Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Switching Control for Adaptive Disturbance Attenuation

Giorgio Battistelli * **Daniele Mari** ** **Daniela Selvi** * **Alberto Tesi** *
Pietro Tesi ***

* *University of Florence, Dipartimento di Ingegneria dell'Informazione (DINFO), Via di Santa Marta 3, 50139 Firenze, Italy (e-mail: giorgio.battistelli@unifi.it , daniela.selvi@unifi.it , alberto.tesi@unifi.it).*
** *Università della Calabria, Dipartimento di Ingegneria Informatica, Modellistica, Elettronica e Sistemistica (DIMES), Via P. Bucci, cubo 42/c, 87036 Arcavacata di Rende (CS), Italy (e-mail: daniele.mari@deis.unical.it)*
*** *University of Groningen, Institute for Technology, Engineering & Management, Faculty of Mathematics and Natural Sciences, Nijenborgh 4, 9747 AG Groningen, The Netherlands (e-mail: p.tesi@rug.nl)*

Abstract: The problem of adaptive disturbance attenuation is addressed in this paper using a switching control approach. A finite family of stabilizing controllers is pre-designed, with the assumption that, for any possible operating condition, at least one controller is able to achieve a prescribed level of attenuation. Then, at each time instant, a supervisory unit selects the controller associated with the best potential performance on the basis of suitably defined test functionals. In the paper, we prove some important properties which are satisfied by the test functionals, and analyze the stability of the overall switched system. Simulation results are provided to show the validity of the proposed method as a solution to the problem.

Keywords: Adaptive control, Switching control, Adaptive disturbance attenuation, Stability of switched systems, Hysteresis switching logic

1. INTRODUCTION

The *Adaptive switching control* approach has been proposed as an alternative solution to the classical *Adaptive control* paradigm (see Morse et al. [1992], Narendra and Balakrishnan [1997], Hespanha et al. [2001, 2003a], Baldi et al. [2010], Battistelli et al. [2012, 2013, 2014]). Instead of designing continuously parametrized controllers, a finite family of controllers is synthesized, each one suitable for a specific operating condition; the choice, at any time instant, of one of them is made by a high-level supervisory unit. As well known, one important feature of this control approach is modularity: the switching rule and the control strategy are designed independently of each other. Specifically, each controller can be synthesized according to any technique, since the design procedure is carried out off-line. The absence of any connection with the switching logic can be useful either when existing control structures should be used, or when specific problems require advanced control strategies. The possible advantages of the switching control approach compared to the classical adaptive one are discussed for example in Hespanha et al. [2003b].

In this paper, the switching control approach is applied to the problem of attenuating the effects of bounded disturbances acting on a system. The plant model is assumed to be linear time-invariant and described by a known transfer function, but a non-negligible uncertainty affects the a priori available disturbance model. Within this setting, robust control approaches could in general fail to provide satisfactory performances in all the possible operating conditions. The proposed solution is based on the assumptions that all the pre-synthesized controllers stabilize the plant, and, for any possible operating condition, at least one of the controllers is able to achieve a certain prescribed performance level. Then, the supervisory unit selects, at each time instant, the controller providing the best potential performance according to the well-known *hysteresis switching logic* (HSL), see Morse et al. [1992]. The potential performance of each controller is quantified in terms of test functionals defined on the basis of the plant input/output data.

The problem of noise attenuation is currently of great interest, as witnessed for example by the recently proposed benchmark on Adaptive Regulation for the rejection of narrow band disturbances, presented in Landau et al. [2013]. In Landau et al. [2011], the authors provide a review on several solutions to the problem of the asymptotic suppression of the effects of unknown and possibly time-varying finite-band disturbances. See also, for example, Guo and Bodson [2009], Karimi and Emedi [2013], and Marino and Tomei [2013]. Several solutions to the problem of noise attenuation for narrow band disturbances employ either indirect or direct approaches. In an indirect procedure the disturbance frequency is estimated and then used in a disturbance cancellation architecture for known frequency; direct approaches instead rely on algorithms providing controllers inherently designed to attenuate the disturbances. Solutions to the problem of adaptive disturbance attenuation are of interest for several applications, such as: active noise control (Kuo and Morgan [1999]); noise cancellation in an acoustic duct (Amara et al. [1999]); eccentricity compensation (Canudas de Wit and Praly [2000]); disk drive control (Chen and Tomizuka [2012]); disturbance torque compensation in electric machines

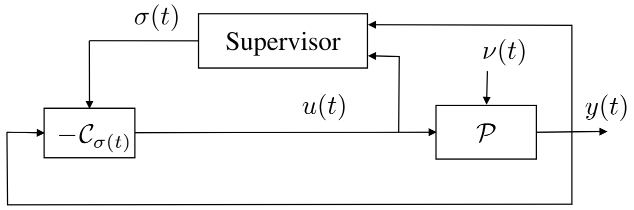

Fig. 1. Overall control scheme.

