Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Dynamic Surface Control of Mobile Wheeled Inverted Pendulum Systems with Nonlinear Disturbance Observer ⋆

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Abstract: In this study, a dynamic surface controller based on a nonlinear disturbance observer is investigated to control mobile wheeled inverted pendulum system. By using a coordinate transformation, the underactuated system is presented as a semi-strict feedback form which is convenient for controller design. A dynamic surface controller together with a nonlinear disturbance observer is designed to stabilize the underactuated plant. The proposed dynamic surface controller with a nonlinear disturbance observer can compensate the external disturbances and the model uncertainties to improve the system performance significantly. The stability of the closed-loop mobile wheeled inverted pendulum system is proved by Lyapunov theorem. Simulation results show that the dynamic surface controller with a nonlinear disturbance observer can suppress the effects of external disturbances and model uncertainties effectively.

1. INTRODUCTION

Recently, many robotic systems based on mobile-wheeled inverted pendulum (MWIP) have become quite popular in the robotic community .

However, the control of naturally unstable MWIP system is a challenge.

Many control techniques have been studied in the past decades for the control of benchmark underactuated systems. The investigated control methods include PID control, intelligent control, optimized model reference adaptive control, neural network based model reference control and so on. The Sliding-Mode Control (SMC) might be a comparatively appropriate approach to deal with uncertain MWIP systems because SMC is robust to both parameter variations and noise disturbances. Huang et al. proposed an SMC control scheme (Huang [2010]), which is based on a novel sliding surface, to realize the velocity control of an MWIP system that suffers from uncertainties. Nevertheless, the disadvantage of the SMC control method is "chattering" phenomenon. An alternative control design method called Multiple Sliding Surface Control (MSS), was developed independently of Integrator Backstepping (IB) but is mathematically very similar. It has been investigated in a lot of applications including underactuated systems, e.g., the Inertia Wheel Pendulum(IWP) (Qaiser [2006]). MSS, however, has the same problem as integrator backstepping in that it leads to an "explosion of terms".

In order to avoid the drawback of both IB and MSS mentioned above, a robust nonlinear control technique called Dynamic Surface Control (DSC) has been developed by Swaroop et al (Swaroop [1997]). Recently, many researchers apply the DSC technique into the control of underactuated mechanical systems, including the underactuated marine vessels (Oh [2008]), the Inertia Wheel Pendulum (IWP) and so on. However, most of aforementioned studies discussed only Class-I underactuated mechanical system as defined by R. Olfati-Saber (Saber [2001], Definition 3.9.1), while the MWIP system does not belong to the Class-I underactuated mechanical system. To facilitate the design of DSC for the MWIP system, in this study we transform the dynamics of an MWIP system into a cascade nonlinear system in semi-strict feedback form by using a new global change of coordinates.

Based on an accurate mathematical model, the DSC can achieve good control performance. However, in the practical MWIP modelling and control there are inevitable unknown modeling errors, frictions and other disturbances, which may deteriorate the control performance of an MWIP nonlinear system. It is found that using a disturbance observer can further improve the robustness of DSC controller.

⋆ This work was supported by the International Science & Technology Cooperation Program of China "Precision Manufacturing Technology and Equipment for Metal Parts" under Grant No.2012DFG70640, the International Science & Technology Cooperation Program of Hubei Province "Joint Research on Green Smart Walking Assistance Rehabilitant Robot" under Grant No. 2012IHA00601 and was supported by Program for New Century Excellent Talents in University (Grant No. $NCET-12-0214$).

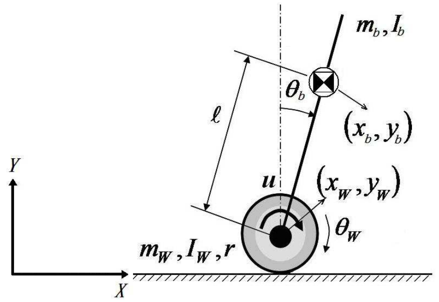

Fig. 1. Mobile Wheeled Inverted Pendulum (MWIP) system model

In this paper, we proposed a dynamic surface control with nonlinear disturbance observer for MWIP system. Appropriate coordinate transformations are also given, as the dynamic model is not in a control design amenable form. A dynamic surface controller is designed for the cascade system and a disturbance observer is designed to compensate the uncertain and disturbance items.

2. SYSTEM FORMULATION

2.1 MWIP System Dynamic Model

The MWIP system is modeled as a one-dimensional (1-D) inverted pendulum that rotates about the wheels' axles. Hence, the body's motion on a plane is determined by the inclination and translational motion. Fig. 1 shows the structure of an MWIP system, where θ_w and θ_b are the wheel's rotation angle and the inclination angle of the body, respectively. To describe the parameters of the MWIP system, some notations should be clarified first (see also Fig. 1), which are as follows:

Table I: Notations

Notation	Decription
m_b, m_w	masses of the body and the wheel
I_b, I_w	moments of inertia of the body and wheel
l	length between the wheel axle and the center of gravity of the body
r	radius of the wheel
D_b	viscous resistance in the driving system
D_w	viscous resistance of the ground
u	rotation torque generated by the motor coaxial with the wheel

