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An Optimal Control Approach to Decentralized Energy-Efficient Coverage Problems

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Abstract: One of the challenges of working with multi-agent systems is the limited energy of the agents, especially when the agents are flying vehicles with limited batteries to save weight. The main contribution of this paper is to formulate the energy-efficient coverage problem as an optimal control problem. The optimal control problem will be related to Lloyd's algorithm. The solution to the optimal control problem is spatially distributed over Delaunay graphs and provides an energy-efficient local controller to maximize the coverage. As a second contribution, by imposing constraints on the parameters of the optimal control problem we guarantee that the agents maintain their energy during the coverage task. As expected, weighting the speed of the agents against the coverage objective will decrease energy consumption in the multi-agent system. Several examples demonstrate the performance of the energy-efficient approach.

1. INTRODUCTION

In the last two decades, multi-agent robotic systems have attracted the attention of researchers from different disciplines. Advancements in the field of wireless networks, consistent progress in production of high speed processors, and achievements in manufacturing miniature mobile robots with manifold sensing capabilities have fueled this attraction. It is believed that groups of cooperative autonomous mobile robots have the potential to take over challenging and often dangerous tasks such as surveillance, search and rescue operations, planetary exploration, and disposal of hazardous waste (see Murray [2007] and the references therein). In this paper, we focus on the problem of deploying a multi-agent system over an area in order to achieve optimal coverage (see Cortés et al. [2004], Sayyaadi and Moarref [2011], and the references therein).

One of the challenges in working with multi-agent systems is their relatively small operating time. Mobile robots (agents) are usually small and also use small actuators. Therefore, considering the weight and size of their frame, actuators, and sensors, they cannot carry heavy batteries. For instance, the flight time of a regular quadrotor (which is a popular vehicle for making robotic swarms) ranges from a few minutes to twenty minutes (Roberts et al. [2007], Hoffmann et al. [2007]). Energy-aware control strategies can have a great impact on the performance of multi-agent systems. However, energy-efficient coverage algorithms have not received many research contributions. In Mei et al. [2006], rectangle scan-lines are used as coverage routes to propose a speed-management method which maximizes the traveling distance of the robots under energy and time constraints. In Kwok and Martínez [2007, 2010], generalized Voronoi partitions (namely *multiplicatively-weighted* and *power-weighted* Voronoi diagrams) and the modified Lloyd's algorithm are used to solve a deployment problem in power-constrained sensor networks. It is shown that this choice of Voronoi regions can have an effect on restricting the speed of the agents in order to save energy (depending on the initial locations of the agents). Song et al. [2013] propose iterative algorithms to reduce the movement step sizes in Lloyd's algorithm and reduce sensor traveling distances. Yan and Mostofi [2012] propose an algorithm to co-optimize communication and motion strategies in a network of mobile robots. In Ghaffarkhah and Mostofi [2012], mixed-integer linear programs are used to design periodic trajectories and power policies for the mobile agents that minimize the total energy (the summation of motion and communication energy) consumption of the mobile agents.

In contrast to the previous work in the literature, the main contribution of this paper is to formulate the coverage problem as an optimal control problem. The optimal control problem will be related to Lloyd's algorithm. The solution to the optimal control problem is spatially distributed over Delaunay graphs and provides an energy-efficient local controller. LaSalle invariance principle is used to prove that the energy-efficient controller locally maximizes the coverage. As a second contribution, by imposing constraints on the parameters of the optimal control problem we guarantee that the agents maintain their energy during the coverage task.

The outline of the paper is as follows. Preliminary information and definitions on Voronoi partitions and coverage control are presented in Section 2. Section 3 presents an optimal control interpretation of the coverage problem and proposes a decentralized optimal solution. Simulation results are presented and discussed in Section 4. Concluding remarks are stated in Section 5.

2. PRELIMINARIES

In this section, we present preliminary notions and definitions on Voronoi partitioning and coverage problem. They are collected from Cortés et al. [2004, 2005].

