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State observation of a class of bilinear systems with large sensor delay ⋆

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Abstract: A predictor bilinear observer is developed for a class of bilinear systems subject to delayed outputs. The developed observer enjoys the property of being convergent for arbitrary large and constant delays. The theory of the observer is presented and approved by numerical simulations of a jacket stirred-tank heater.

Keywords: Time-delay Systems, Bilinear systems, Sensor delay, Observers.

1. INTRODUCTION

Unlike systems governed by ordinary differential equations, time-delay systems are infinite dimensional in nature and time-delay is often a source of instability. Actually, the time delay can be present in the system states, the system inputs or the system outputs with different sizes and characteristics. Since the last two decades several approaches have been devoted to control and filtering of this class of dynamical systems which is also known as hereditary systems or systems with after effect, see Gu et al. [2003], Niculescu [2001], Hale and Lunel [1993]. The significant development of novel strategies dedicated for control and observation of time-delay systems were motivated by the fact that systems with state and output delays are ubiquitous in numerus areas of engineering including but not limited to: sensor networks, process control, and autonomous vehicle control, see e.g., Wang et al. [2013]. To defeat this challenge, the control of time-delay systems has been seen with different looks and quite successfully design procedures have been applied like sliding-mode-based algorithms, convex-optimization-based methods, and predictor stabilizing procedures Krstic [2009]. An overview of some control approaches to time-delay systems is discussed in Richard [2003]. Referring to the abundant literature in control of time-delay systems, the size of the delay plays a key role in the existence of stabilizing controllers Mahmoud [2000]. However, when the state time delay is large and known, predictor-based feedbacks have shown their efficacy in delay compensation of some classes of systems, see e.g. Krstic [2008], Krstic [2009] and the references therein.

In a certain way, observer design is generally seen as the dual problem of state stabilization. Besides the complexity of making an observer convergent with the use of output feedbacks, observer design for time-delay systems has faced additional constraints like the necessity of the perfect knowledge of the time delay, the structure of the system under observation, and the size of the delay. For linear systems, it was shown that state estimation can be achieved whatever the size of the delay if some conservative conditions are met. Delay-dependent conditions, generally stated as convex-optimization problems, have been found less conservative than delay-free conditions. However, the maximum tolerable time delay to fulfill the conditions of existence of an observer is generally small. Among the most well-known approaches to observer design, we cite Lyapunov-based methods that have been developed for special classes of systems like in Ibrir [2011], Cacase et al. [2010], Germani et al. [2001], Boutayeb [2001], Germani and Pepe [2005], and convex-optimization based procedures, see e.g. Mahmoud [2000], Ibrir et al. [2006] and the references therein. A new approach to nonlinear observer design subject to state delay is given in Germani et al. [2002]. An interesting approach based upon prediction has been extended to observer design for linear systems subject to output delay, where the convergence of the observer is assured for large constant delays, see Krstic [2009].

Identifiability and observer design for bilinear systems have been the subject of extensive research contributions see, e.g. Grasselli and Isidori [1981], Bornard et al. [1988], Souley Ali et al. [2006], Sontag et al. [2009]. However, to the best of our knowledge, observer design for arbitrary bilinear systems with long time-delay measurements has not thoroughly studied. In this paper, we extend the design of predictor-observer design to a class of bilinear systems subject to large output delay. We show that the observer error can be made globally asymptotically stable whatever the size of the constant delay. The observer structure is readily constructed as a copy of the system

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dynamics with a correction predictor term that depends on the instantaneous solution of a time-varying Ricatti Equation. We show that the output correction term admits a time-delay state-space realization that facilitates the integration of the observer dynamics in real time. An example of Jacket Stirred-tank heater is considered to illustrate temperature reconstruction for different time delays.