Lagrange's motion equation is used to analyze the dynamics of this system, which leads to a second-order underactuated model given by (Huang [2010])

$$\begin{cases} m_{11}\ddot{\theta}_w + m_{12}\cos(\theta_b)\ddot{\theta}_b \\ = u - (D_w + D_b)\dot{\theta}_w + D_b\dot{\theta}_b + m_{12}\dot{\theta}_b^2\sin(\theta_b), \\ m_{12}\cos(\theta_b)\ddot{\theta}_w + m_{22}\ddot{\theta}_b \\ = -u - D_b\left(\dot{\theta}_b - \dot{\theta}_w\right) + G_b\sin(\theta_b) \end{cases} \quad (1)$$

where parameters m_{11}, m_{12}, m_{22} and G_b satisfy

$$\begin{cases} m_{11} = (m_b + m_w)\ r^2 + I_w \\ m_{12} = m_b l r \\ m_{22} = m_b l^2 + I_b \\ G_b = m_b g l \end{cases} \quad (2)$$

Adding the first equation of the (1) to the second one and considering external disturbance, we have

$$\begin{cases} m_{11}\ddot{\theta}_w + m_{12}\cos(\theta_b)\ddot{\theta}_b \\ = u - (D_w + D_b)\dot{\theta}_w + D_b\dot{\theta}_b + m_{12}\dot{\theta}_b^2\sin(\theta_b) + \tau_{ext1}, \\ (m_{11} + m_{12}\cos(\theta_b))\ddot{\theta}_w + (m_{22} + m_{12}\cos(\theta_b))\ddot{\theta}_b \\ = -D_w\dot{\theta}_w + m_{12}\dot{\theta}_b^2\sin(\theta_b) + G_b\sin(\theta_b) + \tau_{ext2} \end{cases} \quad (3)$$

where τ_{ext1} and τ_{ext2} are used to denote external disturbances.

2.2 Nonlinear Disturbance Observer Design

To improve the robustness and control performance of the closed-loop MWIP control system, it is necessary to apply a nonlinear disturbance observer estimating model uncertainties, frictions and external disturbances. This subsection illustrates the design procedure of a nonlinear disturbance observer in the MWIP system.

Considering the nonlinear underactuated system with disturbance, to simplify the denotation, we rewrite (3) as vector form:

$$M(q)\ddot{q} + N(q,\dot{q}) + F(\dot{q}) = \tau + \tau_{ext} \quad (4)$$

where

$$q = [q_1, q_2]^T = [\theta_w, \theta_b]^T$$

$$M(q) = \begin{bmatrix} m_{11} & m_{12}\cos(q_2) \\ m_{11} + m_{12}\cos(q_2) & m_{22} + m_{12}\cos(q_2) \end{bmatrix},$$

$$N(q,\dot{q}) = \begin{bmatrix} -m_{12}\dot{q}_2^2\sin(q_2) \\ -G_b\sin(q_2) - m_{12}\dot{q}_2^2\sin(q_2) \end{bmatrix},$$

$$F(\dot{q}) = \begin{bmatrix} (D_w + D_b)\dot{q}_1 - D_b\dot{q}_2 \\ D_w\dot{q}_1 \end{bmatrix},$$

$$\tau = \begin{bmatrix} u \\ 0 \end{bmatrix}, \tau_{ext} = \begin{bmatrix} \tau_{ext1} \\ \tau_{ext2} \end{bmatrix}.$$

Now, assume that $\hat{M}(q)$ and $\hat{N}(q,\dot{q})$ are the estimates of the actual $M(q)$ and $N(q,\dot{q})$, and that $\Delta M(q)$ and $\Delta N(q,\dot{q})$ are the corresponding additive uncertainties p-resented in the model of the MWIP. That is we have

$$M(q) = \hat{M}(q) + \Delta M(q), \quad N(q,\dot{q}) = \hat{N}(q,\dot{q}) + \Delta N(q,\dot{q}) \quad (5)$$

The lumped disturbance vector τ_d is defined as

$$\tau_d = [\tau_{d1}, \tau_{d2}]^T = \tau_{ext} - \Delta M(q)\ddot{q} - \Delta N(q,\dot{q}) - F(\dot{q}) \quad (6)$$

By this definition, the effect of all dynamic uncertainties, joint frictions and external disturbances is lumped into a single disturbance vector τ_d. From (4), it is seen that

$$\hat{M}(q)\ddot{q} + \hat{N}(q,\dot{q}) = \tau + \tau_d \quad (7)$$

To estimate the lumped disturbance τ_d, the nonlinear disturbance observer is designed as:

$$\dot{\hat{\tau}}_d = -L\hat{\tau}_d + L(\hat{M}(q)\ddot{q} + \hat{N}(q,\dot{q}) - \tau) \quad (8)$$