2.1 Voronoi partitions and Delaunay graphs

A partition of a set $Q \subset \mathbb{R}^d$ is a subdivision of Q into components that overlap only on sets of measure zero (with respect

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to Lebesgue measure). Given a set Q and n distinct points $P = \{p_1,...,p_n\} \subset Q^n$, the Voronoi partition of Q with generators P, is defined by $V(P) = \{V_i(P)|i \in \{1,...,n\}\}$, where $V_i(P) = \{q \in Q| \|q - p_i\| \leq \|q - p_j\|, \forall p_j \in P\}$. The set of all points closer to a point $p_i \in P$ than to any other point in P, is the interior of a (not necessarily bounded) convex polytope $V_i(P)$ called the Voronoi cell of p_i. The Voronoi partition in which each generator is located at the center of mass (centroid) of its own Voronoi cell is called a *Centroidal Voronoi configuration*. Two Voronoi cells that share a $(d-1)$-dimensional facet are called Voronoi neighbors. The set of indices of the Voronoi neighbors of p_i is denoted by $N(i)$. Clearly, $j \in N(i)$ if and only if $i \in N(j)$.

A graph G is defined by the pair $G(\{p_1,...,p_n\}, \varepsilon_G)$ where $\{p_1,...,p_n\}$ represents the vertices of the graph and ε_G denotes the set of edges in G. A proximity graph is a graph in which the existence of an edge between two distinct vertices is a function of their relative position. The notion of Voronoi partition is related to a special type of proximity graphs called the *Delaunay graph*. In a Delaunay graph G_D, two vertices are connected by an edge if their corresponding Voronoi cells are neighbors.

Given a set Y, consider a mapping T which maps n distinct points in $\mathbb{R}^d$ to n points in Y, i.e. $T : (\mathbb{R}^d)^n \to Y^n$. The mapping T is said to be spatially distributed over a proximity graph G if its j^{th} component T_j can be computed only with the knowledge of the vertex p_j and its neighboring vertices.

2.2 Coverage control

In the coverage (or deployment) problem, the goal is to deploy robotic agents over an area such that coverage is maximized. Consider a convex area $Q \subset \mathbb{R}^d$ and n distinct points $x_i \in Q$, $i \in \{1,\ldots,n\}$, representing the location of the agents. Due to noise and signal attenuation, coverage at a point $q \in Q$ degrades with the square of the distance between q and the agent that provides coverage at q. Let a *performance* function be defined as $f(x_{i(q)}, q) = \|x_{i(q)} - q\|^2$ where $i(q) : Q \to \{1,\ldots,n\}$ indicates the agent that provides coverage at q. Furthermore, let the bounded *density* function $\phi : Q \to \mathbb{R}^+$ denote the priority (importance) of coverage at a point $q \in Q$. A candidate Lyapunov function $V(x)$, where $x = [x_1^T,...,x_n^T]^T$, is defined as

$$V(x) = \int_Q f(x_{i(q)}, q)\phi(q)dq,$$

and provides a measure of how poor the coverage is over Q. Therefore, in order to maximize coverage one needs to minimize $V(x)$. Since $f(x_{i(q)}, q)$ is non-decreasing, in order to minimize V, it is intuitive to assign the task of providing coverage at a point $q \in Q$ to the agent that is closest to q. Hence, using the notion of Voronoi partitions, $V(x)$ is rewritten as

$$V(x) = \sum_{i=1}^{n} \int_{V_i} f(x_i, q)\phi(q)dq = \sum_{i=1}^{n} \int_{V_i} \|x_i - q\|^2 \phi(q)dq, \quad (1)$$

where V_i is the Voronoi cell corresponding to the i^{th} agent. Differentiating V with respect to x_j, we get

$$\begin{aligned} \frac{\partial V}{\partial x_j} &= 2\int_{V_j} (x_j - q)^T \phi(q)dq \\ &= 2\left(\int_{V_j} \phi(q)dq\right)\left(x_j - \frac{\int_{V_j} q\phi(q)dq}{\int_{V_j} \phi(q)dq}\right)^T. \end{aligned} \quad (2)$$

The first term on the right-hand-side of (2) represents the mass of the j^{th} Voronoi cell

$$M_{V_j} = \int_{V_j} \phi(q)dq,$$

and the second term on the right-hand-side of (2) represents the center of mass of the j^{th} Voronoi cell

$$CM_{V_j} = \frac{\int_{V_j} q\phi(q)dq}{\int_{V_j} \phi(q)dq} = \frac{\int_{V_j} q\phi(q)dq}{M_{V_j}}.$$

