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An Event-Triggered Consensus Control with Sampled-Data Mechanism for Multi-agent Systems ⋆

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Abstract: In this paper, an event-triggered control strategy with sampled-data mechanism is proposed to address the consensus problem for multi-agent system. The states of each agent are obtained periodically instead of continuous detection by using sampling technology. Rather than traditional Lyapunov technique, the system stability is analyzed by introducing a disagreement vector whose convergence is proved by contraction mapping, and a necessary condition about the upper bound of sampling period is derived under which systems could achieve consensus. Simulations results are presented to show the effectiveness of proposed event-triggered control strategy with sampled data mechanism compared to the basic one.

Keywords: Event-triggered control, sampled-data control, multi-agent systems, consensus.

1. INTRODUCTION

Event-triggered control of multi-agent systems has been a hot issue and attracted a great number of attentions (Mazo & Tabuada, 2011; Heemels et al., 2008; Wang & Lemmon, 2011(a)). Event-triggered control is an alternative method to time-triggered control. The unique feature of it is that the control signal is updated only when a specific event occurs. The generation of this event is predefined by the requirement of stability and performance. Compared to traditional time-triggered control, event-triggered control presents a better performance and actuates more efficiency in many applications (Astrom & Bernhardsson, 2002); Meng & Chen, 2012). Therefore, there has been an increasing interest in the improvement and application of event-triggered control (Lunze & Lemmon, 2010; Astrom, 2008).

Over last few years, event-triggered control has been widely used in cooperative control of multi-agent systems ((Tang et al., 2011; Wang & Lemmon, 2011(b); Tallapragada & Chopra, 2013), especially in consensus problems. An event-triggered real-scheduling of consensus control is designed by Tabuada (2007), where controller actuation is updated when event condition is triggered. Based on this method, a centralized and a decentralized consensus event-triggered consensus strategies are developed with a specific threshold consists of local information (Dimarogonas & Johansson, 2009; Dimarogonas & Frazzoli, 2012). Seyboth et al. (2013) proposed a new event-triggered strategy in which each agent broadcasts its actual state to neighbors only when its own trigger condition is violated, in which the threshold is a function of time. This approach preserves the decentralized character and do not require the continuous information of neighbors.

Recently, a new method named self-triggered control is proposed. It is an extension of event-triggered control and has been considered in many cases (Heemels & Johansson, 2012; Anta & Tabuada, 2010; Almeida at al. 2010). The greatest feature is that the next triggering time is computed in advance, no information of neighbors is required until the next event occurs. But an important precondition of using self-triggered control is that the system models must be given, because the predicted deadline depends on system model.

Nowadays, a new trend of integrating event-triggered control with sampled-data control is widely discussed in the world, because the continuous measurement of state is hardly achieved. It is more realistic to approximate the continuous supervision by a high fast rate sampling due to sensors intermittent working mechanism. Combining the benefits of both event-triggered control and sampled-data control, some results have been proposed.

Peng & Han (2013) proposed an event-triggered strategy and control co-design for sampled-data control systems to determine whether or not to transmit the sampled data. A strategy called period event-triggered control is presented by striking a balance between sampled-data control and event-triggered control (Heemels et al. 2011). Meng & Chen (2013) proposed an event triggering scheme which is designed based on a quadratic Lyapunov function. An sampled-data event detector is introduced to drive the states to their initial average. It is worth to note

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that existing results on event-triggered strategies mostly used a Lyapunov-based triggering condition and adopt a Lyapunov-type argument. It is known to all, Lyapunov function is hard to select, even sometimes could not be found. If the threshold of event condition is a function of time or other tuning parameters instead of a function of local information, the simple quadratic Lyapunov function is no longer suitable. In addition, how large the sampling period could be chosen to guarantee the stability and act a well performance is the most concerning.

In this paper, an event-triggered strategy with sampled-data mechanism is proposed. Different from previous event-triggered methods, the trigger function is not designed by Lyapunov technique which could intrinsically guarantee the global asymptotic stability of system. It only depends on each agents own sampled state. A disagreement error is introduced to describe the difference between current state and final steady state. The system consensus stability is proved by presenting the convergence of the disagreement vector sequence. Moreover, a necessary condition about the upper bound of sampling period is derived to guarantee the asymptotic convergence.