(Qian et al. [2004]); biomass productivity in a fed-batch reactor (Valentinotti et al. [2003]); disturbance attenuation for Adaptive Optics applications for ground-based telescopes (Agapito et al. [2012]). As for the latter case, the aim is that of reducing the effects of wavefront distortions caused by the atmospheric turbulence, whose profile can vary in accordance with the operating conditions of the telescope (*i.e.* different wind speed). Notice that such distortions are typically characterized by frequency profiles of arbitrary complexity, which can be unsuitably described by means of finite-dimensional models. With this respect, a positive feature of the proposed approach is that it is well-suited to being applied in all the above-mentioned contexts, since the switching mechanism does not depend on the disturbance model which only influences the controller family design. All the proofs are omitted due to space limitation.

2. PROBLEM SETTING

Consider the problem of controlling a single-input single-output (SISO) linear time-invariant (LTI) dynamical system whose input-output behavior can be described by the difference equation

$$A(d)y(t) = B(d)u(t) + \nu(t)\,, \tag{1}$$

where $t \in \mathbb{Z}_+, \mathbb{Z}_+ := \{0, 1, \cdots\}$, denote discrete time instants, $y(t)$ is the system output, $u(t)$ the control input, and $\nu(t)$ an unknown bounded disturbance acting on the system. The polynomials $A(d) = 1 + a_1\, d + \cdots + a_{n_a}\, d^{n_a}$ and $B(d) = b_1\, d + \cdots + b_{n_b}\, d^{n_b}$ in the unit backward shift operator d are known and have strictly Schur greatest common divisor (g.c.d.). We refer to $\mathcal{P}$ as the plant in (1) having transfer function $P(d) = B(d)/A(d)$.

The problem of interest is that of regulating the plant output $y(t)$ around zero by attenuating (and possibly rejecting) the disturbance $\nu(t)$. We assume that the a priori available disturbance model is affected by a non-negligible uncertainty, so that it is not possible to design a single LTI controller able to achieve satisfactory performance in all the possible operating conditions. The proposed solution follows the so-called *Adaptive switching control* paradigm, and is based on the idea of attenuating the disturbance $\nu(t)$ via a family of pre-designed controllers supervised by a high-level switching logic. The understanding is that the family of controllers is designed off-line so that: all controllers stabilize the plant; and, for any possible operating conditions, there exists at least one controller able to achieve a certain prescribed performance level (for example, in terms of disturbance-to-output energy gain). Then, the task of the supervisory unit is that of inferring, on the basis of the plant input/output data, the potential behavior achievable by the use of each candidate controller, and selecting the one providing the best potential performance.

Let now $\mathscr{C} := \{\mathcal{C}_i,\, i \in \overleftarrow{N}\}$ denote the family of pre-designed candidate controllers, where $\overleftarrow{N} := \{1, 2, \cdots, N\}$. The transfer function of the i-th controller is $C_i(d) = S_i(d)/R_i(d)$ with the polynomials $R_i(d)$ and $S_i(d)$ having strictly Schur g.c.d. and $R_i(d)$ such that $R_i(0) = 1$. We denote by $\sigma(\cdot) : \mathbb{Z}_+ \to \overleftarrow{N}$ the controller switching signal, i.e. the signal that identifies which of the candidate controllers belonging to $\mathscr{C}$ is in feedback with the plant at each time instant. Accordingly, the switching controller (or multi-controller) will be denoted by $\mathcal{C}_{\sigma(t)}$. This is a shorthand notation to mean that, on all the time intervals on which $\sigma(t)$ is constant and equal to a certain i, the multi-controller takes the form of a LTI system having transfer function equal to $C_i(d)$. We defer any discussion on the internal structure of the multi-controller to Section 4, where it will be shown how a suitable implementation of such a block always preserves stability under arbitrary switching. The overall control scheme is depicted in Fig. 1.

2.1 Controller selection

With the aim of obtaining the best possible performance given the finite family $\mathscr{C}$ of pre-synthesized controllers, an appropriate switching logic selects $\mathcal{C}_i \in \mathscr{C}$ to be placed in feedback with the plant. To this end, at any time t, a set of test functionals $\Pi(t) := \left\{\Pi_i(t),\, i \in \overleftarrow{N}\right\}$ is computed, each one quantifying the performance achievable by the use of the related candidate controller $\mathcal{C}_i$.

With this respect, the potential performance achievable by a certain controller $\mathcal{C}_i$ can be evaluated by taking into account the mapping from the disturbance to the plant input/output pair of the i-th potential loop $(\mathcal{P}/\mathcal{C}_i)$, defined as the feedback interconnection of the plant $\mathcal{P}$ with the controller $\mathcal{C}_i$. To this end, let us consider the weighted mixed-sensitivity transfer matrix $\Sigma_i(d)$ related to the loop $(\mathcal{P}/\mathcal{C}_i)$ defined as

$$\Sigma_i(d) := \frac{1}{\chi_i(d)}\, L_i(d) \tag{2}$$

where

$$L_i(d) = \begin{bmatrix} R_i(d) \\ -\eta\, S_i(d) \end{bmatrix}$$

and $\chi_i(d) = A(d)\, R_i(d) + B(d)\, S_i(d)$ is the characteristic polynomial of $(\mathcal{P}/\mathcal{C}_i)$. The nonnegative scalar weight η is a design parameter which can be tuned to give more or less importance to the contribution of the control input in the performance evaluation. Further, let