Throughout this paper, we note by $\mathbb{R}$ the set of real numbers. The notation $A > 0$ (resp. $A < 0$) means that the matrix A is positive definite (resp. negative definite). A' is the matrix transpose of A. I stands for the identity matrix of appropriate dimension, $\|\cdot\|_F$ denotes the Frobenius norm, and $\|\cdot\|$ denotes the usual Euclidean norm. We note by $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ the smallest and the largest eigenvalue of the matrix A, respectively. A dynamical system with state variable vector $x(t)$ is input-to-state-stable (ISS) with respect to its control input u, i.e., there exists a class $\mathscr{KL}$ function β and a class $\mathscr{K}$ function γ, such that, for any $x(0)$ and for any input u continuous and bounded on $[0, +\infty)$ the solution exists for all $t \geq 0$ and satisfies

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0) + \gamma\left(\sup_{t_0 \leq \tau \leq t} \|u(\tau)\|\right). \quad (1)$$

2. SYSTEM DESCRIPTION AND OBSERVER DESIGN

Consider the bilinear system

$$\begin{aligned} \dot{x}(t) &= A(u(t))\, x(t) + \psi(u(t)), \\ y(t) &= C\, x(t-\tau), \quad x(0) = x_0, \end{aligned} \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control excitation input, and $y(t) \in \mathbb{R}^p$ is the only measured output. We assume that for $s \in [-\tau, 0[$ the value of $y(s) = \phi(s) \in \mathbb{R}^p$ where $\phi(s)$ is a known vector. The matrix $A(u(t)) \in \mathbb{R}^{n\times n}$ is a well-known real-valued matrix, that is only dependent upon the system input while the input vector $\psi(u(t)) \in \mathbb{R}^n$ is an input-dependent known vector. To complete the system description, the following assumptions are assumed to hold true.

Assumption 1. The pair $\big(A(u(t)), C\big)$ is uniformly observable for all $u(t) \in \mathbb{R}^m$; $t \geq 0$, i.e., there exist a constant Δ and another constant ϱ depending on Δ such that the observability Gramian $J(t-\Delta, t)$ satisfies

$$J(t-\Delta, t) = \int_{t-\Delta}^{t} \psi'(\tau, t) C' C \psi(\tau, t)\, d\tau \geq \varrho I > 0, \quad (3)$$

where $\psi(\tau, t)$ is the state transition matrix of the system:

$$\begin{aligned} &\frac{\partial \psi(\tau, t)}{\partial \tau} = A(u(\tau))\, \psi(\tau, t),\ \psi(t, t) = I, \\ &\psi(\tau, t) = \psi^{-1}(t, \tau). \end{aligned} \quad (4)$$

Assumption 2. The delay τ is constant and known.

Assumption 3. The system output $y(t)$ is continuously measured.

Assumption 4. The excitation input is globally bounded and null for $t < 0$.

The objective of this paper is to design a globally-convergent observer for system (2) whatever the size of the delay τ. Let $Q \in \mathbb{R}^{n\times n}$ be any symmetric and positive definite matrix, and let $P(t)$ be the time solution of the following Algebraic Riccati Equation (ARE):

$$\dot{P}(t) = P(t)\, A'(u(t)) + A(u(t))\, P(t) - P(t) C' C P(t) + Q, \quad (5)$$

where $P(0)$ is chosen as the solution of the following matrix equation:

$$P(0)\, A'(0) + A(0)\, P(0) - P(0) C' C P(0) + Q = 0. \quad (6)$$

From the theory of linear time-varying systems, and based on Assumptions 1-4, it is then concluded that $P(t) > 0$ for all $t \geq 0$. Finally, the observer is readily constructed as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= A(u(t))\, \hat{x}(t) + \psi(u(t)) \\ &\quad + e^{\int_{t-\tau}^{t} A(u(r))\, dr} \mathscr{L}(t-\tau) C z(t-\tau) \\ z(t) &= \hat{x}(t) - x(t) + \int_{t-\tau}^{t} e^{\int_{s}^{t} A(u(r))\, dr} \mathscr{L}(s)\, C\, z(s)\, ds, \end{aligned} \quad (7)$$

where the notation $e^{\mathscr{A}}$ stands for the exponential matrix of $\mathscr{A}$ and the observer gain is given by:

$$\mathscr{L}(s) = \begin{cases} -P(s)\, C' & \text{for } s > 0, \\ -P(0)\, C' & \text{for } -2\tau \leq s \leq 0. \end{cases} \quad (8)$$

