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Nonlinear Robust Control of 3 Phase Inverter with Output LC Filter

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Abstract: Three phase inverters are commonly used to transfer energy from a source to the power grid. The quality of the power delivered to the grid, can be ensured via the use of an output LC filter. However inserting an output filter to an inverter circuitry would introduce new challenges to the controller design due to the additional parametric uncertainties imposed. In this study we present a new model based robust controller for a three phase inverter with output LC filter under the constraint that the output filter parameters are not exactly known. Specifically, *d-q* reference frame model of an inverter with output LC filter is used to develop a nonlinear robust controller that ensures the 3-phase output voltage with desired amplitude and frequency and with lowest harmonic distortion. Stability of the proposed method and the boundedness of the closed–loop system, is established via Lyapunov based tools in conjunction with a robust backstepping procedure. Simulation results are given in order to demonstrate performance and effectiveness of the proposed robust controller.

Keywords: Three phase inverter, output LC filter, model based robust control.

1. INTRODUCTION

In recent years, renewable energy sources are getting more attention Lawrence and Middlekauff [2005]. As a consequence, solar cells and wind tribunes are becoming widely applied in distributed generation systems Eltawil and Zhao [2010]. Generally, electrical power produced by these type of energy sources are stored in chemical (i.e. batteries) or electrical (super capacitors) storage devices and connected to the power grid via a *dc*-to-*ac* inverter system. To connect the inverter output efficiently to the grid, the amplitude and the frequency of the inverter output need to be matched with the grid side Blaabjerg et al. [2006]. An LC low–pass filter is usually connected to the output of the inverter to achieve the lowest Total Harmonic Distortion (THD) Ahmed et al. [2007]. The inverter–filter couple needs to be actively controlled to ensure the lowest THD of output voltage with the desired amplitude and frequency under the different load conditions Kim and Sul [2011].

Power quality is one of the major performance requirement for inverters. Performance of the controller algorithm has direct influence on the power quality (see Prodanovic and Green [2002] and Kovari et al. [2004]). Model–based controllers have been implemented in the literature, to name a few, Kukrer [1996] implement the discrete–time dead–beat control algorithm by using space vector based model of an inverter with output LC filter. In Mattavelli [2005] an improved dead–beat controller with an disturbance observer is proposed and a state-space model of the inverter is used. In Escobar et al. [2007], authors used an adaptive controller based on neural-network identification and deadbeat current regulation. Cortes et al. [2009] proposed a model predictive controller with an observer for the load current estimation. In Kim and Lee [2010], a nonlinear model of an inverter including output LC filter is derived and input-output feedback linearization is applied to the model. In Mu et al. [2011], a nonlinear controller requiring the exact values of the model parameters was proposed. A flatness based control method was proposed in Houari et al. [2012]. Wai et al. [2013] applied a backstepping control design procedure.

In this work, we propose a model based nonlinear robust controller based on the *d-q* reference frame model of an inverter with output LC filter. Despite the lack of exact knowledge of the parameters of the output filter, the proposed robust controller ensures practical 3-phase output voltage tracking to any desired amplitude and frequency. The stability of the overall system is proven via Lyapunov based tools and effectiveness of the proposed method is illustrated through simulation studies.

The rest of the work is organized in the following manner: *d-q* reference frame model of the investigated system is given Section 2. Section 3 presents the objective and details of the design procedure while Section 4 contains the stability analysis. Results of our simulation studies are presented in Section 5 and Section 6 contains some concluding results.

2. INVERTER MODEL

An illustration of the system investigated in this work is presented in Figure 1. The system consists of a three legged full bridged intelligent power module (IPM), a three phase output LC filter and a three phase load. For the sake of robustness, the parameters of the load are assumed to be unknown. In Figure 1, V_{dc} and C_{dc} are the DC link voltage and capacitance, respectively, S_a, S_b, S_c are the switching functions of the gates of IGBTs in the IPM. The gates of the IGBTs are switched by PWM signals according to the control input signal. V_a, V_b, V_c are the output voltages, i_a, i_b, i_c are the output currents of the switching elements. The output voltages and currents of the LC filter are labelled as V_{af}, V_{bf}, V_{cf} and i_{La}, i_{Lb}, i_{Lc}.