$$\varepsilon(t) := A(d)\, y(t) - B(d)\, u(t) \tag{3}$$

denote the prediction error. We note that, for any $t \geq n = \max\{n_a, n_b\}$, the prediction error $\varepsilon(t)$ coincides with the disturbance $\nu(t)$. Then, the hypothetical weighted plant input/output behavioral data $z_i = [y_i \quad \eta\, u_i]^\top$ produced by $(\mathcal{P}/\mathcal{C}_i)$ in response to ν can be computed through the difference equation

$$\chi_i(d)z_i(t) = L_i(d)\varepsilon(t).$$

The above recursion is run from an initial time $t_0 \geq n$ and initialized with zero initial conditions.

On the basis of the foregoing definitions, it is immediate to see that a convenient choice for the test functionals $\Pi_i(t),\ i \in \overleftarrow{N}$ is the following:

$$\Pi_i(t) := \begin{cases} \dfrac{\|\, z_i|_{t-M(t)}^{t} \,\|}{\|\, \varepsilon|_{t-M(t)}^{t} \,\|}\,, & \text{if } \|\, \varepsilon|_{t-M(t)}^{t} \,\| > 0 \\[2ex] 0\,, & \text{if } \|\, \varepsilon|_{t-M(t)}^{t} \,\| = 0 \end{cases}\,, \tag{4}$$

where $\| \varepsilon|_{\underline{t}}^{\bar{t}} \| := \sqrt{\sum_{k=\underline{t}}^{\bar{t}} |\varepsilon(k)|^2}$, with $\underline{t}$ and $\bar{t}$ two generic time instants such that $\underline{t} \leq \bar{t}$, $|\cdot|$ the Euclidean norm and $\varepsilon|_{\underline{t}}^{\bar{t}} := [\varepsilon(\underline{t}) \ldots \varepsilon(\bar{t})]^\top$. We denote by $M(t)$ the *memory* of the test functionals, i.e., the size of the time window $\{t-M(t), t-M(t)+1, \ldots, t\}$ on which the test functionals are computed. As for the choice of the memory $M(t)$, we can distinguish between two different situations. When the disturbance can be assumed to be almost stationary, albeit with an unknown frequency profile, we can use a *persistent memory* and set $M(t) = t - t_0$. On the contrary, if it is expected that the disturbance characteristics vary with time, a *finite memory* such as

$$M(t) = \begin{cases} t - t_0, & \text{if } t < t_0 + M_* \\ M_*, & \text{otherwise} \end{cases} \tag{5}$$

is more appropriate so as to make the test functionals able to promptly reflect any change in the enviromental conditions. In (5), M_* is a positive integer. Clearly, the former scenario gives rise to a learning problem, whereas the latter corresponds to an adaptation problem.

As for the selection of the controller to be placed in the loop, we assume that the switching sequence σ is determined according to the HSL (see Morse et al. [1992])

$$\left.\begin{array}{l} \sigma(t+1) = l(\sigma(t), \Pi(t)), \qquad \sigma(t_0) = i_0 \in \overleftarrow{N} \\ l(i, \Pi(t)) = \begin{cases} i, & \text{if } \Pi_i(t) < \Pi_{i_*(t)}(t) + h \\ i_*(t), & \text{if } \Pi_i(t) \geq \Pi_{i_*(t)}(t) + h \end{cases} \end{array}\right\} \tag{6}$$

where $i_*(t)$ is the smallest index in $\overleftarrow{N}$ such that $\Pi_{i_*(t)}(t) \leq \Pi_i(t)$, $\forall i \in \overleftarrow{N}$, and $h > 0$ is the hysteresis constant. For $t \leq t_0$, $\sigma(t)$ is kept constant and equal to an arbitrary initial value $i_0 \in \overleftarrow{N}$.

3. PROPERTIES OF THE TEST FUNCTIONALS

Within the analysis provided in this section we assume that $\| \varepsilon|_{t-M(t)}^{t} \| > 0$, for any $t \geq t_0$, so that $\Pi_i(t)$ is defined as in the first case of (4).

If the disturbance $\nu(t)$ can be assumed to be quasi-stationary and ergodic and a persistent memory $M(t) = t - t_0$ is adopted, each $\Pi_i(t)$ converges to the gain of $\Sigma_i(d)$ in response to ν; further, since σ is determined according to the HSL, on the basis of the well-known *HSL Lemma* of Morse et al. [1992] it is possible to assert that there exists a finite time $t_f \in \mathbb{Z}_+$ after which no more switching can occur, with the final controller attaining the best achievable performance apart from the hysteresis constant. Details regarding the analysis of the properties of the test functionals when a persistent memory is adopted are omitted for the sake of brevity.

The analysis that follows concerns an adaptation problem wherein a finite memory $M(t)$ defined as in (5) is used. The boundedness of $\Pi_i(t)$, $i \in \overleftarrow{N}$ is guaranteed also in this case thanks to the stability of $\Sigma_i(d)$; however, the finite size of the memory prevents one from ensuring the existence of an asymptotic limit for the test functionals. Specifically, the following result holds.