Equation (2) is now rewritten as

$$\frac{\partial V}{\partial x_j} = 2M_{V_j}(x_j - CM_{V_j})^T.$$

Therefore, any set of points $x_j = CM_{V_j}$, $j \in \{1,\ldots,n\}$, (any Centroidal Voronoi configuration) is a local minimum of V. Since there is no explicit formula for finding Centroidal Voronoi configurations, Lloyd's iterative algorithm (see Du et al. [1999]) is widely used to find Centroidal Voronoi configurations. In Lloyd's algorithm, at any given time, each agent computes the center of mass of its own Voronoi cell and moves towards it. Assuming agents with first order dynamics, the control input $u_j \in \mathbb{R}^d$, $j \in \{1,\ldots,n\}$, is defined as

$$\dot{x}_j = u_j, \quad (3)$$

$$u_j = k_j\left(CM_{V_j} - x_j\right), \quad (4)$$

where $k_j > 0$. Control input (4) is a descent algorithm that makes V a Lyapunov function because

$$\dot{V} = \sum_{j=1}^{n} \frac{\partial V}{\partial x_j}\dot{x}_j = -2\sum_{j=1}^{n} k_j M_{V_j} \|x_j - CM_{V_j}\|^2 \leq 0.$$

Using LaSalle invariance principle (see Khalil [1995]), it can be proved that the multi-agent system (3) with control input (4) converges to a Centroidal Voronoi configuration and provides a locally optimal coverage over the region (see Cortés et al. [2004] for more details).

Remark 1. The coverage problem described in this section only optimizes the final position of the agents and does not consider the control effort used in the process. Therefore, control input (4) may produce inefficiently large inputs that waste energy. In contrast, Section 3 formulates the coverage problem as an optimal control problem. The optimal control problem will be related to Lloyd's algorithm. The solution to the optimal control problem is spatially distributed over Delaunay graphs and provides an energy-efficient local controller to maximize the coverage. By weighting the speed of the agents against the coverage objective we decrease energy consumption in the multi-agent system.

3. MAIN RESULTS

According to Lloyd's algorithm, the control strategy in which each agent moves towards the centroid of its Voronoi cell locally solves the coverage problem. Therefore, we use the weighted distance between an agent and the centroid of its corresponding Voronoi cell as a coverage criterion defined as

$$\begin{aligned} M_{V_i}\|x_i - CM_{V_i}\| &= (\int_{V_i} \phi(q)dq)\|x_i - \frac{\int_{V_i} q\phi(q)dq}{\int_{V_i} \phi(q)dq}\| \\ &= \|\int_{V_i} (x_i - q)\phi(q)dq\|. \end{aligned} \quad (5)$$

The smaller the coverage criterion is, the better coverage is achieved over V_i. By multiplying the distance $\|x_i - CM_{V_i}\|$ by the mass M_{V_i} in the coverage criterion, we distinguish between Voronoi cells based on the priority and importance of the areas they are covering. In other words, if two agents are at the same distance from the centroid of their corresponding Voronoi cells, the coverage criterion is larger for the agent that covers areas with more priority. Using the coverage criterion (5), we propose an optimal control interpretation of the coverage control problem. We show that by decreasing the ratio of the weight on the coverage criterion to the weight on energy consumption, we can achieve a locally optimal coverage using less energy.

Theorem 1. Consider the following optimal control problem for a multi-agent system with bounded Voronoi cells V_i

$$\inf_{u_i,\ i\in\{1,...,n\}} \int_0^\infty \sum_{i=1}^n \left(s_i \| \int_{V_i} (x_i - q)\phi(q)dq\|^2 + r_i {u_i}^T u_i \right) d\tau$$

$$\text{subject to} \quad \dot{x}_i = u_i, \tag{6}$$

with $s_i \geq 0$, $i \in \{1,...,n\}$, and $r_i > 0$, $i \in \{1,...,n\}$. The optimal solution that is spatially distributed over Delaunay graphs is

$$u_i = -\sqrt{s_i/r_i} \int_{V_i} (x_i - q)\phi(q)dq. \tag{7}$$

Moreover, if $s_i r_i = 1$, $\forall i \in \{1,...,n\}$, the resulting value function is

$$V(x) = \sum_{i=1}^n \int_{V_i} \|x_i - q\|^2 \phi(q)dq, \tag{8}$$

and the multi-agent system converges to a Centroidal Voronoi configuration.