The rest of this paper is organized as follows. Section 2 presents some mathematical preliminaries and the problem formulation in this paper. A basic event-triggered control law is also discussed in this part. An event-triggered control upon sampled data is presented in section 3. In section 4, some simulation results are provided. Section 5 concludes the paper.

2. PRELIMINARIES

In this section some facts about graph theory used in this paper are reviewed and could be found in Godsil and Royle (2001), the problem statement is also given.

2.1 Graph Theory and Problem Statement

For a graph G with N nodes $V = \{1, 2, \dots, N\}$ and edge set $E = \{(i, j) \in V \times V | i, j \ adjacent\}$,the Laplacian matrix L is defined as $L = D - A$, where D is the degree matrix and A is the adjacency matrix of G. The set of neighbors of the node i is denoted by $N_i = \{j \in V : (i, j) \in E\}$ .For connected graphs, L has exactly one zero eigenvalue $\lambda_1(G)$ and the smallest non-zero eigenvalue $\lambda_2(G)$ is called algebraic connectivity.

Consider the multi-agent system consisting of N agents with single-integrator dynamic

$$\dot{x}_i(t) = u_i(t), \tag{1}$$

where x_i denotes the state of agent i, u_i denotes the control input for each agent. The consensus control laws are given by

$$u_i = -\sum_{j\in N_i} a_{ij}(x_i - x_j) \tag{2}$$

In this paper, the above control law is redefined so as to take into account event-triggered strategies. For each i, and $t \geq 0$, a measurement error $e_i(t)$ is introduced, The discrete time instants where the events are triggered are defined as when the condition $f(e_i(t)) = 0$ holds. The sequence of event-triggered execution time is denoted by $t_0^i, t_1^i, \dots$.

For the sequence of events $t_0^i, t_1^i, \dots,$ the state value of agent i is a piecewise constant function $\hat{x}_i(t) = x_i(t_k^i),\ t \in [t_k^i, t_{k+1}^i)$, $\hat{x}_i(t)$ is the latest trigger value of the state, and sent to the neighbor at every trigger time. The control law will update immediately with the state, thus the control law could be redefined as

$$u(t) = -L\hat{x}(t) \tag{3}$$

2.2 A Basic Event-triggered Control

In this part, a basic event-triggered control strategy on fixed and directed graph is derived from Seyboth et al(2013) in which the graph is only undirected. And a useful theorem is presented first.

Lemma 1. Suppose L is the Laplacian matrix of a directed, strongly connected graph G. Then for all $t \geq 0$ and all vectors $\upsilon \in R^N$, with $1^T\upsilon = 0$, $L\left(\hat{G}\right) = \frac{1}{2}\left(L + L^T\right)$ it holds that

$$\left\|e^{-Lt}\upsilon\right\| \leq e^{-\lambda_2(\hat{G})t}\left\|\upsilon\right\|$$

Proof: since the graph G is a strongly connected graph, it is diagonalizable with an orthogonal matrix H. As $H^T L H = \frac{1}{2}H^T(L + L^T)H = H^T L(\hat{G})H$,

$$\begin{aligned} e^{-Lt}\upsilon &= Hdiag(1, e^{-\lambda_2(G)t}, ..., e^{-\lambda_N(G)t})H^T\upsilon \\ &= Hdiag(1, e^{-\lambda_2(\hat{G})t}, ..., e^{-\lambda_N(\hat{G})t})H^T\upsilon \end{aligned}$$

As $1^T\upsilon = 0$, so

$$\begin{aligned} \left\|e^{-Lt}\upsilon\right\| &= \left\|Hdiag(0, e^{-\lambda_2(\hat{G})t}, ..., e^{-\lambda_N(\hat{G})t})H^T\upsilon\right\| \\ &\leq \|H\| \left\|diag(0, e^{-\lambda_2(\hat{G})t}, ..., e^{-\lambda_N(\hat{G})t})\right\| \left\|H^T\right\| \|\upsilon\| \\ &= e^{-\lambda_2(\hat{G})t}\|\upsilon\| \end{aligned}$$