Proposition 1. Let the test functionals $\Pi_i(t)$, $i \in \overleftarrow{N}$ be defined as in (4) with $M(t)$ defined as in (5). Then, for any $i \in \overleftarrow{N}$ and $t \geq t_0$

$$\Pi_i(t) \leq \begin{cases} \|\Sigma_i\|_\infty, & \text{if } t < t_0 + M_* \\ \|\Sigma_i\|_\infty + \dfrac{\delta_i C_\nu}{\| \nu|_{t-M_*}^{t} \|}, & \text{if } t \geq t_0 + M_* \end{cases} \tag{7}$$

holds, with δ_i a suitable nonnegative constant and C_ν the magnitude of the disturbance ν. □

In order to establish the results that follow, we consider a periodic signal $s(t)$ with period $T > 0$, and denote the gain of the system $\Sigma_i(d)$ in response to s as

$$\overline{\Pi}_i := \left[\frac{\sum_{\ell=1}^{T} \left| \Sigma_i(e^{-j\frac{2\pi}{T}\ell}) \hat{s}_T\left(\frac{2\pi\ell}{T}\right) \right|^2}{\sum_{\ell=1}^{T} \left| \hat{s}_T\left(\frac{2\pi\ell}{T}\right) \right|^2} \right]^{1/2}, \tag{8}$$

where $\hat{s}_T(2\pi\ell/T)$ are the coefficients of the Discrete Fourier Transform of the sequence $s|_1^T$.

Lemma 1. Let the test functionals $\Pi_i(t)$, $i \in \overleftarrow{N}$ be defined as in (4) with $M(t)$ defined as in (5). Further, let the disturbance $\nu(t)$ coincide with the periodic signal $s(t)$ for any $t \in [t_1, t_2]$ such that $t_2 \geq t_1 + M_*$. Then, for any $t \in [t_1 + M_*, t_2]$ and $i \in \overleftarrow{N}$, there exist suitable nonnegative constants λ_i, δ_i such that

$$|\Pi_i(t) - \overline{\Pi}_i| \leq \frac{[\lambda_i T + \delta_i] C_\nu}{\| \nu|_{t-M_*}^{t} \|}. \tag{9}$$

□

We point out that the constants λ_i and δ_i are related to the forced response of $\Sigma_i(d)$ to ν and, respectively, the free response of $\Sigma_i(d)$ to the initial conditions at time $(t - M_*)$.

It can be noticed from (9) that the larger the interval spanned by the finite memory, the lower the bound on the difference $|\Pi_i(t) - \overline{\Pi}_i|$. In fact, for a generic time instant $t \in [t_1 + M_*, t_2]$, we can assert that

$$\| \nu|_{t-M_*}^{t} \| \geq \left\lfloor \frac{M_*}{T} \right\rfloor \| s|_1^T \|, \tag{10}$$

where $\lfloor M_*/T \rfloor$ denotes the largest integer ξ such that $\xi \leq M_*/T$; the equality in (10) holds if $M_* = \mu T$, with μ a positive integer.

Lemma 1 is useful to derive the following theorem.

Theorem 1. Let the same assumptions as in Lemma 1 hold. Further, let $\overline{\lambda} := \max_{i \in \overleftarrow{N}} \lambda_i$ and $\overline{\delta} := \max_{i \in \overleftarrow{N}} \delta_i$. Finally, assume that the HSL (6) is adopted. Then, if

$$\left\lfloor \frac{M_*}{T} \right\rfloor > 4 \, \frac{\overline{\lambda} T + \overline{\delta}}{h \, \| s|_1^T \|} \, C_\nu \tag{11}$$

holds, there can be at most one switch in the interval $[t_1 + M_*, t_2]$. □

The previous theorem shows that at most one switch can occur in the interval $[t_1 + M_*, t_2]$ if the size M_* of the finite memory is sufficiently large, given a certain h, or, equivalently, if the value of the hysteresis constant h is sufficiently large, given a certain M_*. This means that the smaller the value of M_*, the more prompt the control scheme to react to changes in the disturbance characteristics, but also the more prone to spurious switching. If the disturbance characteristics vary in time, this result holds for each pair (t_1, t_2) such that the hypotheses of the theorem are satisfied; a special case of interest is when the disturbance is piecewise periodic. We point out that, if $t_1 \equiv t_0$, (9) and (11) reduce respectively to

$$|\Pi_i(t) - \overline{\Pi}_i| \le \frac{\lambda_i T C_\nu}{\| \nu|_{t-M_*}^t \|},$$

$$\left\lfloor \frac{M_*}{T} \right\rfloor > \frac{4\overline{\lambda} T C_\nu}{h \| \nu|_1^T \|}.$$

4. MULTI-CONTROLLER IMPLEMENTATION

When the switching stops in finite time, as it happens in the case of persistent memory and quasi-stationary disturbance, stability of the overall feedback scheme follows readily from the fact that the final controller is stabilizing by assumption. Hence, in this case, stability does not depend on the particular implementation of the multi-controller $\mathcal{C}_{\sigma(t)}$. On the other hand, when the switching is persistent, as it should happen when the disturbance characteristics vary with time, special care has to be taken in the multi-controller implementation so as to make sure that stability is preserved. With this respect, Hespanha and Morse [2002] have shown how to implement a multi-controller in order to ensure stability for any arbitrary switching sequence when switching between stabilizing controllers. While the results of Hespanha and Morse [2002] pertain to the continuous-time case, in the following of this section we will briefly discuss how similar considerations hold also for the discrete-time case. The section will be concluded by presenting a specific implementation which turns out to be particularly convenient in some cases of interest.