It is important to note that the input $u(t) = 0$ for $t < 0$; however, the observability condition, as summarized in Assumption 1, assures the existence of the observer gain for $t < 0$ and therefore, the stability of the observation error can be maintained. To prove the observer convergence for the aforementioned settings, let us highlight that the decay to zero of the state $z(t)$ (see (7)), implies the convergence of the observation error $e(t) = \hat{x}(t) - x(t)$. From (2), and (7), the dynamics of the observation error is given by:

$$\dot{e}(t) = A(u(t))\, e(t) + e^{\int_{t-\tau}^{t} A(u(r))\, dr} \mathscr{L}(t-\tau) C z(t-\tau). \quad (9)$$

Note

$$\xi(t) = \int_{t-\tau}^{t} e^{\int_{s}^{t} A(u(r))\, dr} \mathscr{L}(s)\, C\, z(s)\, ds. \quad (10)$$

This yields,

$$\begin{aligned} \dot{\xi}(t) &= \mathscr{L}(t) C z(t) \\ &\quad - e^{\int_{t-\tau}^{t} A(u(r))\, dr} \mathscr{L}(t-\tau) C z(t-\tau) \\ &\quad + \int_{t-\tau}^{t} A(u(t))\, e^{\int_{s}^{t} A(u(r))\, dr} \mathscr{L}(s)\, C\, z(s)\, ds \end{aligned} \quad (11)$$

From (11) and (9), one can get

$$\dot{z}(t) = \dot{e}(t) + \dot{\xi}(t) = \Big(A(u(t)) + \mathscr{L}(t) C\Big) z(t). \quad (12)$$

By taking the Lyapunov function $V = z'(t) P^{-1}(t) z(t)$, one can show that the inverse matrix of P verifies

$$\begin{aligned} \frac{d}{dt} P^{-1}(t) &= -A'(u(t)) P^{-1}(t) - P^{-1}(t) A(u(t)) + C'C \\ &\quad - P^{-1}(t) Q P^{-1}(t) \end{aligned} \quad (13)$$

and hence, $\dot{V} \leq -z'(t)P^{-1}(t)QP^{-1}(t)z(t)$. As a result $\lim_{t\to\infty} z(t) = \lim_{t\to\infty} e(t) = 0$. Remark that system (8) is stable for all t by the choice of the observer gain (8). We have proved the following statement.

Theorem 1. Consider system (2) under the Assumptions 1-4. Then, for all initial conditions $\hat{x}(s)$ and $z(s)$; $-2\tau \leq s \leq 0$ the states of the predictor observer (7) converge asymptotically to those of system (2) when time elapses.

Even the observer is able to reproduce the system states whatever the size of the delay, the predictor term in the observer dynamics evolves the computation of the exponential of a time-varying matrix. The expansion of the exponential matrix depends essentially on the form of the matrix $A(u(t))$. If the matrix $A(u(t))$ is nilpotent for some order k, i.e., $A^k(u(t)) = 0$ then, the expansion of the exponential matrix is finite, and therefore, the integration of its expansion over a limited time interval can be easily done. In the following section, we give an easy and systematic procedure to compute the observer gain in case where $A(u(t))$ is arbitrary.

3. STATE-SPACE REALIZATION OF THE PREDICTOR OBSERVER

The formulation of the predictor observer as stated in the statement of Theorem 1 is very helpful to prove the convergence of the predictor observer, however, the observer correction terms cannot be easily implemented in that form. The purpose of this section is to propose a state-space realization of the observer gains with stability analysis related issues. It will be shown that the observer gains are the solution of a set of retarded differential equations.

3.1 Dynamic realization of the the observer gains

In this section, we give a state-space realization of all the variables of the predictor observer (7). To this end, let us define

$$\alpha(t) = e^{\int_{t-\tau}^{t} A(u(r))\,dr}. \tag{14}$$

Then,

$$\begin{aligned}\dot{\alpha}(t) &= \Big(A(u(t)) - A(u(t-\tau))\Big)\,\alpha(t),\\ \alpha(0) &= e^{\int_{-\tau}^{0} A(u(r))\,dr}.\end{aligned} \tag{15}$$

Let

$$\mu(t) = \int_{t-\tau}^{t} e^{\int_{w-\tau}^{t-\tau} A(u(r))\,dr}\,\mathscr{L}(w-\tau)\,C\,z(w-\tau)\,dw. \tag{16}$$