Consider the system (1) with control law (3), we define the measurement error for each agent as

$$e_i(t) = \hat{x}_i(t) - x_i(t) \tag{4}$$

and denote the stack vector $e(t) = [e_1(t), e_2(t), \dots, e_N(t)]^T$. With the event trigger strategy, the closed-loop system changes to

$$\dot{x}(t) = -L\hat{x}(t) = -L(x(t) + e(t)) \tag{5}$$

The trigger function is given by

$$f_i(e_i(t)) = \|e_i(t)\| - ce^{-\alpha t} \tag{6}$$

with $c > 0, \lambda_2(G) > \alpha > 0$. It is obvious that the trigger condition of each agent have nothing to do with neighbors information. Whether agent i should be triggered depends solely on its own state. If agent i is triggered, its current state will be transmitted to neighbors to update the control law. Otherwise the latest triggered state is used to control the system.

The task is to prove the convergence of the proposed event-triggered consensus law. According to Olfati-Saber (2004), since G is strongly connected and balanced, the state vector x can be decomposed as $x(t) = 1 \cdot a + \delta(t)$, where $a = Ave(x)$ is an invariant quantity and δ is called the disagreement vector and $1^T\delta = 0$, 1 is the vector of ones.

Then

$$\begin{aligned} \dot{\delta}(t) = \dot{x}(t) &= -L(x(t) + e(t)) \\ &= -L(1a + \delta(t) + e(t)) \\ &= -L(\delta(t) + e(t)) \end{aligned} \tag{7}$$

The analytical solution of the disagreement dynamics can be calculated as

$$\delta(t) = e^{-Lt}\delta(0) - \int_0^t e^{-L(t-\tau)} Le(\tau)\, d\tau \tag{8}$$

The disagreement vector is bounded by

$$\|\delta(t)\| \le \left\|e^{-Lt}\delta(0)\right\| + \int_0^t \left\|e^{-L(t-\tau)} Le(\tau)\right\| d\tau \tag{9}$$

Applying Theorem 1,

$$\|\delta(t)\| \le e^{-\lambda_2(G)t}\|\delta(0)\| + \int_0^t e^{-\lambda_2(\hat{G})(t-\tau)}\|L\|\|e(\tau)\| d\tau \tag{10}$$

From the trigger condition, it is known that $\|e_i(t)\| \le ce^{-\alpha t}$, so $\|e(t)\| \le \sqrt{N}ce^{-\alpha t}$, solving the integration leads to

$$\|\delta(t)\| \le e^{-\lambda_2(G)t}\|\delta(0)\| + \frac{\|L\|\sqrt{N}c}{\lambda_2(G)-\alpha}\left(e^{-\alpha t} - e^{-\lambda_2(\hat{G})t}\right) \tag{11}$$

since $\lambda_2(G) > \alpha > 0$, the analysis above could guarantee the multi-agent system convergence to a desired formation asymptotically for $t \to \infty$ with $\|\delta(t)\| \to 0$.

Now it remains to show *Zeno behavior* is excluded. It must be proven that there is an inter-event time between two adjacent trigger times lower-bounded by a positive constant. Thus there will not be an accumulation point of events.

With the definition of measurement error, $\dot{e}_i(t) = -\dot{x}_i(t)$. Observe that

$$\begin{aligned}\|\dot{e}_i(t)\| \le \|\dot{x}_i(t)\| \le \|\dot{x}(t)\| &= \|L\hat{x}(t)\| \\ &\le \|L\|\|\hat{x}(t)\|\end{aligned} \tag{12}$$

As $\hat{x}(t)$ is a piecewise constant, and for any t between the latest trigger time t^* and the next event time for agent i

$$\begin{aligned}\|e_i(t)\| &\le \int_{t*}^t \|\dot{x}(t)\| d\tau \\ &\le \int_{t*}^t \|L\|\|\hat{x}(t)\|\, d\tau \\ &\le \|L\|\|\hat{x}(t)\|\,(t-t^*)\end{aligned} \tag{13}$$

A new trigger event will not be executed until $\|e_i(t)\| > ce^{-\alpha t}$. Hence, a lower bound $T = t - t^*$ on the inter-event time is given by

$$\|L\|\|\hat{x}(t)\| e^{\alpha t^*} T = ce^{-\alpha T} \tag{14}$$

Thus, we conclude that the lower bounded inter-event time T is positive.