Following the reasoning shown in Hespanha and Morse [2002], we adopt the Youla-Kucera parametrization for the controllers. To this aim, we consider a controller $\mathcal{C}_0$ with transfer function $C_0(d) = S_0(d)/R_0(d)$ satisfying the Bezout identity, *i.e.*

$$A(d)R_0(d) + B(d)S_0(d) = 1\,. \tag{12}$$

Then, as well known, each transfer function $C_i(d)$, $i \in \overleftarrow{N}$ can be expressed as

$$C_i(d) = \frac{S_0(d)D_i(d) + A(d)N_i(d)}{R_0(d)D_i(d) - B(d)N_i(d)} \tag{13}$$

where

$$Q_i(d) = \frac{N_i(d)}{D_i(d)} \tag{14}$$

is the transfer function of the Youla parameter $\mathcal{Q}_i$ corresponding to $\mathcal{C}_i$. Recall also that for any stabilizing controller the corresponding Youla parameter turns out to be stable. Hereafter, we will refer to $\mathscr{Q}$ as the finite family composed of $\{\mathcal{Q}_i,\ i \in \overleftarrow{N}\}$.

The proposed multi-controller implementation is based on the idea of switching between the Youla parameters as depicted in Fig. 2. Accordingly the control input $u(t)$ is generated as

$$u(t) = v(t) - \overline{u}(t) \tag{15}$$

where $v(t)$ is the output of the switching system $\mathcal{Q}_{\sigma(t)}$ having input $\overline{e}(t)$, and the signals $\overline{u}(t)$ and $\overline{e}(t)$ are obtained as

$$\begin{bmatrix} \overline{u}(t) \\ \overline{e}(t) \end{bmatrix} = \begin{bmatrix} (R_0(d)-1) & S_0(d) \\ B(d) & -A(d) \end{bmatrix} \begin{bmatrix} u(t) \\ y(t) \end{bmatrix}. \tag{16}$$

As for the switching system $\mathcal{Q}_{\sigma(t)}$, let m be the largest among the orders of the transfer functions of $\mathcal{Q}_i$ in $\mathscr{Q}$ and let us consider, for any $i \in \overleftarrow{N}$, a stabilizable and detectable m-dimensional realization $\{\overline{A}_i, \overline{B}_i, \overline{C}_i, \overline{D}_i\}$ of the transfer function $Q_i(d)$. Then, $\mathcal{Q}_{\sigma(t)}$ takes the form

$$\mathcal{Q}_{\sigma(t)} : \begin{cases} \zeta(t+1) = \overline{A}_{\sigma(t)}\zeta(t) + \overline{B}_{\sigma(t)}\overline{e}(t) \\ v(t) = \overline{C}_{\sigma(t)}\zeta(t) + \overline{D}_{\sigma(t)}\overline{e}(t) \end{cases}. \tag{17}$$

Notice that each $\overline{A}_i$ is asymptotically stable since the Youla parameter $Q_i(d)$ is stable by construction.

The following result can now be stated, in which we resort to the concept of internal stability, meaning that $(\mathcal{P}/\mathcal{C}_{\sigma(t)})$ is internally stable if all the signals in the system remain bounded for any bounded disturbance ν.

Theorem 2. Let the multi-controller $\mathcal{C}_{\sigma(t)}$ be implemented as in (15)-(17). Then, when $\sigma(t) = i$, the frozen-time transfer function between $y(t)$ and $u(t)$ coincides with $-C_i(d)$.

If in addition the matrices $\overline{A}_i$, $i \in \overleftarrow{N}$ admit a common quadratic Lyapunov function, i.e., there exists a positive definite matrix G such that

$$\overline{A}_i^\top G \overline{A}_i - G < 0, \quad i \in \overleftarrow{N}, \tag{18}$$

then the switching system $(\mathcal{P}/\mathcal{C}_{\sigma(t)})$ made up of the feedback interconnection of $\mathcal{P}$ and $\mathcal{C}_{\sigma(t)}$ is internally stable for any switching signal σ. □

In view of Theorem 2, it can be seen that, in order to ensure stability, it is sufficient to choose the realizations $\{\overline{A}_i, \overline{B}_i, \overline{C}_i, \overline{D}_i\}$, $i \in \overleftarrow{N}$ so that condition (18) holds. The following lemma shows that this is always possible.