Then, we have,

$$Cz(t-\tau) = C\hat{x}(t-\tau) - y(t) + C\,\mu(t). \tag{17}$$

This yields,

$$\begin{aligned}\dot{\mu}(t) &= \mathscr{L}(t-\tau)Cz(t-\tau)\\ &- e^{\int_{t-2\tau}^{t-\tau} A(u(r))\,dr}\,\mathscr{L}(t-2\tau)\,C\,z(t-2\tau)\\ &+ A(u(t-\tau))\int_{t-\tau}^{t} e^{\int_{w-\tau}^{t-\tau} A(u(r))\,dr}\,\mathscr{L}(w-\tau)\\ &\qquad\times C\,z(w-\tau)\,dw.\end{aligned} \tag{18}$$

As a consequence,

$$\begin{aligned}\dot{\mu}(t) &= \mathscr{L}(t-\tau)\Big(C\hat{x}(t-\tau) - y(t) + C\mu(t)\Big)\\ &- e^{\int_{t-2\tau}^{t-\tau} A(u(r))\,dr}\,\mathscr{L}(t-2\tau)\times\\ &\Big(C\hat{x}(t-2\tau) - y(t-\tau) + C\mu(t-\tau)\Big) + A(u(t-\tau))\mu(t).\end{aligned} \tag{19}$$

Finally, the dynamics of the μ vector can be expressed as state-space time delay system of the form:

$$\begin{aligned}\dot{\mu}(t) = &\Big[A(u(t-\tau)) + \mathscr{L}(t-\tau)C\Big]\mu(t)\\ &- \alpha(t-\tau)\mathscr{L}(t-2\tau)C\mu(t-\tau)\\ &+ \mathscr{L}(t-\tau)\big(C\hat{x}(t-\tau) - y(t)\big)\\ &- \alpha(t-\tau)\mathscr{L}(t-2\tau)\big(C\hat{x}(t-2\tau) - y(t-\tau)\big),\end{aligned} \tag{20}$$

with $\mu(s) = 0$ for $-\tau \leq s \leq 0$, and $\alpha(s) = e^{\int_{-\tau}^{0} A(u(r))\,dr}$ for $-2\tau \leq s \leq 0$. For a given $C\hat{x}(s) = \hat{\phi}(s)$; $s \leq 0$, the dynamics of the observer (7) is then rewritten as

$$\begin{aligned}\dot{\hat{x}}(t) &= A(u(t))\,\hat{x}(t) + \psi(u(t))\\ &\quad + \alpha(t)\mathscr{L}(t-\tau)\big[C\hat{x}(t-\tau) - y(t) + C\mu(t)\big],\end{aligned} \tag{21}$$

where the dynamics of $\alpha(t)$, $\mathscr{L}(t)$, and $\mu(t)$ are given by (15), (8), and (20), respectively.

3.2 Stability of the "α" and the "μ" systems

Since $u(t)$ is globally bounded then all the entries of the matrix $\alpha(t)$ are bounded. Consequently, $\forall t \geq 0$, $\|\alpha(t)\|$ verifies the following inequality:

$$\|\alpha(t)\| \leq \left\| e^{\int_{t-\tau}^{t} A(u(s))\,ds} \right\|_F \leq n e^{\left\| \int_{t-\tau}^{t} A(u(s))\,ds \right\|_F}. \tag{22}$$

The dynamics of observer (21) is seen as a copy of the system dynamics with two injection terms. The first correction term is the delayed-output injection and the second one is the μ correction term that compensates the error made by the output delay. Our task here is to show that the realization of the μ dynamics given by (20) assures the convergence of the vector $\mu(t)$ to zero when time elapses. From (20), one can write an equivalent representation of the $\mu(t+\tau)$, that is

$$\begin{aligned}\dot{\mu}(t+\tau) = &\Big[A(u(t)) + \mathscr{L}(t)C\Big]\mu(t+\tau)\\ &- \alpha(t)\mathscr{L}(t-\tau)z(t) + \mathscr{L}(t)C\,e(t).\end{aligned} \tag{23}$$

From (23), it can be seen that the dynamics of $\mu(t+\tau)$ is composed by a stable dynamics perturbed by the input