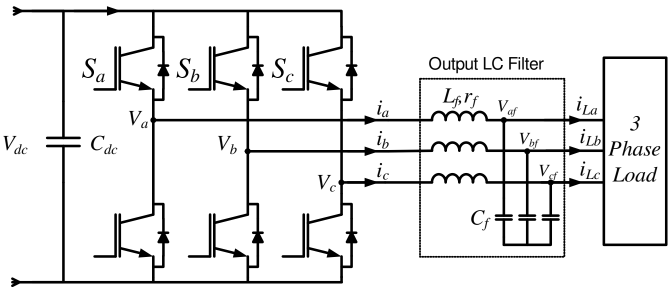

Fig. 1. Inverter with an output LC Filter

As the output of the inverter is periodic, the d–q synchronous reference frame representation of the voltage and the current dynamics of the LC Filter shown in Figure 1 can be written as Houari et al. [2012]

$$C_f \frac{d}{dt} V_{Cd} = C_f \omega V_{Cq} + i_d - i_{Ld} \tag{1}$$

$$C_f \frac{d}{dt} V_{Cq} = -C_f \omega V_{Cd} + i_q - i_{Lq} \tag{2}$$

$$L_f \frac{d}{dt} i_d = -r_f i_d + L_f \omega i_q + V_d - V_{Cd} \tag{3}$$

$$L_f \frac{d}{dt} i_q = -r_f i_q - L_f \omega i_d + V_q - V_{Cq} \tag{4}$$

where $C_f \in \mathbb{R}$ is the filter capacitance, $L_f \in \mathbb{R}$ is the filter inductance, $r_f \in \mathbb{R}$ is the resistance of the filter inductance, and $\omega(t) \in \mathbb{R}$ is the angular frequency. Note that, C_f, L_f, r_f are considered to be uncertain parameters.

After some mathematical manipulations, the d–q frame dynamics in (1)–(4) is rewritten in the following compact form

$$C_f \dot{V}_C = C_f W_{ss} V_C + I - I_L \tag{5}$$

$$L_f \dot{I} = -r_f I + L_f W_{ss} I + U - V_C. \tag{6}$$

In (5) and (6), $V_C(t), I_L(t), I(t), U(t) \in \mathbb{R}^2$ represent the system output, the load current, the filter input current, and the control input, respectively, defined as

$$V_C \triangleq \begin{bmatrix} V_{Cd} \\ V_{Cq} \end{bmatrix}, \; I_L \triangleq \begin{bmatrix} i_{Ld} \\ i_{Lq} \end{bmatrix}, \; I \triangleq \begin{bmatrix} i_d \\ i_q \end{bmatrix}, \; U \triangleq \begin{bmatrix} V_d \\ V_q \end{bmatrix} \tag{7}$$

and $W_{ss}(\omega) \in \mathbb{R}^{2\times 2}$ is an auxiliary skew–symmetric matrix defined as

$$W_{ss} \triangleq \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}. \tag{8}$$

3. PROBLEM STATEMENT AND CONTROLLER FORMULATION

Our aim is to design a controller that ensures the reference input tracking of the output voltage of the inverter with minimum harmonics provided that input current, load current and output voltage of the inverter are measurable, under the restriction that filter parameters are not available. In order to quantify the performance of the controller, we define the output voltage tracking error, $e(t) \in \mathbb{R}^2$, as the difference between the reference output voltage, denoted by $V_{Cref}(t) \in \mathbb{R}^2$, with the actual output voltage, as follows

$$e \triangleq V_{Cref} - V_C. \tag{9}$$

The reference output voltage along with its first and second time derivatives are assumed to be bounded functions of time. Taking the time derivative of (9), multiplying by C_f, inserting the voltage dynamics presented in (5), and finally adding and subtracting $C_f W_{ss} V_{Cref}$, the following open–loop error dynamics is obtained

$$C_f \dot{e} = C_f \dot{V}_{Cref} + C_f W_{ss} e - C_f W_{ss} V_{Cref} - I + I_L. \tag{10}$$