Lemma 2. Given any finite set of asymptotically stable transfer functions $Q_i(d)$ related to a family $\mathscr{Q} = \{\mathcal{Q}_i,\ i \in \overleftarrow{N}\}$ and any symmetric positive definite matrix G (whose order m is the largest order of the transfer functions of the systems in $\mathscr{Q}$), there exist stabilizable and detectable m-dimensional realizations $\{\overline{A}_i, \overline{B}_i, \overline{C}_i, \overline{D}_i\}$ for each $Q_i(d)$ such that (18) holds. □

4.1 A simplified implementation

In some circumstances, it may be convenient to design the controller family $\mathscr{C}$ so that all the Youla parameters $Q_i(d)$ associated with the controllers $\mathcal{C}_i \in \mathscr{C}$ share the same denominator, i.e.,

$$Q_i(d) = \frac{N_i(d)}{D(d)} \tag{19}$$

with $D(d)$ a given stable polynomial. In fact, it is immediate to see that, with such a choice, the weighted mixed-sensitivity $\Sigma_i(d)$ takes the form

$$\Sigma_i(d) = \frac{1}{D(d)} \begin{bmatrix} R_0(d)D(d) - B(d)N_i(d) \\ -\eta(S_0(d)D(d) + A(d)N_i(d)) \end{bmatrix}$$

which depends affinely on the coefficients of the numerator $N_i(d)$. Then, for example, the common characteristic polynomial $D(d)$ can be chosen so as to ensure a desired transient behavior, whereas each numerator $N_i(d)$ can be designed by optimizing a H_2 performance objective defined with respect to a certain disturbance frequency profile (different for any $i \in \overleftarrow{N}$).

Further, the choice in (19) has also the positive feature of allowing a simplified implementation of the multi-controller. More specifically, in this case it is not necessary to look for realizations $\{\overline{A}_i, \overline{B}_i, \overline{C}_i, \overline{D}_i\}$ satisfying condition (18), but the switching system $\mathcal{Q}_{\sigma(t)}$ can be simply implemented through the difference equation

$$\mathcal{Q}_{\sigma(t)} : D(d)v(t) = N_{\sigma(t)}(d)\overline{e}(t)\,. \tag{20}$$

In fact, the following result holds.

Proposition 2. Let the multi-controller $\mathcal{C}_{\sigma(t)}$ be implemented as in (15),(16),(20). Then the switching system $(\mathcal{P}/\mathcal{C}_{\sigma(t)})$ is internally stable for any switching signal σ. □

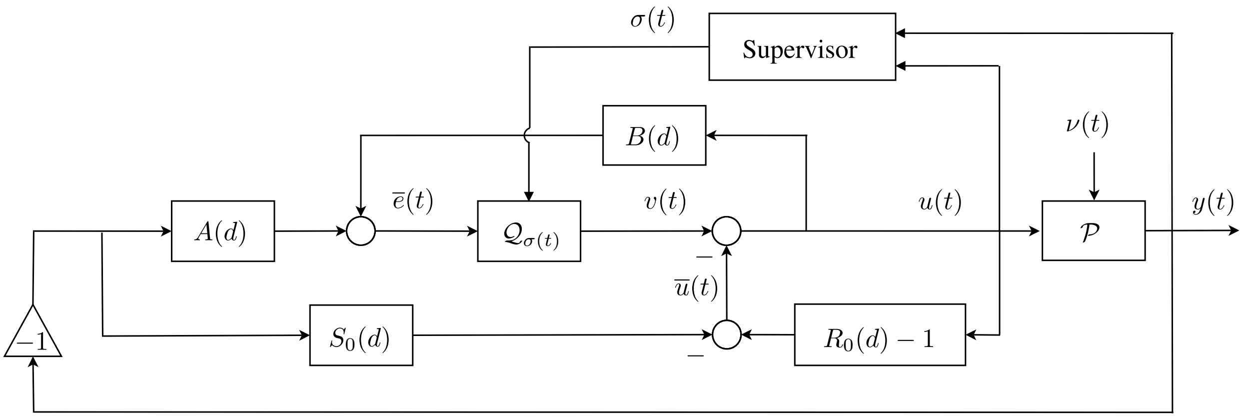

Fig. 2. Multi-controller implementation.

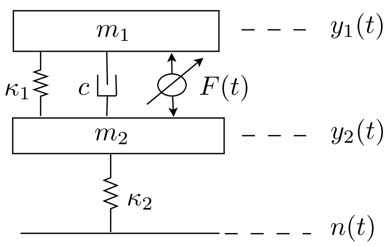

Fig. 3. Quarter-car model scheme.

This latter implementation is the one followed in the simulation set-up presented in the next section.