Define $\tilde{\tau}_d = \tau_d - \hat{\tau}_d$ as the disturbance tracking error and using (8), it is observed that

$$\dot{\hat{\tau}}_d = L\tilde{\tau}_d \quad (9)$$

or, equivalently

$$\dot{\tilde{\tau}}_d = \dot{\tau}_d - L\tilde{\tau}_d \quad (10)$$

In general, there is no prior information about the derivative of the disturbance τ_d. When the disturbance varies slowly relative to the observer dynamics, it is reasonable to suppose that $\dot{\tau}_d = 0$. Therefore, we have

$$\dot{\tilde{\tau}}_d = -\dot{\hat{\tau}}_d = -L\tilde{\tau}_d \tag{11}$$

Let us define an auxiliary variable $z = [z_1, z_2]^T = \hat{\tau}_d - p(q,\dot{q})$, where $\frac{d}{dt}p(q,\dot{q}) = L(q,\dot{q})\hat{M}(q)\ddot{q}$. And substitute it to (8), the observer can be designed as

$$\begin{aligned} &\dot{z} = L(q,\dot{q})\left\{\hat{N}(q,\dot{q}) - \tau - p(q,\dot{q}) - z\right\} \\ &\hat{\tau}_d = z + p(q,\dot{q}) \end{aligned} \tag{12}$$

In this study, the following disturbance observer gain matrix $L(q,\dot{q})$ and vector $p(q,\dot{q})$ is used as

$$\begin{cases} L(q) = X\hat{M}^{-1}(q) \\ p(\dot{q}) = X\dot{q} \end{cases} \tag{13}$$

where X is a constant invertible matrix to be determined, that is

$$X = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}, c_i \geq 0, i = 1,2,3,4 \tag{14}$$

Substituting (13) and (14) into (12), and using (4) we have

$$\begin{cases} \dot{z}_1 = A^{-1}\left[(c_2D_2 - c_1D_1)\right. \\ \quad \cdot\left(\hat{m}_{12}\dot{\theta}_b^2\sin(\theta_b) + u + c_1\dot{\theta}_w + c_2\dot{\theta}_b + z_1\right) \\ \quad - (c_2\hat{m}_{11} - c_1\hat{m}_{12}\cos(\theta_b))\left(\hat{G}_b\sin(\theta_b)\right. \\ \quad \left.\left. + \hat{m}_{12}\dot{\theta}_b^2\sin(\theta_b) + c_3\dot{\theta}_w + c_4\dot{\theta}_b + z_2\right)\right] \\ \dot{z}_2 = A^{-1}\left[(c_4D_2 - c_3D_1)\right. \\ \quad \cdot\left(\hat{m}_{12}\dot{\theta}_b^2\sin(\theta_b) + u + c_1\dot{\theta}_w + c_2\dot{\theta}_b + z_1\right) \\ \quad - (c_4\hat{m}_{11} - c_3\hat{m}_{12}\cos(\theta_b))\left(\hat{G}_b\sin(\theta_b)\right. \\ \quad \left.\left. + \hat{m}_{12}\dot{\theta}_b^2\sin(\theta_b) + c_3\dot{\theta}_w + c_4\dot{\theta}_b + z_2\right)\right] \\ \hat{\tau}_{d1} = z_1 + c_1\dot{\theta}_w + c_2\dot{\theta}_b \\ \hat{\tau}_{d2} = z_2 + c_3\dot{\theta}_w + c_4\dot{\theta}_b \end{cases} \tag{15}$$

where

$$\begin{cases} A = \hat{m}_{11}\hat{m}_{22} - [\hat{m}_{12}\cos(\theta_b)]^2 \\ D_1 = \hat{m}_{22} + \hat{m}_{12}\cos(\theta_b) \\ D_2 = \hat{m}_{11} + \hat{m}_{12}\cos(\theta_b) \end{cases}$$

3. CONTROLLER DESIGN

In general, the DSC control design requests the strict or semi-strict feedback form of the nonlinear system model.

The dynamics of a Class-I underactuated mechanical system may be transformed into a cascade nonlinear system in strict feedback form. However, the dynamics of a Class-II underactuated system may only be transformed into a cascade nonlinear system in a quadratic nontriangular form. Similarly, the dynamics of a MWIP-type underactuated system may not be transformed into a cascade nonlinear system in strict feedback form. This makes it difficult to design a DSC controller for the MWIP system.

In order to solve this problem, we propose a coordinate transformation approach to transform an MWIP model into a cascade nonlinear system in semi-strict feedback form.

Firstly, let us introduce the following variables

$$\begin{cases} x_1 = (\hat{m}_{11} + \hat{m}_{12}\cos(x_2))\,x_4 \\ \quad + (\hat{m}_{22} + \hat{m}_{12}\cos(x_2))\,x_3 \\ x_2 = q_2 = \theta_b \\ x_3 = \dot{q}_2 = \dot{\theta}_b \\ x_4 = \dot{q}_1 = \dot{\theta}_w \end{cases} \tag{16}$$

the system model (3) can then be rewritten as

$$\begin{cases} \hat{m}_{11}\dot{x}_4 + \hat{m}_{12}\cos(x_2)\,\dot{x}_3 \\ = u + \hat{m}_{12}x_3^2\sin(x_2) + \tau_{d1} \\ \\ (\hat{m}_{11} + \hat{m}_{12}\cos(x_2))\,\dot{x}_4 + (\hat{m}_{22} + \hat{m}_{12}\cos(x_2))\,\dot{x}_3 \\ = \hat{m}_{12}x_3^2\sin(x_2) + \hat{G}_b\sin(x_2) + \tau_{d2} \end{cases} \tag{17}$$

