to 450mm to increase productivity. This requires increased dimensions of the wafer stage, which again underlines the importance of a lightweight system design. As a result of a lightweight system design, next-generation wafer stages predominantly exhibit flexible dynamical behavior at lower frequencies, see Fig. 1. This has important consequences on the positioning performance of the wafer stage.

Next-generation wafer stages are: (a) inherently multivariable, since the flexible dynamical behavior is generally not aligned with the motion DOFs, (b) are envisaged to be designed with many actuators and sensors to actively control flexible dynamical behavior, whereas traditionally the number of inputs and outputs equals the number of motion DOFs. In next generation wafer stages, a dynamical relation exists between measured and performance variables, since the sensors are generally located at the edge of the wafer stage, while the performance is required

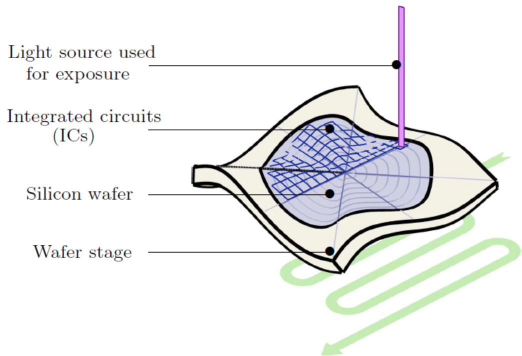

Fig. 1. Schematic of a light-weight wafer scanner, van Herpen (2014).

on the die under exposure on the wafer, which we call the Point Of Interest (POI). In contrast, the flexible dynamical behavior is often neglected in traditional wafer stages, leading to an assumed static geometric relation between measured and performance variables.

The application of lightweight wafer stages motivates the usage of model based control and observer design, since: (a) a model-based design provides a systematic control design procedure for multivariable systems; (b) a model is essential to investigate and achieve the limits of performance; (c) a model-based observer design procedure enables the estimation of unmeasured performance variables at POI from the measured variables through the use of a model (Oomen et al., 2014).

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For estimation of the position at POI, we need an accurate standard plant model which has disturbance (w), performance (z), input (u) and output (y) channels, see Fig. 2. There are two different approaches to obtain the model of a wafer stage. The first approach uses identification techniques. However, identification techniques can at best identify a model from measured inputs to measured outputs i.e. G_{yu} in Fig. 2. So identification techniques cannot identify performance channels in the standard plant i.e G_{zu} and G_{zw} in Fig. 2. Furthermore, to the best of the authors knowledge, a reliable Multi Input-Multi Output (MIMO) parametric identification approach to fully and precisely identify a high-dimensional MIMO system with many Input-Output (IO) channels and many state variables is not available. This is caused by several reasons like difficulty in model order selection, in criteria selection and complexity of finding a computationally reliable identification model. Oomen et al. (2011), present and apply a framework for system identification for robust inferential control that can deal with unmeasured performance variables at POI, but their goal is not to create an accurate standard plant model.

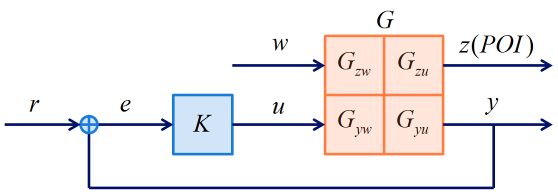

Fig. 2. Standard Plant diagram.

The second approach uses analytical FE models, but also here there are some issues: (a) FE models are generally of high order, so the resulting FE dynamic model is not directly useful, (b) there are differences between dynamic properties of the real structure and the FE model. Despite the aforementioned issues, the second approach seems more promising because with a FE model we have access to a complete standard plant, i.e. G_{yu}, G_{yw}, G_{zu} and G_{zw} in Fig. 2. In order to overcome the mentioned issues, we need to reduce the order of the model and then, update the model based on real structure measurements.

In practice, there are always differences between a Finite Element (FE) model of a system and the real system. The differences could be due to simplification of the model structure, additional facilities in the real hardware, uncertainty in the physical properties, discretization errors, and so on. The area known as *model updating* is concerned with the correction of FE models by processing records of dynamic responses from real structures, (Mottershead and Friswell, 1993). FE model updating is often required to identify and correct the uncertain parameters of a FE model and is usually posed as an optimisation problem, (Jaishi and Ren, 2007). In many cases, the dynamics of the hardware will be examined by performing an experimental modal analysis, to see how much the true hardware complies with the numerical FE model.