$-\alpha(t)\mathscr{L}(t-\tau)z(t) + \mathscr{L}(t)C\,e(t)$ that vanishes to zero according to the result of Theorem 1. Therefore, $\mu(t+\tau)$ will converge to zero when $t \to \infty$. To prove this fact, it is sufficient to take the Lyapunov function $W(\mu,\tau) = \mu'(t+\tau)P^{-1}(t)\mu(t+\tau)$ and show that system (23) is Input-to-State-Stable (ISS) with respect to $z(t)$ and $e(t)$. The first derivative of $W(\mu,\tau)$ along the trajectories of system (23) is given by:

$$\begin{aligned}\dot W(\mu,\tau) &= \dot\mu'(t+\tau)P^{-1}(t)\,\mu(t+\tau)\\ &\quad+ \mu'(t+\tau)P^{-1}(t)\dot\mu(t+\tau)\\ &\quad+ \mu'(t+\tau)\,\frac{d}{dt}P^{-1}(t)\,\mu(t+\tau)\\ &= -\mu'(t+\tau)P^{-1}(t)QP^{-1}(t)\,\mu(t+\tau)\\ &\quad- \mu'(t+\tau)C'C\,\mu(t+\tau) - \mu'(t+\tau)C'C\,e(t)\\ &\quad- 2\mu'(t+\tau)\,P^{-1}(t)\alpha(t)\mathscr{L}(t-\tau)z(t).\end{aligned} \tag{24}$$

Using the following inequalities:

$$-\mu'(t+\tau)C'C\,e(t) \le \mu'(t+\tau)C'C\,\mu(t+\tau) + e'(t)C'Ce(t), \tag{25}$$

and

$$\begin{aligned}&-2\mu'(t+\tau)\,P^{-1}(t)\alpha(t)\mathscr{L}(t-\tau)z(t)\\ &\le \frac{1}{2}\mu'(t+\tau)P^{-1}(t)QP^{-1}(t)\mu(t+\tau)\\ &+ 2z'(t)\mathscr{L}'(t-\tau)\alpha'(t)Q^{-1}\alpha(t)\mathscr{L}(t-\tau)z(t).\end{aligned} \tag{26}$$

This yields,

$$\begin{aligned}\dot W(\mu,\tau) &\le -\frac{1}{2}\mu'(t+\tau)P^{-1}(t)QP^{-1}(t)\,\mu(t+\tau)\\ &\quad+ e'(t)C'Ce(t)\\ &\quad 2z'(t)\mathscr{L}'(t-\tau)\alpha'(t)Q^{-1}\alpha(t)\mathscr{L}(t-\tau)z(t).\end{aligned} \tag{27}$$

The boundedness of $\mu(t+\tau)$ depends on the boundedness of all the entries of the matrix $\alpha(t)$ and the boundedness of the matrix $P(t)$ as well. Since $\|\alpha(t)\|$ is bounded as in (22) and $\|P(t)\|$ is bounded by the uniform observability condition of the pair $(A(u(t),C)$ (see Lemma 1 in Chen and Kao [1997] for more details) then, there exit two constants $c_1 = \lambda_{\min}(P^{-1}(t)QP^{-1}(t))$ and $c_2 = \lambda_{\max}(Q^{-1})\|P(t-\tau)\|^2\|\alpha(t)\|^2$ such that

$$\dot W(\mu,\tau) \le -\frac{c_1}{2}\|\mu(t+\tau)\|^2 + \|Ce(t)\|^2 + 2c_2\|Cz(t)\|^2, \tag{28}$$

which implies the ISS property of system (23) with respect to $Ce(t)$ and $Cz(t)$. Since $e(t)$ and $z(t)$ converge to zero by the result of Theorem 1 then, $\lim_{t\to+\infty}\mu(t+\tau) = \lim_{t\to+\infty}\mu(t) = 0$.