Proof. The Hamilton-Jacobi-Bellman equation for (6) is

$$\inf_{u_i,\ i\in\{1,...,n\}} H(L(x,u), \frac{\partial V}{\partial x}) = 0, \tag{9}$$

where

$$L = \sum_{i=1}^n \left(s_i \| \int_{V_i} (x_i - q)\phi(q)dq\|^2 + r_i {u_i}^T u_i \right), \tag{10}$$

$$\frac{\partial V}{\partial x} = \left[\frac{\partial V}{\partial x_1}, ..., \frac{\partial V}{\partial x_n} \right],$$

and

$$H = L + \frac{\partial V}{\partial x}\dot{x} \tag{11a}$$

$$= \sum_{i=1}^n \left(s_i \| \int_{V_i} (x_i - q)\phi(q)dq\|^2 + r_i {u_i}^T u_i + \frac{\partial V}{\partial x_i} u_i \right). \tag{11b}$$

Differentiating H with respect to u_i, we get a necessary condition for optimality

$$\frac{\partial H}{\partial u_i} = 2r_i u_i^T + \frac{\partial V}{\partial x_i} = 0 \Rightarrow \frac{\partial V}{\partial x_i} = -2r_i u_i^T. \tag{12}$$

Replacing (12) in (9) and (11b), we can write

$$\sum_{i=1}^n \left(s_i \| \int_{V_i} (x_i - q)\phi(q)dq\|^2 - r_i {u_i}^T u_i \right) = 0. \tag{13}$$

Equation (13) has infinitely many solutions for u_i. For the following solution

$$u_i = -\sqrt{s_i/r_i} \int_{V_i} (x_i - q)\phi(q)dq,\ \forall i \in \{1,...,n\}, \tag{14}$$

the i^{th} agent needs to know only the relative positions of its Voronoi neighbors $N(i)$ (to be able to compute its own Voronoi cell) and the density function over its own Voronoi cell. Therefore, the control algorithm (14) is spatially distributed over Delaunay graphs. Integrating (12) yields

$$V = -2r_1 \int u_1^T dx_1 + f_1(x_{j\neq 1}),$$

$$\vdots$$

$$V = -2r_n \int u_n^T dx_n + f_n(x_{j\neq n}).$$

Therefore,

$$V = -2\sum_{i=1}^n r_i \int u_i^T dx_i. \tag{15}$$

Hence, the Value function corresponding to (14) is computed as

$$V = 2\sum_{i=1}^n \sqrt{s_i r_i} \int \int_{V_i} (x_i - q)^T \phi(q)dq\, dx_i. \tag{16}$$

Based on the conservation of mass law in Cortés et al. [2005] (a generalization of Leibniz integral rule), we can write

$$\frac{\partial}{\partial x_i} \int_{V_i} \|x_i - q\|^2 \phi(q)dq = \int_{V_i} \frac{\partial}{\partial x_i} \|x_i - q\|^2 \phi(q)dq + \int_{\partial V_i} \|x_i - \gamma\|^2 \phi(\gamma) n^T(\gamma) \frac{\partial \gamma}{\partial x_i} d\gamma, \tag{17}$$

where ∂V_i is the boundary of V_i, $\gamma : S \times Q^n \to Q$ with $S \subset \mathbb{R}$ is a parameterization of ∂V_i, and $n(\gamma)$ denotes the unit outward normal to ∂V_i. Solving for the first integral on the right hand side, equation (17) can be simplified to

$$\begin{aligned}\int_{V_i} \frac{\partial}{\partial x_i} \|x_i - q\|^2 \phi(q)dq =& \frac{\partial}{\partial x_i} \int_{V_i} \|x_i - q\|^2 \phi(q)dq \\ &- \int_{\partial V_i} \|x_i - \gamma\|^2 \phi(\gamma) n^T(\gamma) \frac{\partial \gamma}{\partial x_i} d\gamma \\ \Leftrightarrow \int_{V_i} 2(x_i - q)^T \phi(q)dq =& \frac{\partial}{\partial x_i} \int_{V_i} \|x_i - q\|^2 \phi(q)dq \\ &- \int_{\partial V_i} \|x_i - \gamma\|^2 \phi(\gamma) n^T(\gamma) \frac{\partial \gamma}{\partial x_i} d\gamma.\end{aligned} \tag{18}$$

Replacing (18) in (16), we get

$$\begin{aligned}V =& \sum_{i=1}^n \sqrt{s_i r_i} \left(\int_{V_i} \|x_i - q\|^2 \phi(q)dq \right. \\ &\left. - \int \left(\int_{\partial V_i} \|x_i - \gamma\|^2 \phi(\gamma) n^T(\gamma) \frac{\partial \gamma}{\partial x_i} d\gamma \right) dx_i \right) \\ =& \sum_{i=1}^n \sqrt{s_i r_i} \left(\int_{V_i} \|x_i - q\|^2 \phi(q)dq \right. \\ &\left. - \int \left(\sum_{j\in N(i)} \int_{V_i \cap V_j} \|x_i - \gamma_{ij}\|^2 \phi(\gamma_{ij}) n^T(\gamma_{ij}) \frac{\partial \gamma_{ij}}{\partial x_i} d\gamma_{ij} \right) dx_i \right).\end{aligned} \tag{19}$$