In literature, model updating techniques are presented which try to update the physical parameters of the FE model like parameters in discrete mass, damping, stiffness elements, plate thickness, and so on. A survey has been given in Mottershead and Friswell (1993). Friswell and Penny (1990) studied updating model parameters from frequency domain data via reduced order models constructed from a Taylor series expansion of the model differential equation. Imergun et al. (1995b) further investigated the FE model updating based on frequency response measurements. The approach is studied on a medium-sized FE model of a plate-beam structure which is modeled using about 500 DOFs in Imergun et al. (1995a). This work has been completed later in the comprehensive work of Grafe (1998). Jaishi and Ren (2005) demonstrate a comparative study on the influence of different possible residuals in the objective function. Frequency residual only, mode shape related functions only, modal flexibility residuals only, and their combinations are studied independently. D'Ambrogio and Fregolent (2000) present an updating technique that includes anti-resonances in the definition of the output residuals.

Most techniques have been applied and verified on non-industrial FE models with a limited number of DOFs. In fact, they become computationally expensive for large-scale system models. However, Jaishi and Ren (2005) applied model updating on a model of a bridge with 14060 DOFs and Mottershead et al. (2011) applied sensitivity-based model updating technique on a helicopter air-frame model.

The goal of this research is to obtain an updated Reduced order FE model of a wafer stage, which can be used for POI estimation and model-based control. The original FE model of a wafer stage has a large number of DOFs. In our approach, first the FE model is reduced, and then some physical parameters of the reduced FE model are updated based on the eigenvalue sensitivity technique. Having a reduced FE model, and model updating technique, facilitates the investigation of the dynamic influence of changing for example the location of actuators and sensors in the structure. Since even small differences in hardware properties in similar lithographic machines may already be relevant for high-performance motion control design, it is useful to be able to obtain machine-specific models. This also relaxes the robustness requirements in the control design, paving the way to improved machine performance.

The remainder of this paper is organised as follows. In Section 2, a modal-based approach to construct the reduced FE model is described. Then, formulation of the model updating technique, which has been used in this research, is discussed in Section 3. To illustrate our approach, a FE model of a plate instead of a real wafer stage is introduced in Section 4.1 which will be reduced in Section 4.2. The results on the updated reduced plate FE model are discussed in Section 4.3. An outlook regarding controller and observer design is given in Section 5 and finally, conclusions are given in Section 6.

2. FINITE ELEMENT MODEL REDUCTION

Updating a FE model has several complexities: (a) it is computationally expensive to update a complex FE model of an industrial problem, which typically has a number of DOFs in the order $n \approx 10^6$, (b) assuming a correct model structure, in order to update a FE model, the updating parameters i.e. design parameters have to be selected; this means we have to find parameters in the FE model, which cause the discrepancy between the real structure and the analytical model, which is not easy most of the times. (c) Moreover, the updated parameters should preferably have some physical interpretation, so that we can understand the result of the mathematical updating process. A possible solution to (a) is to reduce the size of the FE model by selecting only modes in a frequency band of interest, see (3). Then, the reduced set of generalized DOFs should be replaced by desired physical DOFs where physical properties of the FE model can be updated. Desired DOFs can also include DOFs, which are involved in the location of the actuators/sensors or DOFs associated with the POI in the performance channel, see (6).

There are a number of approaches for model order reduction, see Besselink et al. (2013) and see Schilders et al. (2008). In this paper, we follow a standard modal model reduction by selecting only low frequency dominant modes and then reconstruct the FE mass, stiffness and damping matrices based on the selected desired DOFs. In future research, we will include the compliance of the truncated higher modes in the model.

A linear flexible structure with viscous damping can be represented by the following second-order matrix differential equation:

$$M\ddot{q}_n + D\dot{q}_n + Kq_n = f, \tag{1}$$

where $q_n \in \mathbb{R}^{n\times 1}$ is a vector with n DOFs, with $M \in \mathbb{R}^{n\times n}$ positive-definite mass matrix, $D \in \mathbb{R}^{n\times n}$ a positive semi-definite damping matrix, and $K \in \mathbb{R}^{n\times n}$ a positive semi-definite stiffness matrix.