4. NUMERICAL SIMULATION

A jacket stirred-tank heater is shown in Figure 1 where a hot fluid is circulated through the jacket of the Continuous Stirred Tank Reactor. Heat flows between the jacket and the vessel which increases the energy content of the fluid inside the vessel. The heat transferred to the vessel fluid from the jacket fluid is $UA(T_j - T)$, where U is the overall heat transfer coefficient, A is the area for heat transfer, T_j is the temperature of the jacket fluid and T the temperature of the vessel fluid. Assuming that the volume and the density are constant, $F_i = F$. The energy balances

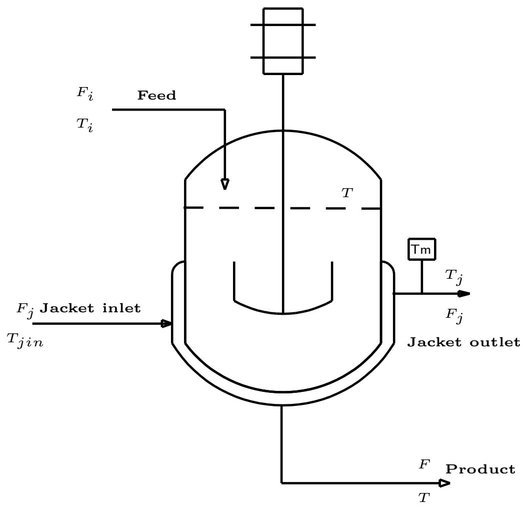

Fig. 1. The Jacket Stirred-tank heater

on the vessel and fluids result in the following differential equations Bequette [2003]

$$\begin{aligned}\frac{dT}{dt} &= \frac{F}{V}(T_i - T) + \frac{UA}{V\rho\,c_p}(T_j - T),\\ \frac{dT_j}{dt} &= \frac{F_j}{V_j}(T_{jin} - T_j) - \frac{UA}{V_j\rho_j\,c_{pj}}(T_j - T),\end{aligned} \tag{29}$$

where V and V_j represent the volumes of the vessel and the jacket, F and F_j represent feed rates, c_p and c_{pj} represent the specific heat capacity, ρ and ρ_j represent densities and the input temperatures are T_i and T_{jin}. The output of this system is a time delayed measurement of the jacket temperature represented by τ. Allowing $x_1(t) = T$, $x_2(t) = T_j$, $u_1(t) = F$, $u_2(t) = T_i$, $u_3(t) = F_j$ and $u_4(t) = T_{jin}$ the system given by Eq. (29) can be rewritten as

$$\begin{aligned}\dot x_1(t) &= -\left(\frac{1}{V}u_1(t) + \frac{UA}{V\rho\,c_p}\right)x_1(t) + \frac{UA}{V\rho\,c_p}x_2(t)\\ &\quad+ \frac{1}{V}u_1(t)\,u_2(t),\\ \dot x_2(t) &= \frac{UA}{V_j\,\rho_j\,c_{pj}}x_1(t) - \left(\frac{1}{V_j}u_3(t) + \frac{UA}{V_j\rho_j\,c_{pj}}\right)x_2(t),\\ &\quad+ \frac{1}{V_j}u_3(t)\,u_4(t),\\ y(t) &= x_2(t-\tau).\end{aligned} \tag{30}$$

In this simulation $\hat x(s)$ and $y(s)$ are null for $-2\tau \le s < 0$. The system parameters are: $V = 20\,\mathrm{ft}^3$, $V_j = 5.5\,\mathrm{ft}^3$, $U = 61.3\,\mathrm{Btu}/(\mathrm{ft}^2\cdot{}^\circ F\cdot\mathrm{min})$, $A = 3\,\mathrm{ft}^2$, $\rho = 60.49\,\mathrm{lb}/\mathrm{ft}^3$, $\rho_j = 61.3\,\mathrm{lb}/\mathrm{ft}^3$, $c_p = 0.5\,\mathrm{Btu}/({}^\circ F\cdot\mathrm{lb})$, and $c_{pj} = 1\,\mathrm{Btu}/({}^\circ F\cdot\mathrm{lb})$. The simulation is performed with the following initial conditions of the observer and the heater system: $\hat x_0 = [105 \quad 54]'$, $\mu(0) = [0 \quad 0]'$, and $x_0 = [21 \quad 85]'$; where $u_1(t) = 10.5 + 0.5\sin(0.5\,t)\,\mathrm{ft}^3/\mathrm{min}$, $u_2(t) = 68\ {}^\circ F$; $\forall t$, $u_3(t) = 15.5 + 0.5\cos^2(0.1\,t)\,\mathrm{ft}^3/\mathrm{min}$ $u_4(t) = 150\ {}^\circ F$; $\forall t$. The performance of the time-delay observer is tested for a small and large constant delay. For $\tau = 5$ (sec), and $Q = I$, we have

$$P(0) = \begin{pmatrix}0.63131 & 0.09026\\ 0.09026 & 0.1595\end{pmatrix}. \tag{31}$$