3. A NEW EVENT-TRIGGERED CONTROL WITH SAMPLED-DATA MECHANISM

Event-triggered formulations require the constant monitoring of a trigger condition. Such continuous state detection and continuous trigger condition checking will be hardly accomplished. In this part, we integrate the sampled data with event-triggered strategy to relax this requirement. The introduction of the sampled state measurement renders the state of each agent obtained by periodic sampling,and the event detector is worked only at each sample time.

Each agent consists of a typical structure with an event triggered control upon sampled data as shown in Fig. 1,

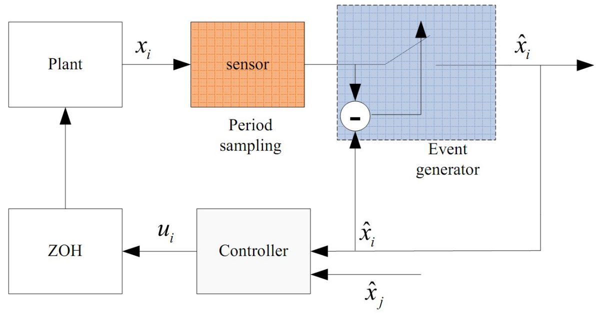

Fig. 1. Conceptual framework of proposed sampled-data based event-triggered control.

where and an event generator is located between the sensor and the controller. The event generator detects errors between current and latest sampled data. Whether or not the sampled data should be sent to neighbors is determined by a specified threshold. If an agents threshold is violated or its neighbor is triggered, the control signal is updated by the newest sampled data, otherwise the control signal is held by a zero-order hold (ZOH) operator.

It is defined that t_k^i is the kth event instant for agent i and h is the sampling period for all agents synchronized by a timer. The event condition for agent i has the following form

$$\left\|e_i(t_k^i + nh)\right\| = \left\|x_i(t_k^i) - x_i\left(t_k^i + nh\right)\right\| \le ce^{-\alpha\left(t_k^i + nh\right)} \tag{15}$$

$t_0^i = 0$ is the initial time, and all measurements $x_i(t_k^i)$ are from the sample states $x_i(mh)$, that is to say, the event is only triggered at sampling time. This means that the inter-event time $\left\{t_{k+1}^i - t_k^i\right\}$ is lower bounded by sampling period h.

Remark 1. The advantages of the event condition in (15) are obvious. First, the event detectors are distributed, each agent only needs its own state to decide the trigger time, the actuator can be triggered asynchronously. Second, the sampled event detector can remarkably reduce sensor energy consumption and network communications as the condition is only checked at discrete sampling times. Finally, the sampled event detector guarantees that the next trigger time is at least one period later. It should be specially noted that when $c \le 0$, the condition is triggered at every sampling time , the event-triggered control turns into a sampled-data control.

The topology is also assumed to be fixed and directed. Under the control law given in the previous section, the closed-loop system for agent i can be obtained as

$$\dot{x}_i(t) = -\sum_{j\in N_i}\left(x_i(t_k^i) - x_j(t_{k'}^j)\right) \tag{16}$$

where $t_{k'}^j = \max\left\{t \middle| t \in \left\{t_k^j, k = 0, 1, \ldots\right\}, t \le t_k^i + nh\right\}$. Notice that for $t \in \left[t_k^i + nh, t_k^i + nh + h\right)$,

$$\begin{aligned}x_i(t_k^i) &= x_i(t_k^i + nh) + e_i(t_k^i + nh) \\ x_j(t_{k'}^j) &= x_j(t_k^i + nh) + e_j(t_k^i + nh)\end{aligned} \tag{17}$$

The equation (16) above can be written in stack vector form as follow for $t \in [mh, mh + h)$

$$\dot{x}(t) = -L(x(mh) + e(mh)) \tag{18}$$

Similar to equation (7),

$$\begin{aligned}\dot{\delta}(t) = \dot{x}(t) &= -L(x(mh) + e(mh)) \\ &= -L(1a + \delta(mh) + e(mh)) \\ &= -L(\delta(mh) + e(mh))\end{aligned} \tag{19}$$