Assuming proportional damping, the following eigenvalue problem can be solved:

$$\big(-\omega_i^2 M + K\big)\phi_i = 0. \tag{2}$$

We select a subset of the eigenvector matrix ϕ to include the k dominant mass normalized low frequency modes:

$$\phi_{(n,k)} = [\phi_1 \ \phi_2 \ \cdots \ \phi_k]\,. \tag{3}$$

These may include rigid body modes. By applying the transformation,

$$q_n = \phi_{(n,k)} p_k, \tag{4}$$

a low-order modal form of the original dynamical model can be generated:

$$I_{(k,k)}\ddot{p}_k + 2Z_{(k,k)}\Omega_{(k,k)}\dot{p}_k + \Omega^2_{(k,k)}p_k = \phi^T_{(n,k)}f, \tag{5}$$

where Ω and Z are diagonal matrices with angular eigenfrequencies and dimensionless modal damping factors, respectively. Assume that q_n is partitioned as $q_n = [q_k^T \ q_s^T]^T$, where q_k contains the desired DOFs and q_s contains the remaining DOFs. Then, the eigenvector matrix $\phi_{(n,k)}$ will be partitioned accordingly,

$$q_n = \begin{bmatrix} q_k \\ q_s \end{bmatrix} = \phi_{(n,k)}p_k = \begin{bmatrix} \phi_{(k,k)} \\ \phi_{(s,k)} \end{bmatrix} p_k. \tag{6}$$

From (6), it follows that

$$p_k = \phi^{-1}_{(k,k)} q_k. \tag{7}$$

By substituting (7) into (5), followed by pre-multiplication with $\phi^{-T}_{(k,k)}$ we obtain

$$\begin{aligned} &\overbrace{\big(\phi^{-T}_{(k,k)} I_{(k,k)} \phi^{-1}_{(k,k)}\big)}^{M^*} \ddot{q}_k + \\ &\overbrace{\big(\phi^{-T}_{(k,k)} 2Z_{(k,k)}\Omega_{(k,k)} \phi^{-1}_{(k,k)}\big)}^{D^*} \dot{q}_k + \\ &\overbrace{\big(\phi^{-T}_{(k,k)} \Omega^2_{(k,k)} \phi^{-1}_{(k,k)}\big)}^{K^*} q_k = \overbrace{\phi^{-T}_{(k,k)}\phi^T_{(n,k)} f}^{f^*}, \end{aligned} \tag{8}$$

which contains the reduced mass, damping, and stiffness matrices: M^*, D^*, and K^*, relates to q_k. As a results, a reduced FE model of a dynamical system has been generated, which has almost the same dynamic input-output behavior as the original model in the frequency range $\omega \in [0, \omega_k]$. Using this reduced-order FE model strongly improves the efficiency of the subsequent model updating compared to using the original high-order FE model.

It has to be mentioned that the interpretation of the entries of the reduced mass, damping, and stiffness matrices in general is not straightforward. However, since input-output behavior is often more important for control, this need not be problematic.

3. MODEL UPDATING

Having a reduced FE model in hand, we can start updating the model based on experimental measurements. Mottershead et al. (2011) claim that the sensitivity methods are probably the most successful among the many approaches to the problem of updating FE models of engineering structures. The claim is partly based on the successful application to a large-scale FE model of a helicopter airframe. The sensitivity method is based on the minimization of modal residuals in an iterative procedure based on the following equation,

$$[S]_i \left\{\Delta p\right\}_i = \{\varepsilon\}_i, \tag{9}$$

where $[S]$ is the sensitivity matrix, $\{\Delta p\}$ represents the changes in the updating parameters, $\{\varepsilon\}$ are the residuals, i.e. the differences between the measured and predicted dynamic properties and i indicates the iteration. Here, existing differences in the eigenvalues and possibly eigenvectors are contained in the residual vector $\{\varepsilon\}$, whereas the sensitivity matrix $[S]$ embodies the first-order sensitivities of eigenvalues or eigenvectors or both with respect to the selected updating parameters.

In this research, we formulate the model updating problem based on the sensitivity method and minimization of the eigenvalue residuals (de Kraker, 2013). Assume that we want to update $p \in \mathbb{R}^{q\times 1}$ design parameters in the FE model and that we have experimental information about m eigenvalues, i.e. $\lambda_{exp} \in \mathbb{C}^{m\times 1}$, then we define the deviation from the corresponding numerical eigenvalues $\lambda_{num} \in \mathbb{C}^{m\times 1}$, in each iteration, as follows:

$$\varepsilon_i = \Delta\lambda_i = \lambda_{exp} - (\lambda_{num})_i. \tag{10}$$