To insert the dynamics of filter input current $I(t)$, we will apply a backstepping procedure by defining an auxiliary control input like term, denoted by $I_d(t) \in \mathbb{R}^2$. Adding and subtracting this term to (10) yields

$$C_f \dot{e} = C_f \dot{V}_{Cref} + C_f W_{ss} e - C_f W_{ss} V_{Cref} + I_L + z - I_d \tag{11}$$

where $z(t) \in \mathbb{R}^2$ is an auxiliary error designed to quantify the mismatch between the filter input current $I(t)$ and the auxiliary term $I_d(t)$ in the sense that

$$z \triangleq I_d - I. \tag{12}$$

Based on the subsequent analysis, the auxiliary control input like term $I_d(t)$ is designed as

$$I_d = K_e e + \hat{C}_f \left(\dot{V}_{Cref} - W_{ss} V_{Cref} \right) + I_L + V_{R1} \tag{13}$$

where $K_e \in \mathbb{R}^{2\times 2}$ is a diagonal, positive definite, constant gain matrix, $\hat{C}_f \in \mathbb{R}$ represents the constant best–guess estimate of C_f, and $V_{R1}(e) \in \mathbb{R}^2$ is a robustifying term designed in the following manner

$$V_{R1} = \frac{\rho_1^2}{\epsilon_1} e \tag{14}$$

where $\epsilon_1 \in \mathbb{R}$ is a positive constant and $\rho_1 \in \mathbb{R}$ is a positive bounding constant that satisfies

$$\rho_1 > \left\| \widetilde{C}_f \left(\dot{V}_{Cref} - W_{ss} V_{Cref} \right) \right\| \tag{15}$$

with $\widetilde{C}_f \triangleq C_f - \hat{C}_f \in \mathbb{R}$ being the constant parameter estimation error. Inserting (13) into the (11), the closed–loop dynamics of the voltage subsystem is obtained as

$$C_f \dot{e} = C_f W_{ss} e - K_e e + \widetilde{C}_f \left(\dot{V}_{Cref} - W_{ss} V_{Cref} \right) + z - \frac{\rho_1^2}{\epsilon_1} e. \tag{16}$$

The backstepping design procedure applied requires the dynamics of the auxiliary error signal $z(t)$ to be investigated. To obtain the dynamics of $z(t)$, we take the time derivative of (12), multiply it by L_f, insert the time derivative of (13) and the current dynamics given by (6), and then add and subtract $L_f W_{ss} I_d$ term, to obtain

$$L_f \dot{z} = Y\varphi + L_f W_{ss} z + V_C - U \quad (17)$$

where $Y(t)$ is a regressor matrix and φ is an unknown constant parameter vector with their multiplication being defined as

$$\begin{aligned} Y\varphi = & r_f I - \frac{L_f}{C_f}(I - I_L) \\ & + L_f \left\{ \left(K_e + \frac{\rho_1^2}{\epsilon_1} \right) \left(\dot{V}_{Cref} - W_{ss} V_C \right) \right. \\ & + \hat{C}_f \left(\ddot{V}_{Cref} - \dot{W}_{ss} V_{Cref} - W_{ss} \dot{V}_{Cref} \right) \\ & \left. + \dot{I}_L - W_{ss} I_d \right\}. \end{aligned} \quad (18)$$

Notice that, to calculate the regressor matrix $Y(t)$ the time derivative of $I_L(t)$ is required. While this may be considered as a drawback of the proposed work, it is remarked that since we design a robust controller numerical derivative of $I_L(t)$ can be utilized in the regressor matrix and the subsequently robustifying term [*i.e.*, (20)] will compensate for this mismatch as well.