5. SIMULATION RESULTS

In this section we show simulation results obtained by applying the proposed solution to the active quarter-car model depicted in Fig. 3 subject to sinusoidal road excitations. The system can be described by means of the following equations:

$$\begin{aligned} m_1\ddot{y}_1(t) &= -\kappa_1(y_1(t)-y_2(t)) \\ &\quad -c(\dot{y}_1(t)-\dot{y}_2(t))+F(t) \qquad (21)\\ m_2\ddot{y}_2(t) &= \kappa_1(y_1(t)-y_2(t))+c(\dot{y}_1(t)-\dot{y}_2(t)) \\ &\quad -\kappa_2(y_2(t)-n(t))-F(t) \qquad (22)\end{aligned}$$

where, with abuse of notation, we use the letter t to indicate continuous time. The objective is to regulate y_1 around zero in the presence of the sinusoidal road excitation n. By discretizing the model described by (21) and (22), we obtain a discrete-time model as the one shown in (1), where y is represented by the displacement y_1 around its equilibrium point, u is represented by the force F, and ν is replaced by a moving average of samples of the disturbance n. The sampling time T_s used in the discretization procedure was set to $10^{-3}\,s$.

The considered parameters refer to the motorcycle model of Savaresi et al. [2010] and are summarized in Table 1. For the simulation tests we assumed that the magnitude of the disturbance n was 0.1 m, while its frequency f was unknown and could vary in time within the range $\Omega = [0,\ 10]\ Hz$. Four controllers $\mathcal{C}_i$, $i = 1, 2, 3, 4$, were synthesized, each one able to guarantee a certain level of attenuation in a subset $\Omega_i \in \Omega$. We show in Fig. 4 the Bode Diagram of the transfer functions from n to y_1, obtained when each of the controllers is in feedback with the plant.

Table 1. Parameter set used in the simulation tests.

suspended mass	m_1	117 kg
suspension stiffness	κ_1	30000 N/m
suspension damping	c	3000 N/m/s
tire mass	m_2	30 kg
tire stiffness	κ_2	200000 N/m

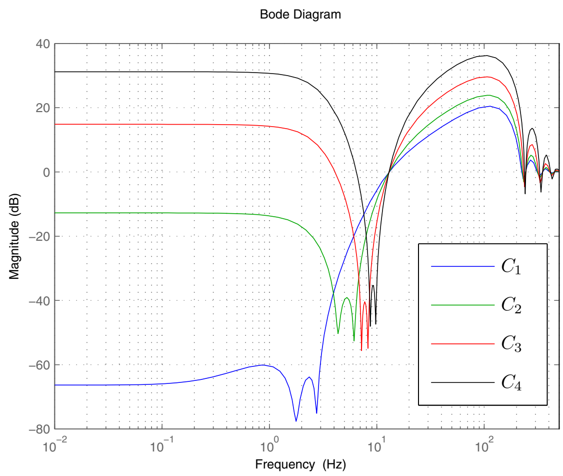

Fig. 4. Bode diagram of the the transfer functions from n to y_1 related to the controllers $\mathcal{C}_i$, $i = 1, 2, 3, 4$.

We adopted a finite memory with $M(t)$ defined as in (5) and $M_* = 2000$ time samples. As for the switching logic, we set $h = 5 \times 10^{-3}$. The parameter η was set to zero; this choice allows one to immediately identify the best controller to be selected by the supervisory unit as the one corresponding to the maximum attenuation level achieved by the transfer functions in Fig. 4 at the frequency f of the sinusoidal disturbance.

We show in Fig. 5 the results obtained by assuming that the frequency f is such that

$$f = \begin{cases} 2.6\,Hz\ , & t \in [0, 10)\,s \\ 5.8\,Hz\ , & t \in [10, 20)\,s\ . \\ 7.3\,Hz\ , & t \in [20, 30)\,s \end{cases} \qquad (23)$$

After a short learning time the supervisory unit selects the controller which provides the best performance level in correspondence to each value of f.

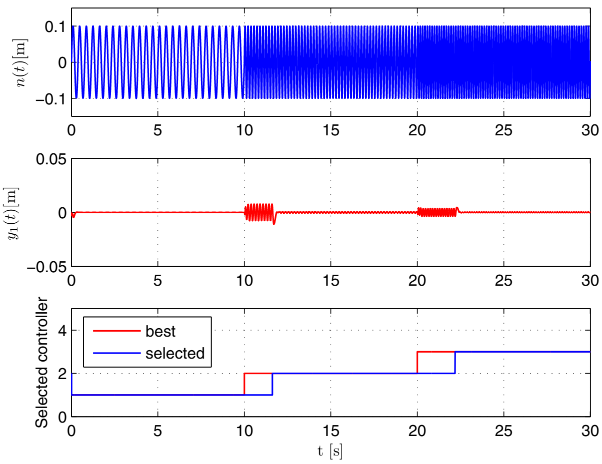

Fig. 5. Sinusoidal road excitation with frequency assuming the values shown in (23).

6. CONCLUSION

In this paper the *Adaptive switching control* paradigm is applied to the problem of disturbance attenuation in the case of a non-negligible uncertainty affecting the a-priori disturbance model. A pre-designed family of stabilizing controllers provides, for each possible operating condition, at least one controller able to achieve a prescribed level of attenuation. The potential performance of each controller is evaluated in terms of test functionals defined on the basis of the plant input/output data. At each time instant, a supervisory unit selects the controller providing the best potential performance according to the *hysteresis switching logic*. Under certain hypotheses, the switching is proved to stop in a finite time and stability follows from the fact that each of the controllers is stabilizing; when the switching is persistent - as in the case of a disturbance whose characteristics vary with time - stability is preserved by adopting a particular implementation for the multi-controller. Simulation results are provided and show that the proposed solution is able to achieve a high performance level.