Then, entries S_{ab} of the sensitivity matrix S in (9) are defined as,

$$(S_{ab})_i = \frac{\partial \lambda_{anum}}{\partial p_b}|_{p_i}, \tag{11}$$

which are derivatives of the numerical eigenvalues with respect to the design parameters p_b for a specific value of these design parameters in the corresponding iteration. In fact we are dealing with complex equations with real unknowns. Since $[S]_i$ is a rectangular matrix, (11) in general cannot be solved directly, so we apply a least squares approach based on minimization of the scalar real error measure,

$$e_i = \Delta\lambda_i^H W \Delta\lambda_i, \tag{12}$$

where a real positive definite diagonal matrix W is added for the possibility of different weighting of the errors in the eigenvalues. Then, minimizing (12) leads to

$$Re\left[S_i^H W S_i\right]\Delta p_i = Re\left[S_i^H W \Delta\lambda_i\right], \tag{13}$$

and a new estimate for the updating parameter is found by

$$p_{i+1} = p_i + \Delta p_i. \tag{14}$$

Recalculation of (13) based on the newly obtained parameters in several iterations will result in convergence of the design parameters to the optimal values.

4. ILUSTRATIVE EXAMPLE

To better illustrate our approach, a FE model of a planar plate instead of a real wafer stage is introduced in the next section.

4.1 A Planar Plate Finite Element Model

Fig. 3 shows the FE model of an isotropic aluminum plate, which is a very simplified model of a wafer stage in lithography machines. It consists of 8×8 Kirchhoff plate elements. Each element has four nodes, and each node has three DOFs: transversal displacement Z, and rotations R_x, R_y. The system mass, damping, and stiffness matrices have dimension 243×243. Geometric and physical properties of the plate are listed in Table 1. The original nodes are indicated by the dark grey dots in the picture. The green circles indicate the locations of the four out-of-plane actuators $u_1, \cdots, u_4$ while the four blue crosses at the corners of the plate indicate the locations of the four out-of-plane sensors $y_1, \cdots, y_4$. A modal damping of 0.1% is assumed for all modes in the model.

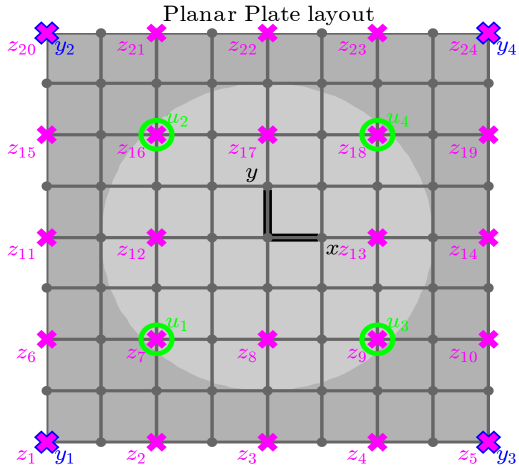

Fig. 3. FE plate model.

4.2 Reduction of Plate Model

A reduced modal model (5) of the plate is created by selecting the lowest $k = 24$ modes including the rigid body modes. Subsequently, a reduced FE model (8) based on 24 desired DOFs indicated by purple crosses in Fig. 3 is created. From each desired node the z-DOF is selected.

As an example, in Fig. 4, a comparison between a Bode magnitude plot based on the original FE model and on the reduced FE model is made. Clearly, up to $2.7kHz$ the reduced FE model has almost the same input-output behavior as the original FE model. The model has 3 rigid body modes. The torsion mode shape, which is the fourth mode and the first elastic mode, of both original and the reduced FE model are depicted in Fig. 5 and Fig. 6, respectively. Both the eigenfrequency (see the first resonance in Fig. 4) and the mode shape are obviously identical.

4.3 Model Updating Of The Reduced Plate Model

Given the reduced plate model, assume that an extra actuator with a point mass is added to the original plate at the location indicated by z_8 in z direction, see Fig. 3. We would like to examine its influence on the dynamic behavior of the plate. One way is to include the added actuator in the original high-order FE model, but this approach is generally expensive and time-consuming. The alternative examined here is to update the reduced FE model. This also better mimics a practically realistic situation where an actuator is added to existing hardware and an accurate model for control of the modified system must be made. Note that the added mass does not affect the first elastic torsional mode, because it is located on the nodal line of this mode.