Based on the subsequent stability analysis, the control input $U(t)$ is designed to have the following form

$$U = e + K_z z + V_C + Y\hat{\varphi} + V_{R2} \quad (19)$$

where $K_z \in \mathbb{R}^{2\times 2}$ is a constant, diagonal, positive definite gain matrix, $\hat{\varphi}$ is the constant best–guess estimate of the unknown parameter vector φ, and $V_{R2}(\cdot) \in \mathbb{R}^2$ is a robustifying term designed in the following form

$$V_{R2} = \frac{\rho_2^2}{\epsilon_2} z \quad (20)$$

where $\epsilon_2 \in \mathbb{R}$ is a positive constant and $\rho_2(\|e\|, \|z\|) \in \mathbb{R}$ is a positive bounding function which satisfies

$$\rho_2 > \|Y\widetilde{\varphi}\| \quad (21)$$

with the constant parameter estimation error defined as $\widetilde{\varphi} \triangleq \varphi - \hat{\varphi}$. After substituting (19) and (20) into the open–loop error dynamics in (17), the closed–loop dynamics of $z(t)$ is obtained as follows

$$L_f \dot{z} = L_f W_{ss} z + Y\widetilde{\varphi} - e - K_z z - \frac{\rho_2^2}{\epsilon_2} z. \quad (22)$$

4. STABILITY ANALYSIS

At this stage we are ready to state the following Theorem:

Theorem 1. The robust controller of (19) and the auxiliary control input (13) with the robustifying terms (14) and (20) guarantee uniform ultimate boundedness of the inverter output voltage tracking error $e(t)$ in the sense that

$$\|e(t)\| \leq \sqrt{\frac{a}{b}\|x(0)\|^2 \exp(-\beta t) + \frac{2\epsilon}{b\beta}(1 - \exp(-\beta t))} \quad (23)$$

where $x(t) \triangleq \left[e^T \;\; z^T \right]^T \in \mathbb{R}^4$ is the combined error signal, a, b, β, $\epsilon \in \mathbb{R}$ are positive scalars defined explicitly as

$$a \triangleq \max\{C_f, L_f\} \quad (24)$$

$$b \triangleq \min\{C_f, L_f\} \quad (25)$$

$$\beta \triangleq \frac{2\min\{\lambda_{\min}(K_e), \lambda_{\min}(K_z)\}}{\max\{C_f, L_f\}} \quad (26)$$

$$\epsilon \triangleq \frac{\epsilon_1}{4} + \frac{\epsilon_2}{4} \quad (27)$$

where ϵ_1, ϵ_2, K_e and K_z were previously defined, and the notation $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ are used to denote maximum and minimum eigenvalue of a matrix, respectively.

Proof. To prove the theorem, we define a non–negative Lyapunov function, denoted by $V(e, z) \in \mathbb{R}$, in the following manner

$$V \triangleq \frac{C_f}{2} e^T e + \frac{L_f}{2} z^T z \quad (28)$$

which can be bounded from below and above as

$$\frac{1}{2}\min\{C_f, L_f\}\|x\|^2 \leq V \leq \frac{1}{2}\max\{C_f, L_f\}\|x\|^2. \quad (29)$$

The time derivative of the above Lyapunov function is obtained as

$$\dot{V} = C_f e^T \dot{e} + L_f z^T \dot{z} \quad (30)$$

and after substituting the closed–loop dynamics in (16) and (22), simplifying the resulting equation by canceling the common terms, we reach

$$\begin{aligned} \dot{V} = & -e^T K_e e - z^T K_z z + C_f e^T W_{ss} e + L_f z^T W_{ss} z \\ & + e^T \widetilde{C}_f \left(\dot{V}_{Cref} - W_{ss} V_{Cref} \right) - \frac{\rho_1^2}{\epsilon_1}\|e\|^2 \\ & + z^T Y\widetilde{\varphi} - \frac{\rho_2^2}{\epsilon_2}\|z\|^2. \end{aligned} \quad (31)$$