Since $\|x_i - \gamma_{ij}\| = \|x_j - \gamma_{ji}\|$, $\frac{\partial \gamma_{ij}}{\partial x_i} = \frac{\partial \gamma_{ji}}{\partial x_j}$, and $n(\gamma_{ij}) = -n(\gamma_{ji})$, the following equation holds for any $i, j \in \{1, \ldots, n\}$,

$$\int_{V_i \cap V_j} \|x_i - \gamma_{ij}\|^2 \phi(\gamma_{ij}) n^T(\gamma_{ij}) \frac{\partial \gamma_{ij}}{\partial x_i} d\gamma_{ij} = - \int_{V_j \cap V_i} \|x_j - \gamma_{ji}\|^2 \phi(\gamma_{ji}) n^T(\gamma_{ji}) \frac{\partial \gamma_{ji}}{\partial x_j} d\gamma_{ji}. \tag{20}$$

Based on (20) and assuming $s_i r_i = 1$, $\forall i \in \{1,...,n\}$, the term

$$-\sum_{i=1}^n \sqrt{s_i r_i} \int \left(\sum_{j\in N(i)} \int_{V_i \cap V_j} \|x_i - \gamma_{ij}\|^2 \phi(\gamma_{ij}) n^T(\gamma_{ij}) \frac{\partial \gamma_{ij}}{\partial x_i} d\gamma_{ij} \right) dx_i$$

in (19) vanishes and V is simplified to

$$V=\sum_{i=1}^{n}\int_{V_i}\|x_i-q\|^2\phi(q)dq. \tag{21}$$

The value function (21) is positive. Based on (9) and (11a), with u_i defined in (14), we have $\dot{V}=-L$. Therefore, equations (10) and (14) yield

$$\dot{V}=-L=-2\sum_{i=1}^{n}\left(s_i\|\int_{V_i}(x_i-q)\phi(q)dq\|^2\right)\leq 0. \tag{22}$$

Hence, using LaSalle invariance principle (see Khalil [1995]), we can conclude that the multi-agent system converges to the largest invariant subset of a set $S\subset Q$ with the property $\dot{V}=0$. But according to (22), $\dot{V}=0$ only when

$$\int_{V_i}(x_i-q)\phi(q)dq=M_{V_i}(x_i-CM_{V_i})=0,\ \forall i\in\{1,\ldots,n\}, \tag{23}$$

which is the set of Centroidal Voronoi configurations. In other words, the set S is equal to the set of Centroidal Voronoi configurations. According to (14) and (23), the control input is equal to zero at Centroidal Voronoi configurations. Therefore, the set S is an invariant set and based on LaSalle invariance principle (see Khalil [1995]), the multi-agent system converges to one of the Centroidal Voronoi configurations. □

Remark 2. The first term in the running cost of the optimization problem (6) is the coverage criterion for the i^{th} agent defined in (5). Note that the coverage criterion for the i^{th} agent is equal to zero if the agent is located at the centroid of its corresponding Voronoi cell. The second term in the running cost represents the speed of each agent.

Remark 3. The control input u_i is derived according to (13) to cancel the coverage criterion. Therefore, the coverage criterion is the force that generates u_i and pushes the i^{th} agent towards the centroid of its Voronoi cell (see (14)). The coverage criterion is proportional to the distance between the agent and its centroid. Hence, an agent moves faster (slower) when it is far from (close to) its corresponding centroid. Amplifying this force, by increasing s_i/r_i, increases the agent's speed towards the centroid. On the other hand, decreasing the value of s_i/r_i reduces the speed of the agent. Since energy consumption is directly related to speed, by choosing smaller (larger) values for s_i/r_i, the multi-agent system uses less (more) energy.

Similar to Kwok and Martínez [2007, 2010] we assume that energy consumption increases with the square of the speed. The following proposition imposes a constraint on s_i/r_i, $\forall i\in\{1,...,n\}$, to guarantee that the multi-agent system performs the coverage control task in a given time without consuming all its energy.