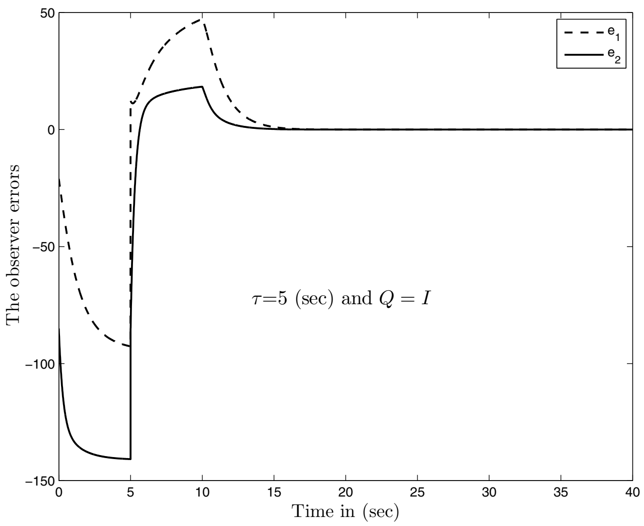

Fig. 2. The observer errors for $\tau = 5$ (sec), $Q = I$

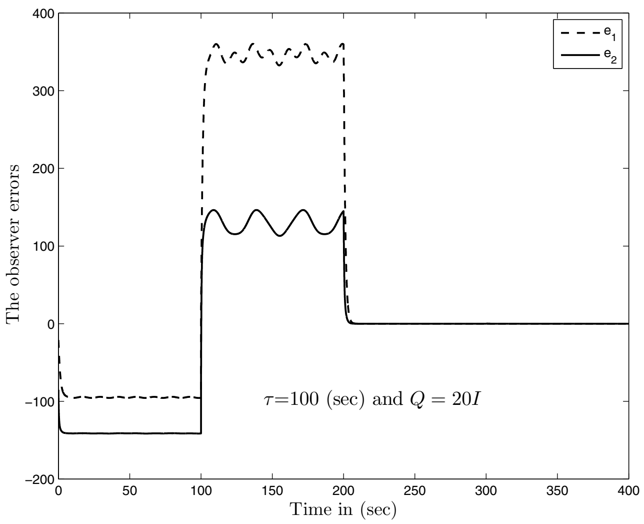

Fig. 3. The observer errors for $\tau = 100$ (sec), $Q = 20I$

The observer is conceived by integrating simultaneously the dynamical equations (5), (15), (20) and (21). As shown in Fig. 2, the observer states converge rapidly to the true states when time is greater than the time delay. This observation is explained by the availability of measurements when $t - \tau \geq 0$. The second simulation is performed for $\tau = 100$ (sec) which represents a significant time delay that affects the observation process. In this simulation we set $Q = 20I$ which gives

$$P(0) = \begin{pmatrix} 11.7457 & 1.08992 \\ 1.08992 & 2.33750 \end{pmatrix}. \quad (32)$$

Notice that the observation errors, depicted in Fig. 3, enter a small neighborhood of the origin after $\tau = 200$ (sec). From this numerical simulation, we stress that the large amount of time delay does not prevent the stability of the observation error even a high-gain output injection is employed to correct the observer dynamics ($Q = 20I$). The oscillations recorded for $t \leq 200$ (sec) are mainly due to the sinusoidal employed input and the retarded dynamics of the μ-state and α-state systems.

5. CONCLUSION

A generalization of predictor observers for a class of bilinear systems is given. Numerical simulations have shown the efficiency of the developed observation procedure for different sizes of time delays. Even the developed observer is globally convergent for arbitrary large delay, the knowledge of the time delay remains necessary for the construction of the observer. The generalization of predictor-observer design for larger classes of systems is under investigation.

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