Table 1. Aluminum Plate Properties

Property	Quantity	Unit
Plate Length	0.6	m
Plate Width	0.6	m
Plate Thickness	0.0272	m
Plate Density (ρ)	2702	kg/m^2
Plate Youngs Modulus (E_p)	70e9	Pa
Plate Poisson Ratio (ν)	0.35	$-$

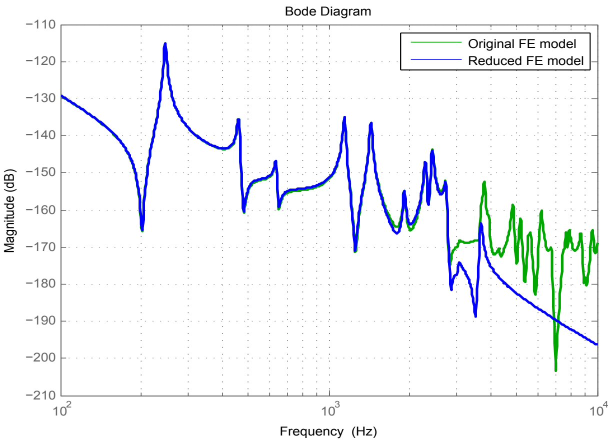

Fig. 4. Bode plot of the original FE model (green) and the reduced model (blue) with input u_1 and output $y_1 = z_1$. $0\ dB \sim 1\ m/N$.

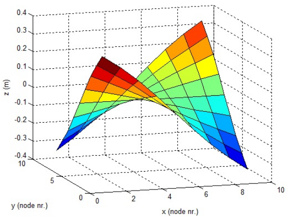

Fig. 5. Torsion mode of the original FE model.

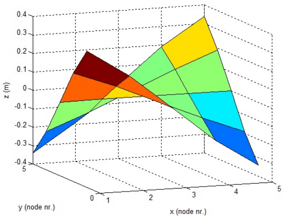

Fig. 6. Torsion mode of the reduced FE model.

Two simulated experimental eigenvalues have been extracted from the original FE model, after adding the extra actuator to the model, which are $\lambda_{12} = -2.85e+01 - 2.87e+03i$ and $\lambda_{14} = -3.93e+01 - 3.97e+03i$ corresponding to the 6^{th} and 7^{th} mode, which are saddle and bending modes, respectively. Note that The 5^{th} mode is not observable in the transfer function from u_1 to z_1. Then, we can construct $\Delta\lambda_i$ in (10) after calculating the corresponding eigenvalues of the reduced model without the extra mass. Since the extra actuator mass has been added to the original plate at the location indicated by z_8, the design parameter to be updated is $M^{\star}_{(8,8)}$: the mass element corresponding to z_8 in the reduced model.

By solving (13) in 8 iterations, the numerical eigenvalues converge to the experimental eigenvalues. Fig. 7 shows the convergence of the real and imaginary part of the 12^{th} and 14^{th} numerical eigenvalues to the measured eigenvalues. The residual error e_i defined in (12) converges to zero, see Fig. 8. As a result of least squares optimization, the delta-mass $\Delta M^{\star}_{(8,8)}$ corresponding to the mass of the extra actuator located at z_8 converges to the optimal value of $0.5\ kg$, see Fig. 9. It is also shown in the figure that an artificial introduced delta-damping $\Delta B^{\star}_{(8,8)}$ corresponding to the location of the extra actuator converges to the optimal value of $0\ Ns/m$, which means the extra actuator did not change the damping at the location which was added, as it should.

From a control perspective, the input-output behavior of a system is crucial, since it characterizes the main properties for control design. Fig. 10 shows three FRF's: (a) for the initial reduced FE model in blue, (b) for the updated reduced FEM model in red, and (c) for the simulated FRF measurement in green. The FRF of the updated reduced FE model matches quit good with the measurement (see Fig.11) up to the frequency range of interest which means the effect of the extra actuator has been compensated in the updated model. So, the updated reduced model can very well simulate the behavior of the planar plate with the extra actuator.

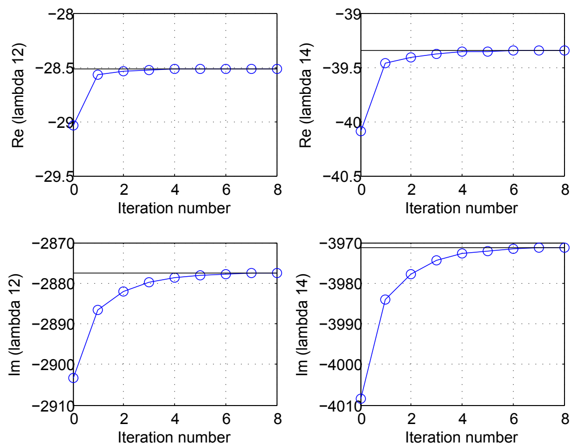

Fig. 7. Convergence of the numerical eigenvalues: λ_{12} and λ_{14} (blue) to the measured eigenvalues (black).