Proposition 1. Suppose that the energy consumption rate for the i^{th} agent is $\dot{E}_i=-\|u_i\|^2$ and that each agent moves according to control law (14). Also assume that each agent starts with an initial energy $E_i(0)$ and that the agents have at most T seconds to move in a region Q and enhance the coverage. The following rule for s_i/r_i, $i\in\{1,...,n\}$, guarantees that the agents do not run out of energy during the deployment

$$\frac{s_i}{r_i}<\frac{E_i(0)}{T(D_QM_Q)^2}, \tag{24}$$

where D_Q is the diameter of the circumcircle of Q (i.e. the circle which passes through all the vertices of Q), and $M_Q=\int_Q\phi(q)dq$ is the mass of Q.

Proof. Inequality (24) yields

$$\frac{s_i}{r_i}T(D_QM_Q)^2<E_i(0). \tag{25}$$

Since $\|x_i-q\|<D_Q$ and $\int_{V_i}\phi(q)dq=M_{V_i}<M_Q$, $\forall i\in\{1,\ldots,n\}$, the following inequalities are valid

$$\|\int_{V_i}(x_i-q)\phi(q)dq\|\leq\int_{V_i}\|x_i-q\|\phi(q)dq<D_QM_Q,$$

and

$$\int_0^T\|\int_{V_i}(x_i-q)\phi(q)dq\|^2dt<T(D_QM_Q)^2.$$

Therefore, inequality (25) yields

$$\frac{s_i}{r_i}\int_0^T\|\int_{V_i}(x_i-q)\phi(q)dq\|^2dt<E_i(0)=E_i(T)-\int_0^T\dot{E}_i(t)dt. \tag{26}$$

Since $\dot{E}_i=-\|u_i\|^2$, inequality (26) and control input (14) yield

$$\frac{s_i}{r_i}\int_0^T\|\int_{V_i}(x_i-q)\phi(q)dq\|^2dt<E_i(T)+\frac{s_i}{r_i}\int_0^T\|\int_{V_i}(x_i-q)\phi(q)dq\|^2dt.$$

Therefore, $E_i(T)>0$. In other words, the agents do not run out of energy during the deployment. □

In order to implement Proposition 1, the agents need to have an estimate of the size of the area they are deployed on (namely, D_Q and M_Q). Although this information is considered as global data, in some applications, the agents can be provided with them prior to deployment. Based on the results of Proposition 1, the ratio s_i/r_i is directly proportional to the available energy and inversely proportional to the convergence time.

4. SIMULATION RESULTS

In this section, we use simulations to demonstrate the performance of control law (14) for coverage control in three different scenarios. During the deployment tasks, the multi-agent system evolves inside a closed convex polygon $Q\subset\mathbb{R}^2$ with density $\phi(q)$ at a point $q\in Q$. In our presentation, the vertices of Q are at $[(1,0),(5,0),(8,7),(6,8),(0,7)]$ and the density function is assumed to be uniform (i.e. $\phi(q)=1,\forall q\in Q$). We consider a multi-agent network that is comprised of seven autonomous agents. The agents are modeled as dimensionless points and their locations are determined by the first-order dynamical system (3) and local controller (14). The agents are assumed to be equipped with sensors that allow them to locate their immediate Voronoi neighbors and acquire information about their own Voronoi cell (here, the density function). Each agent relies only on these local data to move towards its next target according to (14).

Through the simulations, we study the effect of weighting the speed of the agents against the coverage criterion on energy consumption. Table 1 shows the value of $s_i/r_i,\forall i\in\{1,...,7\}$, and the convergence time in each of the three scenarios. Simulation results are summarized in Fig. 1, where each row is devoted to a different scenario.

The first column of Fig. 1 illustrates the evolution of the multi-agent network in each simulation. The initial positions of the agents are located inside the circle with center at $(3,2)$ and radius 1.5. In all the simulations, the agents start from identical initial locations and then scatter over Q to maximize the coverage. The red lines mark the trace of the agents during the coverage control task. The final locations of the agents are

Table 1. Ratio of the weight on coverage criterion (s_i) and the weight on control input (r_i) in different simulations

Simulation Number	$s_i/r_i, \forall i \in \{1,...,7\}$ Agent Number							Convergence Time (s)
	1	2	3	4	5	6	7	
I	1	1	1	1	1	1	1	4.18
II	100	100	100	100	100	100	100	0.42
III	1	1	1	1	0.001	0.001	0.001	193.48