Following expressions can easily be reached as a direct consequence of the skew–symmetric structure of $W_{ss}(\omega)$

$$e^T W_{ss} e = 0 \text{ and } z^T W_{ss} z = 0. \quad (32)$$

Applying the upper bounds given in (15) and (21) into the second and third lines of (31), respectively, and then competing to squares result in

$$\rho_1\|e\| - \frac{\rho_1^2}{\epsilon_1}\|e\|^2 = -\left(\frac{\sqrt{\epsilon_1}}{2} - \frac{1}{\sqrt{\epsilon_1}}\rho_1\|e\| \right)^2 + \frac{\epsilon_1}{4} \leq \frac{\epsilon_1}{4} \quad (33)$$

$$\rho_2\|z\| - \frac{\rho_2^2}{\epsilon_2}\|z\|^2 = -\left(\frac{\sqrt{\epsilon_2}}{2} - \frac{1}{\sqrt{\epsilon_2}}\rho_2\|z\| \right)^2 + \frac{\epsilon_2}{4} \leq \frac{\epsilon_2}{4}. \quad (34)$$

Combining the above derivations allow us to upper bound the right–hand side of (31) as

$$\dot{V} \leq -\min\{\lambda_{\min}(K_e), \lambda_{\min}(K_z)\}\|x\|^2 + \epsilon \quad (35)$$

where ϵ was previously defined in (27). Using the definition of $x(t)$ and the upper bound of $V(t)$ given in (29), the upper bound of $\dot{V}(t)$ given (35) can be reformulated as

$$\dot{V} \leq -\beta V + \epsilon \quad (36)$$

where β was previously defined in (26). The solution of the above differential inequality yields

$$V(t) \leq V(0)\exp(-\beta t) + \frac{\epsilon}{\beta}(1 - \exp(-\beta t)) \quad (37)$$

and from direct application of (29), we can obtain the following upper bound for $x(t)$

$$\|x(t)\| \leq \sqrt{\frac{a}{b}\|x(0)\|^2 \exp(-\beta t) + \frac{2\epsilon}{b\beta}(1 - \exp(-\beta t))} \quad (38)$$

where a and b were previously defined in (24) and (25), respectively. Based on (38) and the definition of $x(t)$, it can be shown that the voltage error term $e(t)$ is bounded as stated in (23). Moreover, applying standard signal chasing argument, we can show that all signals in the closed–loop error system are bounded.

5. SIMULATION RESULTS

In order to illustrate the performance of the purposed controller scheme we have performed simulations using MATLAB Simulink simulation tool. DC link voltage is taken as 620 V and DC link capacitor is assumed to be large enough to keep the DC link voltage constant. Parameters of the low–pass LC filters are taken from Mu et al. [2011] as

$$L_f = 15mH,\ C_f = 22\mu F,\ r_f = 0.2\Omega,\ R_L = 20\Omega \qquad (39)$$

where R_L is a resistive load connected to the output of the filter. The amplitude of the reference phase to phase voltage is set to 380 V and the frequency is 50 Hz. In order to avoid high jumps in the control signal at the beginning, reference signal is designed to be have a soft start. Controller gain matrices are selected as

$$\begin{array}{ll} K_e = diag\{2.5, 1.93\} & K_z = diag\{26, 3.14\} \\ \rho_1 = diag\{0.5, 0.2\} & \rho_2 = diag\{2, 2\} \\ \epsilon_1 = 0.1 & \epsilon_2 = 1. \end{array} \qquad (40)$$

Simulation results are shown in Figures 2-5. Fig. 2 shows the error signals in d-q synchronous reference frame while three phase output voltage errors are represented in Fig. 3. Fig. 4 presents the input voltage of the output LC filter. Finally, output voltage wave forms of the inverter with output LC filter is presented in Fig. 5.