It follows from (16) and (17) that we have:

$$\dot{x}_1 = -\hat{m}_{12}\sin(x_2)\,x_3x_4 + \hat{G}_b\sin(x_2) + \tau_{d2} \tag{18a}$$

$$\begin{cases} \dot{x}_2 = \dfrac{x_1}{\hat{m}_{22} + \hat{m}_{12}\cos(x_2)} - \dfrac{\hat{m}_{11} + \hat{m}_{12}\cos(x_2)}{\hat{m}_{22} + \hat{m}_{12}\cos(x_2)}x_4 \\ \\ \dot{x}_4 = \left[A^{-1}\hat{m}_{12}\hat{m}_{22}\sin(x_2)\right. \\ \quad \cdot\left(\dfrac{x_1 - (\hat{m}_{11} + \hat{m}_{12}\cos(x_2))\,x_4}{\hat{m}_{22} + \hat{m}_{12}\cos(x_2)}\right)^2 \\ \quad \left. - \hat{m}_{12}\hat{G}_b\sin(x_2)\cos(x_2)\right] \\ \quad + A^{-1}(\hat{m}_{22} + \hat{m}_{12}\cos(x_2))\,u \\ \quad + A^{-1}\left[(\hat{m}_{22} + \hat{m}_{12}\cos(x_2))\,\tau_{d1}\right. \\ \quad \left. - \hat{m}_{12}\cos(x_2)\,\tau_{d2}\right] \end{cases} \tag{18b}$$

After coordinate transformation the MWIP system model is represented in a semi-strict feedback form as cascade of a outer (18b) similar to the work in Yang [2007] and a core (18a) subsystem.

Assumption 1. The uncertainties and disturbance of the system are bounded and satisfy

$$|\tau_{d_1}| \leq \bar{d}_1, |\tau_{d_2}| \leq \bar{d}_2, |\tilde{\tau}_{d1}| \leq \xi_1, |\tilde{\tau}_{d2}| \leq \xi_2$$

where $\bar{d}_1, \bar{d}_2, \xi_1$ and ξ_2 are known bounds.

Our purpose is to design a control u forcing x_2 to be stabilized around zero. Together with the proposed disturbance observer, for MWIP system (18) we design a new Dynamic Surface Controller with Nonlinear Disturbance Observer (DSCNDO) as follows:

$$u = u_{DSC} - u_{\tau_d} \tag{19}$$

$$u_{\tau_d} = u_{\tau_{d1}} - u_{\tau_{d2}} \tag{20}$$

where

$$\begin{cases} u_{\tau_{d1}} = \hat{\tau}_{d1} \\ u_{\tau_{d2}} = \dfrac{\hat{m}_{12}\cos(x_2)}{\hat{m}_{22} + \hat{m}_{12}\cos(x_2)}\hat{\tau}_{d2} \end{cases} \tag{21}$$

For convenience of the mathematical derivation, we define following notations in advance:

$$\begin{cases} M_{C12} = \hat{m}_{12}\cos(x_2) = \hat{m}_{12}\cos(S_1), \\ G_{S2} = \hat{G}_b\sin(x_2) = \hat{G}_b\sin(S_1), \\ M_{S12} = \hat{m}_{12}\sin(x_2) = \hat{m}_{12}\sin(S_1), \\ \bar{A} = \hat{m}_{11}\hat{m}_{22} - \hat{m}_{12}^2 \end{cases}$$

The pure DSC component of DSCNDO, u_{DSC}, can be obtained through the following procedure:
Step 1: Design the virtual control law $\bar{x}_4$

1) Define the first dynamic surface

$$S_1 = x_2 - x_d = x_2 - 0 = x_2 \tag{22}$$

Then, from the first equation of (18b) the derivative of S_1 can be expressed as

$$\dot{S}_1 = \dot{x}_2 = \frac{x_1}{\hat{m}_{22} + M_{C12}} - \frac{\hat{m}_{11} + M_{C12}}{\hat{m}_{22} + M_{C12}} x_4 \tag{23}$$

2) Select the virtual control law $\bar{x}_4$,

$$\bar{x}_4 = \frac{\hat{m}_{22} + M_{C12}}{\hat{m}_{11} + M_{C12}} \left(\frac{1}{\hat{m}_{22} + M_{C12}} x_1 + k_1 S_1 \right) \tag{24}$$

where $k_1 > 0$.

3) Input $\bar{x}_4$ to a first order filter, then

$$\tau_4 \dot{x}_{4d} + x_{4d} = \bar{x}_4, x_{4d}(0) = \bar{x}_4(0) \tag{25}$$

where $\tau_4 > 0$ is the filter time constant.