REFERENCES

G. Agapito, G. Battistelli, D. Mari, D. Selvi, A. Tesi, and P. Tesi. Frequency based design of modal controllers for adaptive optics systems. *Optics Express*, 20(24):27108–27122, 2012.

F.B. Amara, P.T. Kabamba, and A.G. Ulsoy. Adaptive sinusoidal disturbance rejection in linear discrete-time systems–Part II: Experiments. *Journal of Dynamic Systems, Measurement, and Control*, 121(4):655–659, 1999.

S. Baldi, G. Battistelli, E. Mosca, and P. Tesi. Multi-model unfalsified adaptive switching supervisory control. *Automatica*, 46(2):249–259, 2010.

G. Battistelli, J.P. Hespanha, and P. Tesi. Supervisory control of switched nonlinear systems. *International Journal of Adaptive Control and Signal Processing*, 26(8):723–738, 2012.

G. Battistelli, J.P. Hespanha, E. Mosca, and P. Tesi. Model-free adaptive switching control of time-varying plants. *IEEE Transactions on Automatic Control*, 58(5):1208–1220, 2013.

G. Battistelli, E. Mosca, and P. Tesi. Adaptive memory in multi-model switching control of uncertain plants. *Automatica*, 50(3):874–882, 2014.

C. Canudas de Wit and L. Praly. Adaptive eccentricity compensation. *IEEE Transactions on Control Systems Technology*, 8(5):757–766, 2000.

X. Chen and M. Tomizuka. A minimum parameter adaptive approach for rejecting multiple narrow-band disturbances with application to hard disk drives. *IEEE Transactions on Control Systems Technology*, 20(2):408–415, 2012.

X. Guo and M. Bodson. Analysis and implementation of an adaptive algorithm for the rejection of multiple sinusoidal disturbances. *IEEE Transactions on Control Systems Technology*, 17(1):40–50, 2009.

J. Hespanha, D. Liberzon, A. S. Morse, B. D. O. Anderson, T. S. Brinsmead, and F. De Bruyne. Multiple model adaptive control, part 2: switching. *International Journal of Robust Nonlinear Control*, 11(5):479–496, 2001.

J. P. Hespanha, D. Liberzon, and A. S. Morse. Hysteresis-based switching algorithms for supervisory control of uncertain systems. *Automatica*, 39(2):263–272, 2003a.

J. P. Hespanha, D. Liberzon, and A. S. Morse. Overcoming the limitations of adaptive control by means of logic-based switching. *Systems & Control Letters*, 49(1):49–65, 2003b.

J.P. Hespanha and A.S. Morse. Switching between stabilizing controllers. *Automatica*, 38(11):1905–1917, 2002.

A. Karimi and Z. Emedi. H_∞ gain-scheduled controller design for rejection of time-varying narrow-band disturbances applied to a benchmark problem. *European Journal of Control*, 19(4):279–288, 2013.

S.M. Kuo and D.R. Morgan. Active noise control: a tutorial review. *Proceedings of the IEEE*, 87(6):943–973, 1999.

I.D. Landau, M. Alma, A. Constantinescu, J.J. Martinez, and M. Noë. Adaptive regulation – rejection of unknown multiple narrow band disturbances (a review on algorithms and applications). *Control Engineering Practice*, 19(10):1168–1181, 2011.

I.D. Landau, A. Castellanos Silva, T.B. Airimitoaie, G. Buche, and M. Noë. Benchmark on adaptive regulation – rejection of unknown/time-varying multiple narrow band disturbances. *European Journal of Control*, 19(4):237–252, 2013.

R. Marino and P. Tomei. Disturbance cancellation for linear systems by adaptive internal models. *Automatica*, 49(5):1494–1500, 2013.

A. S. Morse, D. Q. Mayne, and G. C. Goodwin. Applications of hysteresis switching in parameter adaptive control. *IEEE Transactions on Automatic Control*, 37(9):1343–1354, 1992.

K.S. Narendra and J. Balakrishnan. Adaptive control using multiple models. *IEEE Transactions on Automatic Control*, 42(2):171–187, 1997.

W. Qian, S. K. Panda, and J.X. Xu. Torque ripple minimization in PM synchronous motors using iterative learning control. *IEEE Transactions on Power Electronics*, 19(2):272–279, 2004.

S.M. Savaresi, C. Poussot-Vassal, C. Spelta, O. Sename, and L. Dugard. *Semi-Active Suspension Control Design for Vehicles*. Butterworth-Heinemann (Elsevier), 2010.

S. Valentinotti, B. Srinivasan, U. Holmberg, D. Bonvin, C. Cannizzaro, M. Rhiel, and U. von Stockar. Optimal operation of fed-batch fermentations via adaptive control of overflow metabolite. *Control Engineering Practice*, 11(6):665–674, 2003.