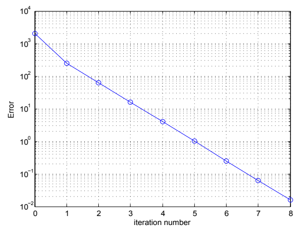

Fig. 8. The residial error based on (12).

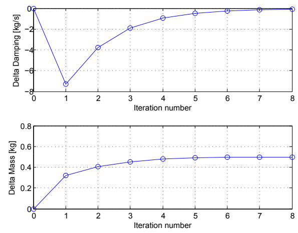

Fig. 9. The estimated mass and damping factor.

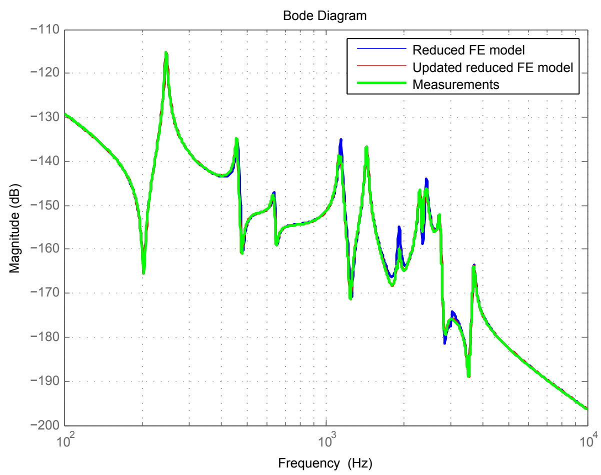

Fig. 10. Bode plot of the reduced and updated reduced FE model vs simulated FRF measurement when the input and output channels are u_1 and z_1, respectively.

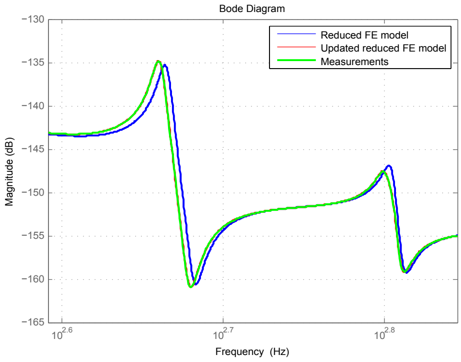

Fig. 11. Zoomed plot of Fig. 10 containing second and third resonances, saddle and bending modes, respectively.

As a result of adding an extra actuator mass to the system, eigenfrequencies and mode shapes have changed. For instance, the damped eigenfrequency corresponding to the 6^{th} mode, which is the saddle mode, is changed, see the first resonance in Fig. 11. It has changed from $461Hz$ to $455Hz$. The corresponding mode shape has also changed. The saddle mode shape of the updated reduced FE model is slightly different than the saddle mode shape of the original or reduced FE model due to the added extra actuator.

The modes of the reduced updated FE model now can be used to update the mode shapes of the original FE model via (7) and (4). Then, the updated original FE model can be used as an accurate standard plant to analyze responses at the POI.

5. OUTLOOK FOR CONTROL

Nowadays, high precision motion control systems like the wafer stages in lithography machines are confronted with servo specifications in the order of sub-nanometers. This means that highly accurate motion control systems are needed which heavily rely on state-of-the-art model-based control design, requiring accurate plant models. The approach presented here helps to create sufficiently accurate reduced-order machine-specific dynamic models which can be used for: (a) robust controller tuning and model-based design, (b) advanced observer design for point-of-interest (POI) estimation, and (c) actuator/sensor placement where the optimal locations of the possible additional actuator/sensor need to be calculated.

6. CONCLUSION

An approach has been described that uses model updating techniques for complex systems with a large number of DOFs. A reduced FE model is constructed, which consists of reduced-order mass, damping, and stiffness matrices. Model updating is conducted for this reduced-order model. For a planar plate FE model, it is shown that an extra actuator mass can be accounted for. In this way, machine-specific updated parametric models are obtained that preserve the input-output behavior of the real system. This paves the way for machine-specific and model-based control design, towards sub-nanometer wafer stage performance.

ACKNOWLEDGEMENTS

This work is part of the Pieken in de Delta 2008 program funded by SenterNovem. The first author acknowleges valuable discussions with Jeroen de Best.

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