Step 2: Design the actual control law u_{DSC}

1) Define the second dynamic surface,

$$S_2 = x_4 - x_{4d} \tag{26}$$

Then, from the second equation of (18b), (19)-(21) and (25), the derivative of S_2 can be expressed as

$$\begin{aligned} \dot{S}_2 = {} & A^{-1} (\hat{m}_{22} + M_{C12}) u_{DSC} \\ & + A^{-1} \left[\hat{m}_{12}\hat{m}_{22} \sin(x_2) \left(\frac{x_1 - (\hat{m}_{11} + M_{C12}) x_4}{\hat{m}_{22} + M_{C12}} \right)^2 \right. \\ & \left. - M_{C12} G_{S2} \right] - \frac{\bar{x}_4 - x_{4d}}{\tau_4} \\ & + A^{-1} \left[(\hat{m}_{22} + M_{C12}) \tilde{\tau}_{d1} - M_{C12} \tilde{\tau}_{d2} \right] \end{aligned} \tag{27}$$

2) Select the control law u_{DSC} as follows,

$$\begin{aligned} u_{DSC} = {} & \frac{A}{\hat{m}_{22} + M_{C12}} \left(\frac{\bar{x}_4 - x_{4d}}{\tau_4} - k_2 S_2 \right) \\ & - \frac{\hat{m}_{12}\hat{m}_{22} \sin(x_2) (x_1 - (\hat{m}_{11} + M_{C12}) x_4)^2}{(\hat{m}_{22} + M_{C12})^3} \\ & + \frac{M_{C12} G_{S2}}{\hat{m}_{22} + M_{C12}} \end{aligned} \tag{28}$$

where $k_2 > 0$.

If the filter error is defined as follows:

$$e = x_{4d} - \bar{x}_4 \tag{29}$$

By combining the above equation with (24), we have

$$e = x_{4d} - \frac{\hat{m}_{22}+M_{C12}}{\hat{m}_{11}+M_{C12}} \left(\frac{x_1}{\hat{m}_{22}+M_{C12}} + k_1 S_1 \right) \tag{30}$$

From (23)-(26) and (29), the derivative of S_1 can be written as

$$\dot{S}_1 = -\frac{\hat{m}_{11}+M_{C12}}{\hat{m}_{22}+M_{C12}} (S_2 + e) - k_1 S_1 \tag{31}$$

From (27) and (28), we obtain

$$\dot{S}_2 = -k_2 S_2 + A^{-1} (\hat{m}_{22}+M_{C12}) \tilde{\tau}_{d1} - A^{-1} M_{C12} \tilde{\tau}_{d2} \tag{32}$$

Then, from (18a),(24)-(26) and (29)-(31), the derivative of e is given by

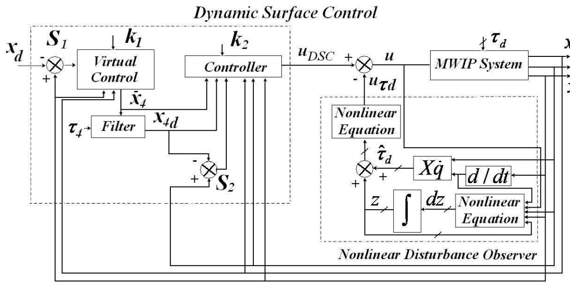

Fig. 2. Block diagram of MWIP system with DSCNDO.

$$\begin{aligned} \dot{e} = {} & -\frac{e}{\tau_4} - \left[\frac{\dot{x}_1 (\hat{m}_{11}+M_{C12}) + x_1 x_3 M_{S12}}{(\hat{m}_{11}+M_{C12})^2} \right. \\ & \left. + \frac{(\hat{m}_{22} - \hat{m}_{11}) M_{S12} x_3}{(\hat{m}_{11}+M_{C12})^2} k_1 S_1 + k_1 \dot{S}_1 \frac{\hat{m}_{22}+M_{C12}}{\hat{m}_{11}+M_{C12}} \right] \end{aligned} \tag{33}$$

The whole control system block diagram is shown in Fig.2

4. STABILITY ANALYSIS

To analyze the stability of system (18) controlled by the proposed DSCNDO, we first introduce the following lemma.

Lemma 1. (Qu [1994]) For a nonlinear system (18), suppose that a positive definite function $V(x(t)) = V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \le -\varsigma V + C$$

where constants ς and C satisfy $\varsigma > 0$ and $C \ge 0$. Then, for any given t_0, function $V(t)$ satisfies the following inequality:

$$0 \le V(t) \le [C - (C - \varsigma V(t_0)) \exp(-\varsigma(t - t_0))] / \varsigma, \forall t \ge t_0$$

Thus, the state of the system (18) is uniformly and ultimately bounded.

The stability analysis of the whole system is concluded in the following theorem:

Theorem 2. Considering a system (18) with modelling errors, external disturbance, unknown payloads and frictions, there exists a set of the surface gains k_1, k_2, the filter time constant τ_4 and the observer gains c_1, c_2, c_3, c_4 satisfying

$$\gamma = \min(a_1, a_2, a_3, a_4, a_5) > 0, \exists \gamma \tag{34}$$

where

$$\begin{cases} a_1 = k_1 - \dfrac{\hat{m}_{11} + \hat{m}_{12}}{\hat{m}_{22}} \\ a_2 = k_2 - \dfrac{k_1}{2} - \dfrac{\hat{m}_{11} + \hat{m}_{12}}{2\hat{m}_{22}} \\ a_3 = \dfrac{1}{\tau_4} - \dfrac{3k_1}{2} - \dfrac{\hat{m}_{11} + \hat{m}_{12}}{2\hat{m}_{22}} \\ a_4 = \dfrac{c_1}{\hat{m}_{11}} - \dfrac{(\hat{m}_{11} + \hat{m}_{12}) c_2}{\bar{A}} - 1 \\ a_5 = \dfrac{c_4}{\hat{m}_{22}} - \dfrac{\hat{m}_{12} c_3}{\bar{A}} - 1 \end{cases} \tag{35}$$

such that the nonlinear disturbance observer based dynamic surface controller guarantees: Based on the control law (19)-(21) and (28), system (18) is semi-globally uniformly and ultimately bounded.