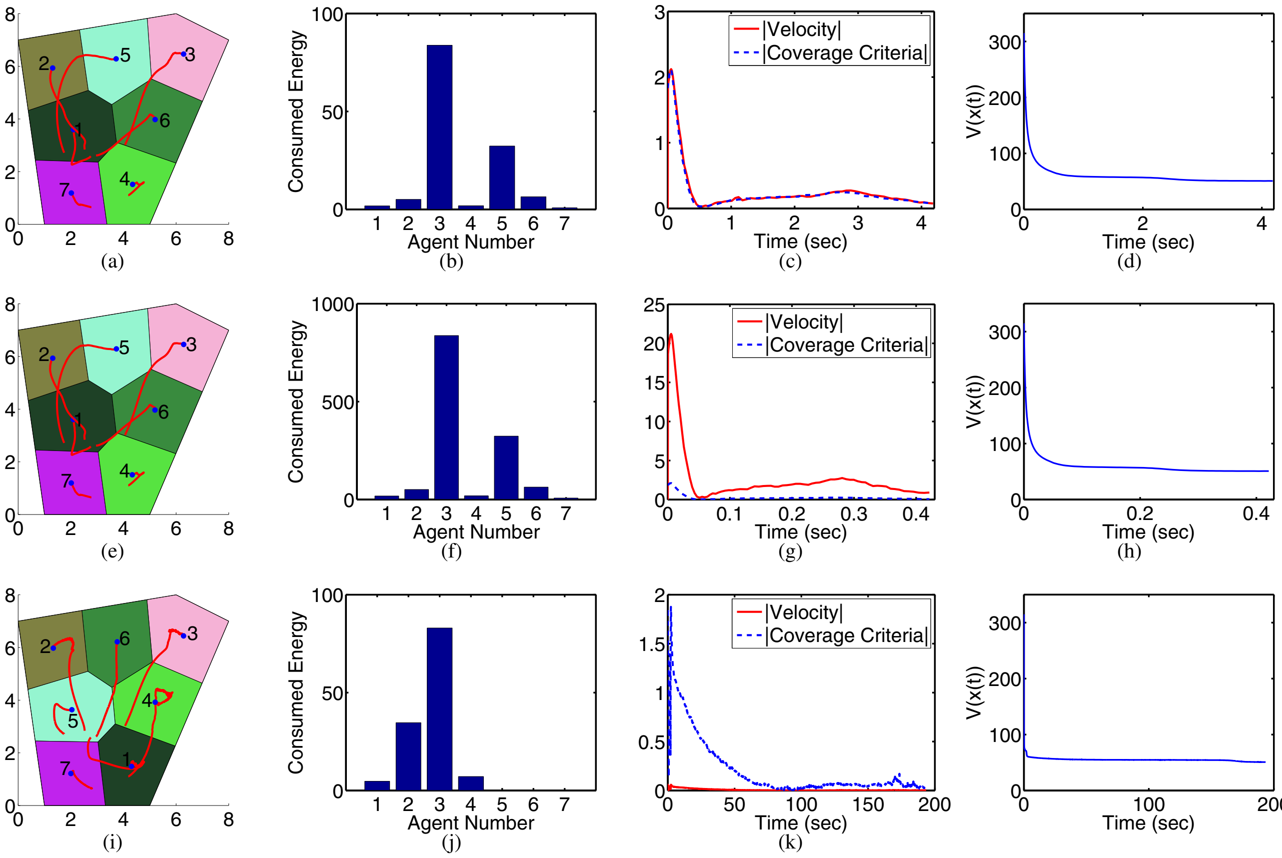

Fig. 1. Coverage control over a region with uniform density. Rows 1 to 3 correspond to Simulation I to Simulation III, respectively (cf. Table 1). The first column shows the final location of the agents as well as their trace. The second column shows the consumed energy during the coverage control task for each agent. The third column shows the speed and the coverage criterion, $\|\int_{V_i}(x_i - q)\phi(q)dq\|$, for the 7$^{\text{th}}$ agent during each simulation. The value function is plotted on the fourth column.

shown with blue dots and their corresponding Voronoi regions are plotted. As evident from the first column of Fig. 1, the multi-agent system converges to a Centroidal Voronoi configuration, which is in consistence with the results of Theorem 1.