Then solve this equation

$$\begin{aligned}\int_0^t \delta(s)ds = &\int_0^h -L(\delta(0) + e(0))ds + \\ &\int_h^{2h} -L(\delta(h) + e(h))ds + \cdots \\ &+ \int_{mh}^t -L(\delta(mh) + e(mh))ds\end{aligned} \tag{20}$$

where $mh < t < (m+1)h$

The solution is

$$\begin{aligned}\delta(t) &= \delta(mh) + \int_{mh}^t -L(\delta(mh) + e(mh))ds \\ &= (1 - (t - mh)L)\delta(mh) - (t - mh)Le(mh)\end{aligned} \tag{21}$$

and $\delta(mh) = (1 - hL)\delta\left((m-1)h\right) - hLe\left((m-1)h\right)$

To solve the sequence, the equation of $\delta(mh)$ only depends on its initial value and measurement errors of every sampling time. And all of these are known constants.

$$\begin{aligned}\delta(mh) &= (1 - hL)^m \delta(0) - hLe\left((m-1)h\right) \\ &- \cdots - (1 - hL)^{m-1-n} hLe\left(nh\right) - \cdots \\ &- (1 - hL)^{m-1} hLe\left(0\right)\end{aligned} \tag{22}$$

Then the bound is calculated to be

$$\begin{aligned}\|\delta(mh)\| \le &\left\|(1 - hL)^m \delta(0)\right\| + \left\|hLe\left((m-1)h\right)\right\| \\ &+ \left\|(1 - hL)hLe\left((m-2)h\right)\right\| \\ &+ \cdots + \left\|(1 - hL)^{m-1} hLe\left(0\right)\right\| \\ \le &\left\|(1 - hL)^m\right\| \left\|\delta(0)\right\| + \left\|hL\right\| \left\|e\left((m-1)h\right)\right\| \\ &+ \cdots + \left\|(1 - hL)^{m-1-n} hL\right\| \left\|e\left(nh\right)\right\| + \cdots \\ &+ \left\|(1 - hL)^{m-1} hL\right\| \left\|e\left(0\right)\right\|\end{aligned} \tag{23}$$

The purpose is to prove the asymptotic convergence of all agents, that is to say, $\lim_{t \to \infty} \|\delta(t)\| = 0$ should be guaranteed. So

$$\begin{aligned}&\|\delta(t)\| \to 0 \Leftrightarrow \|\delta(mh)\| \to 0 \\ &\Leftrightarrow \begin{cases}\left\|(1 - hL)^m\right\| \left\|\delta(0)\right\| \to 0, \\ \left\|(1 - hL)^{m-1-n} hL\right\| \left\|e\left(nh\right)\right\| \to 0.\end{cases}\end{aligned} \tag{24}$$

As $t \to \infty$,we have $m \to \infty$, $\|e(mh)\| \le \sqrt{N} ce^{-\alpha mh} \to 0$,, and $\|\delta(0)\|, \|e\left(mh\right)\|$ are constants. The only solution could guarantee the condition (24) is

$$|I - hL|^m \to 0 \Leftrightarrow |I - hL| < 1$$

Hence, the following theorem can be concluded.

Theorem 1. Consider a multi-agent system consisted of one-order integrators over a directed graph with the protocol in (2) and triggered by event condition (6). Then all agents converges together asymptotically if $|I - hL| < 1 \Rightarrow 0 < h < \min_{i=(1,2\ldots,n)} \frac{2Re(\lambda_i)}{|\lambda_i|^2}$.

Remark 2. According to Xie et al.(2009a), the maximum sampling period to solve the consensus problem and guarantee the stability is $\min_{\lambda \in \Lambda^+(L)} \frac{2Re(\lambda)}{|\lambda|^2}$ this is consistent with our results. If the topology is undirected, the period limitation can be obtained as $2/\lambda_n$. In other words, when the sampling period is $2/\lambda_n$, the system must trigger at every sampling instant, otherwise the system will not be stable.