Proof. Choose the following Lyapunov function candidate

$$V=\frac{1}{2}S_1^2+\frac{1}{2}S_2^2+\frac{1}{2}e^2+\frac{1}{2}\tilde{\tau}_d^T\tilde{\tau}_d \tag{36}$$

Using (7), (8), (11) and (13)-(14) we have

$$\begin{aligned}(\frac{1}{2}\tilde{\tau}_d^T\tilde{\tau}_d)' = A^{-1}&\left\{(\hat{m}_{11}+M_{C12})\,c_2\tilde{\tau}_{d1}^2-(\hat{m}_{22}+M_{C12})\,c_1\tilde{\tau}_{d1}^2\right.\\&-[(\hat{m}_{22}+M_{C12})\,c_3+\hat{m}_{11}c_2]\,\tilde{\tau}_{d1}\tilde{\tau}_{d2}\\&+[(\hat{m}_{11}+M_{C12})\,c_4+M_{C12}c_1]\,\tilde{\tau}_{d1}\tilde{\tau}_{d2}\\&\left.+M_{C12}c_3\tilde{\tau}_{d2}^2-\hat{m}_{11}c_4\tilde{\tau}_{d2}^2\right\}\end{aligned} \tag{37}$$

In addition, we take into account

$$\begin{aligned}&\hat{m}_{11}\hat{m}_{22}-\hat{m}_{12}^2=[(\hat{m}_b+\hat{m}_w)\hat{r}^2+\hat{I}_w](\hat{m}_b l^2+\hat{I}_b)-\hat{m}_b^2\hat{l}^2\hat{r}^2\\&=\hat{m}_w\hat{m}_b\hat{l}^2\hat{r}^2+(\hat{m}_b+\hat{m}_w)\hat{r}^2\hat{I}_b+\hat{m}_b\hat{l}^2\hat{I}_w+\hat{I}_w\hat{I}_b>0\end{aligned} \tag{38}$$

From (37) and (38) we have

$$\begin{aligned}\frac{d}{dt}\left(\frac{1}{2}\tilde{\tau}_d^T\tilde{\tau}_d\right)\le&\left(\frac{(\hat{m}_{11}+\hat{m}_{12})\,c_2}{\bar{A}}-\frac{c_1}{\hat{m}_{11}}\right)\tilde{\tau}_{d1}^2\\&+\left(\frac{\hat{m}_{12}c_3}{\bar{A}}-\frac{c_4}{\hat{m}_{22}}\right)\tilde{\tau}_{d2}^2\\&+A^{-1}[(c_1+c_4)\,M_{C12}+c_4\hat{m}_{11}\\&-(\hat{m}_{11}c_2+\hat{m}_{22}c_3+M_{C12}c_3)]\,\tilde{\tau}_{d1}\tilde{\tau}_{d2}\end{aligned} \tag{39}$$

According to (31)-(33) and (39), we have

$$\begin{aligned}\dot{V}\le&-k_1S_1^2-k_2S_2^2-\frac{e^2}{\tau_4}+k_1eS_2+k_1e^2\\&-\left(\frac{c_1}{\hat{m}_{11}}-\frac{(\hat{m}_{11}+\hat{m}_{12})\,c_2}{\bar{A}}\right)\tilde{\tau}_{d1}^2\\&-\left(\frac{c_4}{\hat{m}_{22}}-\frac{\hat{m}_{12}c_3}{\bar{A}}\right)\tilde{\tau}_{d2}^2\\&-\frac{\hat{m}_{11}+M_{C12}}{\hat{m}_{22}+M_{C12}}\,(S_2+e)\,S_1\\&+A^{-1}\,(\hat{m}_{22}+M_{C12})\,S_2\tilde{\tau}_{d1}\\&-A^{-1}M_{C12}S_2\tilde{\tau}_{d2}-\frac{G_{S2}e}{\hat{m}_{11}+M_{C12}}\\&-\frac{e\tau_{d2}}{\hat{m}_{11}+M_{C12}}-\frac{M_{S12}\,(S_2+e)^2\,e}{\hat{m}_{22}+M_{C12}}\\&-\frac{M_{S12}k_1S_1\,[2\,(S_2+e)+k_1S_1]\,e}{\hat{m}_{11}+M_{C12}}\\&+\frac{(\hat{m}_{22}-\hat{m}_{11})\,M_{S12}k_1S_1\,(S_2+e)\,e}{(\hat{m}_{11}+M_{C12})\,(\hat{m}_{22}+M_{C12})}\\&+\frac{(\hat{m}_{22}+M_{C12})\,k_1^2S_1e}{\hat{m}_{11}+M_{C12}}\\&+A^{-1}\,[(c_1+c_4)M_{C12}+c_4\hat{m}_{11}\\&-(\hat{m}_{11}c_2+\hat{m}_{22}c_3+M_{C12}c_3)]\,\tilde{\tau}_{d1}\tilde{\tau}_{d2}\end{aligned} \tag{40}$$