In the first scenario, s_i/r_i is equal to 1 for all the agents. In the second scenario, s_i/r_i is equal to 100 for each agent. Note that in these two simulations, the agents take almost the same path towards their destinations. However, as expected from (14), the agents move and converge $\sqrt{100}/\sqrt{1} = 10$ times faster in Simulation II than in Simulation I. In Simulation III, agents 5, 6, and 7 are assumed to have very little energy. Therefore, we decrease s_j/r_j, $j \in \{5,6,7\}$, in order to reduce their fuel consumption. The price for saving energy is a longer convergence time, as seen in Table 1. Here, some of the agents end up at different locations than the previous cases (e.g. agent 1 goes to the location that was occupied by agent 4 in the previous simulations). However, the multi-agent system converges to the same Centroidal Voronoi configuration (see Fig. 1(i)). Note that in general, Centroidal Voronoi configurations are not unique (Du et al. [1999]), and the system can converge to any of them.

The second column of Fig. 1 shows the amount of energy consumed by each agent during the deployment task ($-\int_0^T \dot{E}_i(t)dt = \int_0^T \|u_i\|^2 dt$). As expected, each agent expends nearly 10 times more energy in Simulation II than in Simulation I. In Simulation III, the speeds of agents 5, 6, and 7 are greatly weighted against their coverage criteria. Hence, they expend much less energy in this case (compare Fig. 1(j) and Fig. 1(b)).

The third column of Fig. 1 illustrates the speed and the coverage criterion, $\|\int_{V_i}(x_i - q)\phi(q)dq\|$, of the 7$^{\text{th}}$ agent in each simulation. In Simulation I, the ratio $s_7/r_7 = 1$ and therefore the two curves overlap. In Simulation II, the coverage criterion

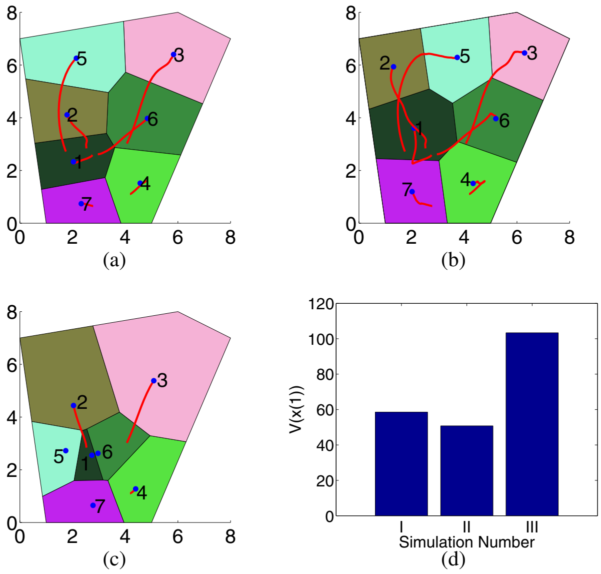

Fig. 2. Network configuration and value function after 1 second of simulation. (a)-(c) show the location of the agents at $t = 1$ (s) for Simulation I-III. (d) illustrates the value function at $t = 1$ (s) for each of the simulations.

is weighted against the speed of the agent (i.e. $s_7/r_7 = 100$). In other words, the agent's priority is to decrease the coverage criterion as fast as possible, at the cost of spending more energy. This allows the speed of the 7$^{\text{th}}$ agent to get $\sqrt{s_7/r_7} = 10$ times greater than its coverage criterion (see Fig. 1(g)). In Simulation III, $s_7/r_7 = 0.001$ which restricts the speed of the 7$^{\text{th}}$ agent and allows the coverage criterion to be greater than the speed.

The change in the value function during each coverage control task is plotted in the forth column of Fig. 1. Since the multi-agent network converges to the same Centroidal Voronoi configuration in all the cases, the value function decreases to the same value 50.8 in all figures.

Fig. 2(a), Fig. 2(b), and Fig. 2(c) illustrate the configuration of the network in the first, second, and third scenario after 1 second of simulation. Fig. 2(d) shows the value function for each of the simulations at that time, i.e. $V(x(1))$. As expected, in equal times, the networks in which the ratio of the weight on coverage criterion to the weight on energy consumption is larger, evolve faster and provide a better coverage. Of course, the energy consumption in such networks is more than the others.

5. CONCLUSIONS

In this paper, we presented an optimal control interpretation of the coverage control problem. The solution to the optimal control problem is spatially distributed over Delaunay graphs and provides an energy-efficient local controller to maximize the coverage. By imposing constraints on the parameters of the optimal control problem we guaranteed that the agents maintain their energy during the deployment task. It was shown that weighting the control input against the coverage criterion decreases energy consumption in the multi-agent system.

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