4. SIMULATION RESULTS

In order to illustrate the effectiveness of the proposed event-triggered control with sampled data, we compare it to the basic event-triggered implementation of (6). It is supposed that all initial parameters of the simulation are the same. Consider a network of three agents whose Laplacain matrix is given by

$$L = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

The eigenvalues of the matrix are $(0, 1.5 \pm 0.886i)$, then the upper bound of period h is equal to 1. The sampling period h for all agents are chosen as $h = 0.2$ within the upper bound. The initial condition are chosen to $x_1 = [0, 7]$, $x_2 = [2, 1]$, $x_3 = [3, 5]$. For the simulation, the threshold used is $0.8e^{-0.3t}$.

For the basic event-triggered control, the simulation results are shown in Fig. 2-4. It can be seen in Fig.2 that each agent converges to the consensus point. Fig.3 describes the evolution of state as it reaches consensus. Fig.4 shows the norm of measurement error $\|e_i(t)\|$ for each agent. We compare it to the proposed event-triggered control upon sampled data whose simulations are shown in Fig. 5-7.

From Fig.2 and Fig.5, it is seen that both of two event-triggered strategies reach consensus at their weighted average. Due to introducing the sampled data into event-triggered control, the node trajectory in fig. 5 is not as smooth as the one displayed in fig.2. but it does not affect the consensus of nodes. By comparing Fig.3 and Fig.6, It is shown that the convergence time in proposed method is nearly the same as the basic event-triggered method, about 6s in both Fig.3 and Fig.6.The only minor difference is that the oscillations of proposed method are a little larger than the basic one.

An obvious difference in two strategies is exhibited by Fig.4 and Fig.7. It can be found that the event condition is triggered more intensive with basic event-triggered method than with proposed one, which means the number of control updates is greatly reduced by our method. Another obvious difference is in Fig.7 : the event is not triggered even though the measurement error $\|e_i(t)\|$ surpasses the threshold until the next sampling time for the proposed method. In this figure, Though the measurement error $\|e1\|$ at 5.1s is beyond the error threshold, the control action isn't triggered until to 5.2s. Because the trigger condition is only verified at sampling time. The detector is not active between periods even when the threshold is violated and the event is not triggered until next sampling time 5.2s, as $h = 0.2$,but the reduction of control action does not affect the consensus convergence if theorem 3 is satisfied. As a summary, the event-triggered control upon sampled data exhibits a similarly good convergence performance in consensus control while greatly reducing the resource consumption.

5. CONCLUSION

In this paper, we have proposed an event-triggered control with sampled-data mechanism to make multi-agent systems consensus, which combines the benefits of sampled-data control and event-triggered control. A disagreement

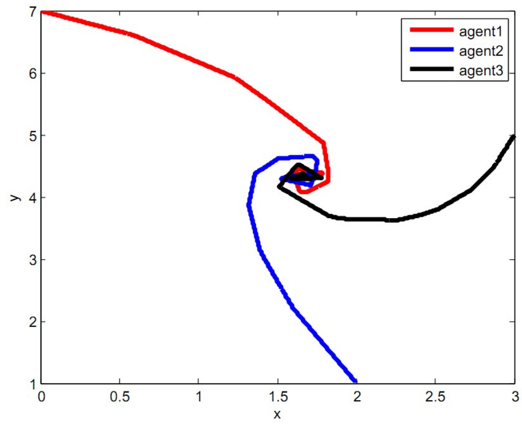

Fig. 2. Consensus result with basic event-triggered control.

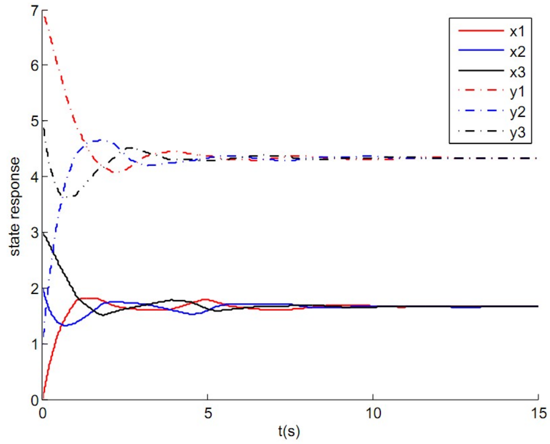

Fig. 3. Evolution of each state with basic event-triggered control.