Including Young's inequality explicitly would do no harm here.

$$\begin{aligned}\dot{V}\le&-\left(k_1-\frac{\hat{m}_{11}+\hat{m}_{12}}{\hat{m}_{22}}\right)S_1^2\\&-\left(k_2-\frac{k_1}{2}-\frac{\hat{m}_{11}+\hat{m}_{12}}{2\hat{m}_{22}}\right)S_2^2\\&-\left(\frac{1}{\tau_4}-\frac{3k_1}{2}-\frac{\hat{m}_{11}+\hat{m}_{12}}{2\hat{m}_{22}}\right)e^2\\&-\left(\frac{c_1}{\hat{m}_{11}}-\frac{(\hat{m}_{11}+\hat{m}_{12})\,c_2}{\bar{A}}-1\right)\tilde{\tau}_{d1}^2\\&-\left(\frac{c_4}{\hat{m}_{22}}-\frac{\hat{m}_{12}c_3}{\bar{A}}-1\right)\tilde{\tau}_{d2}^2\\&+\frac{1}{4}A^{-2}S_2^2\left[(\hat{m}_{22}+M_{C12})^2+M_{C12}^2\right]\\&-\frac{G_{S2}e+M_{S12}k_1S_1\,[2\,(S_2+e)+k_1S_1]\,e}{\hat{m}_{11}+M_{C12}}\\&+\frac{e^2}{4\,(\hat{m}_{11}+M_{C12})^2}-\frac{M_{S12}\,(S_2+e)^2\,e}{\hat{m}_{22}+M_{C12}}\\&+\frac{(\hat{m}_{22}-\hat{m}_{11})\,M_{S12}k_1S_1\,(S_2+e)\,e}{(\hat{m}_{11}+M_{C12})\,(\hat{m}_{22}+M_{C12})}\\&+\frac{(\hat{m}_{22}+M_{C12})\,k_1^2S_1e}{\hat{m}_{11}+M_{C12}}\\&+A^{-2}\,[(c_1+c_4)\,M_{C12}+c_4\hat{m}_{11}\\&-(\hat{m}_{11}c_2+\hat{m}_{22}c_3+M_{C12}c_3)]^2+\tau_{d2}^2\\&+\frac{1}{4}\tilde{\tau}_{d1}^2\tilde{\tau}_{d2}^2\end{aligned} \tag{41}$$

Thus, there exists a nonnegative continuous function $\phi_1(\cdot)$ satisfying

$$\begin{aligned}0\le&\left|\frac{1}{4}A^{-2}S_2^2\left[(\hat{m}_{22}+M_{C12})^2+M_{C12}^2\right]\right.\\&-\frac{G_{S2}e+M_{S12}k_1S_1\,[2\,(S_2+e)+k_1S_1]\,e}{\hat{m}_{11}+M_{C12}}\\&+\frac{e^2}{4\,(\hat{m}_{11}+M_{C12})^2}-\frac{M_{S12}\,(S_2+e)^2\,e}{\hat{m}_{22}+M_{C12}}\\&+\frac{(\hat{m}_{22}-\hat{m}_{11})\,M_{S12}k_1S_1\,(S_2+e)\,e}{(\hat{m}_{11}+M_{C12})\,(\hat{m}_{22}+M_{C12})}\\&+\frac{(\hat{m}_{22}+M_{C12})\,k_1^2S_1e}{\hat{m}_{11}+M_{C12}}\\&+A^{-2}\,[(c_1+c_4)\,M_{C12}+c_4\hat{m}_{11}\\&\left.-(\hat{m}_{11}c_2+\hat{m}_{22}c_3+M_{C12}c_3)]^2\right|\\\le&\;\varphi_1\,(k_1,S_1,S_2,c_1,c_2,c_3,c_4,e)\end{aligned} \tag{42}$$

Given any $p>0$, let us introduce a set $\Omega := \left\{S_1^2+S_2^2+e^2\le 2p\right\}$. Apparently set Ω is compact in R^3. Therefore, the continuous function $\phi_1(\cdot)$ has a maximum, say M on Ω. It follows that

$$\begin{aligned}\dot{V}\le&-\left(k_1-\frac{\hat{m}_{11}+\hat{m}_{12}}{\hat{m}_{22}}\right)S_1^2\\&-\left(k_2-\frac{k_1}{2}-\frac{\hat{m}_{11}+\hat{m}_{12}}{2\hat{m}_{22}}\right)S_2^2\\&-\left(\frac{1}{\tau_4}-\frac{3k_1}{2}-\frac{\hat{m}_{11}+\hat{m}_{12}}{2\hat{m}_{22}}\right)e^2\\&-\left(\frac{c_1}{\hat{m}_{11}}-\frac{(\hat{m}_{11}+\hat{m}_{12})\,c_2}{\bar{A}}-1\right)\tilde{\tau}_{d1}^2\\&-\left(\frac{c_4}{\hat{m}_{22}}-\frac{\hat{m}_{12}c_3}{\bar{A}}-1\right)\tilde{\tau}_{d2}^2+\bar{d}_2^2+\frac{1}{4}\xi_1^2\xi_2^2+M\end{aligned} \tag{43}$$