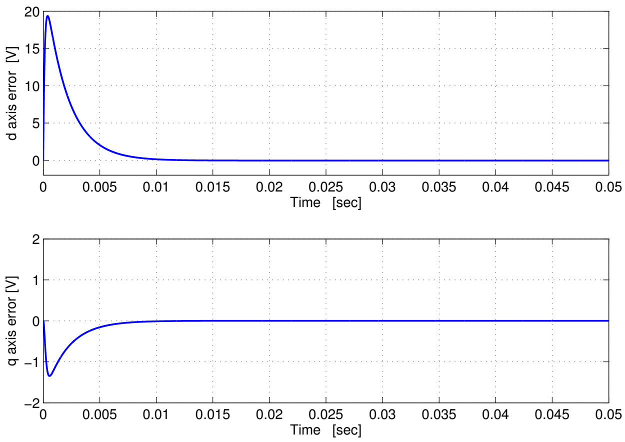

Fig. 2. Error signals in d-q reference frame

6. CONCLUSION

In this study, we present a new full state feedback nonlinear robust controller for inverters with output LC filter with uncertain system parameters. Despite the lack of exact knowledge of the filter parameters, proposed controller ensures the desired output wave form with the desired amplitude and frequency for the stand alone and grid connected applications. Stability of the closed–loop system and boundedness of the signals in the closed–loop system is proven via Lyapunov based tools with the help of backstepping procedure. In order to illustrate the performance of the proposed controller, simulation studies have been performed and results of the simulations have been presented.

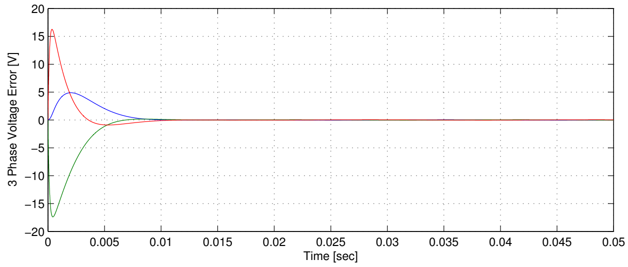

Fig. 3. Error signals in 3 Phase

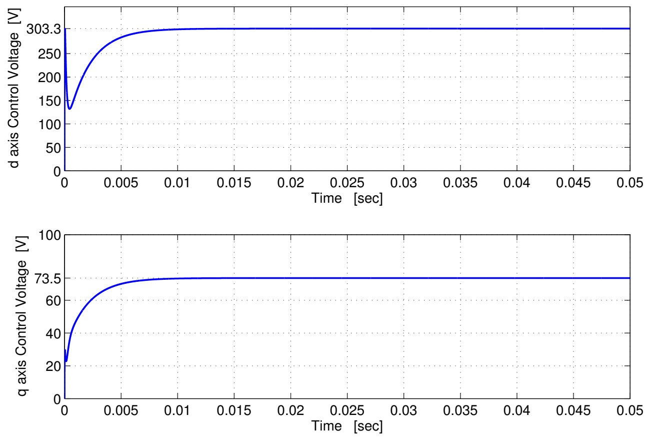

Fig. 4. Control signals in d-q reference frame

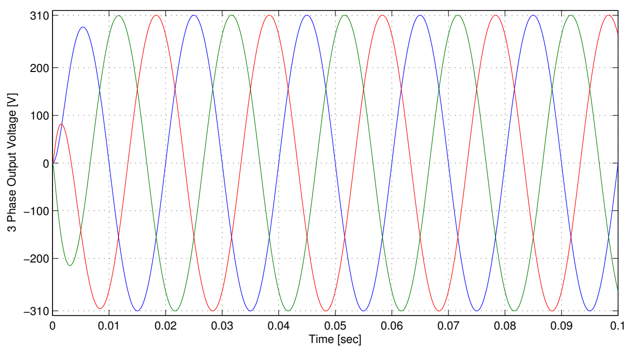

Fig. 5. The output voltage waveform of the inverter

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