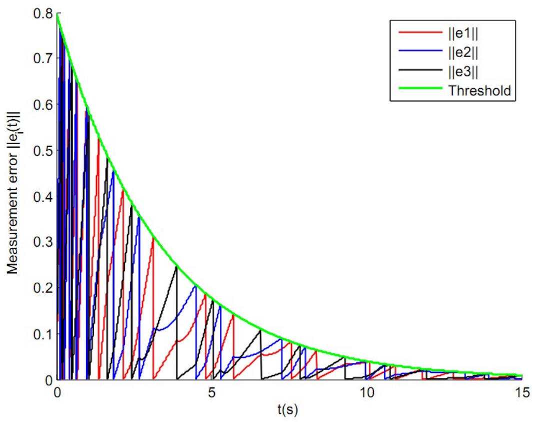

Fig. 4. Evolution of Measurement error with basic event-triggered control.

vector is introduced to provide another method rather than seeking Lyapunov funcion to prove the asymptotically convergence. Through the simulations, we get that there is hardly a difference between the basic event-triggered consensus control and proposed method, while the proposed method remarkably reduces the updates. An upper

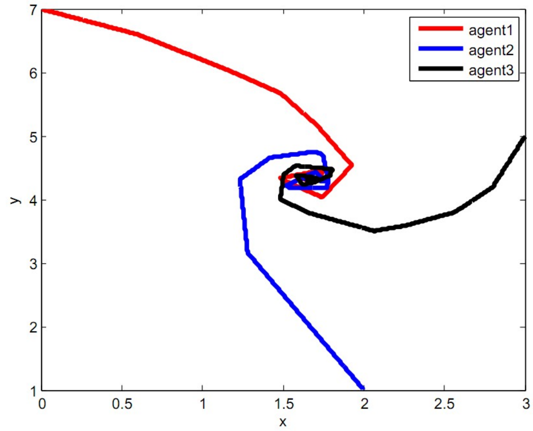

Fig. 5. Consensus result with proposed event-triggered control.

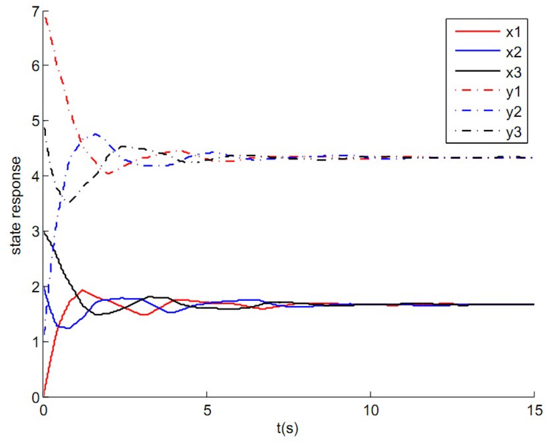

Fig. 6. Evolution of each state with proposed event-triggered control.

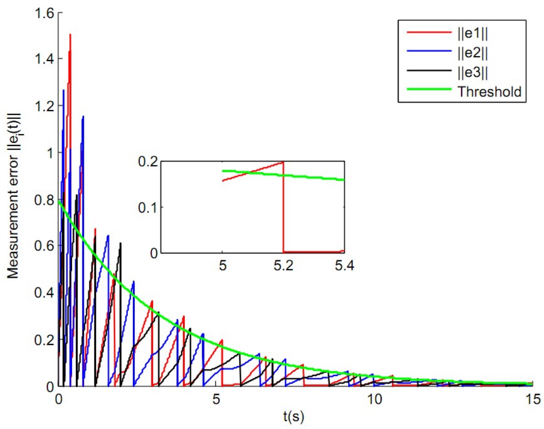

Fig. 7. Evolution of Measurement error with proposed event-triggered control.

bound of sampling period is derived according to the stability condition. The future work is to extend the proposed method to multi-agent system with communication delay and disturbances.

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