According to Lemma 1, if the following inequalities are satisfied:

$$a_i>0, i=1,2,3,4,5 \tag{44}$$

then we have

$$\dot{V} \leq -2\gamma V + M_1 \quad (45)$$

where

$$M_1 = M + \frac{\xi_1^2 \xi_2^2}{4} + \frac{\bar{d}_2^2}{2\hat{m}_{12}} > 0 \quad (46)$$

$$\gamma = \min(a_1, a_2, a_3, a_4, a_5) > 0 \quad (47)$$

By the selections of k_1, k_2, τ_4 and c_1, c_2, c_3, c_4, we can make $\gamma > M_1/2p$. This results in $\dot{V} \leq 0$ on $V = p$. Thus, $V \leq p$ is an invariant set, i.e., if $V(0) \leq p$ then $V(t) \leq p$ for all $t \geq 0$. Therefore, $V(t)$ is bounded, so are S_1, S_2, e and $\tilde{\tau}_{d1}, \tilde{\tau}_{d2}$. According to above analysis and Lemma 1, the overall closed-loop control system is semi-globally uniformly and ultimately bounded. This completes the proof.

5. SIMULATION STUDY

In order to verify the performance of the proposed controller, we will provide some simulations in this section. In the simulation, the parameters of the MWIP system are given in Table II.

TABLE II. Physical Parameters of MWIP System

Parameter	Value	Parameter	Value
m_w	$29.0[Kg]$	I_w	$0.6[Kg \cdot m^2]$
r	$0.254[m]$	$\hat{m}_b$	$210.6[Kg]$
$\hat{I}_b$	$55.0[Kg \cdot m^2]$	$\hat{l}$	$0.267[m]$
m_b	$310.6[Kg]$	I_b	$65.0[Kg \cdot m^2]$
l	$0.317[m]$	D_b	$0.1[N \cdot s/m]$

*$\hat{m}_b$ is the nominal mass of the body. $\hat{I}_b$ is the nominal moment of inertia of the body, and $\hat{l}$ is the nominal length between the wheel axle and the center of gravity of the body.

As Table II shows, the actual physical parameters of the body are different from those of the nominal system. The dissipation parameter D_w is assumed to be Gaussian random variable with known covariance$(0.2N \cdot s/m)$ and mean value$(0.5N \cdot s/m)$. And the external disturbances are assumed as:

$$\tau_{ext1} = 30\sin(2t + \pi/2)(N \cdot m), \tau_{ext2} = 0.2\sin(t)(N \cdot m).$$

An obvious equilibrium of the MWIP system can be easily obtained:

$$x^* = [x_2{}^*, x_3{}^*, x_4{}^*]^T = [0, 0, 0]^T$$

Based on (35) and Table II, the controller parameters are chosen as

$$k_1 = 6, k_2 = 6, \tau_4 = 0.1, c_1 = c_4 = 100, c_2 = c_3 = 0.$$

Let us consider the equilibrium control effect of the DSC and DSCNDO controller with considering any uncertainties and disturbances. The initial conditions are chosen as $x(0) = [-0.1745, 0, 0]^T$. The simulation result of MWIP system by employing DSC and DSCNDO control strategies is shown in Fig. 3.

As shown in Fig. 3, although the DSC can make the state variables to be semi-globally uniformly and ultimately bounded, it does not guarantee that the states converge to the desired value. There are small oscillations in the response trajectories all the time due to the external disturbances.Furthermore, compare to a pure DSC, the control performance seems better when using a DSCNDO because the amplitudes of oscillations are significantly reduced.

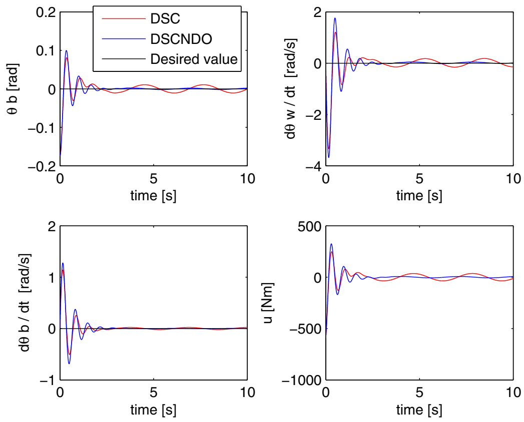

Fig. 3. The simulation results of MWIP system by employing DSC, DSCNDO control strategies$(\theta_b(0) = -10°)$

6. CONCLUSION

An MWIP system belongs to neither the Class-I nor the Class-II underactuated mechanical systems. Application of the DSC to the MWIP-type underactuated mechanical system is difficult because the DSC requires strict feedback form of the dynamic model. In this study, by using necessary coordinate transformation the DSC of the MWIP is achieved. Also, the proposed DSCNDO improves the robustness of the whole system. The effectiveness of proposed methods is verified through simulations.

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