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Model Predictive Control Combined with Model Discrimination and Fault Detection ⋆

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Abstract: In this paper, an output feedback controller in the structure of model predictive control is developed focusing on model discrimination and fault detection and isolation in addition to a tracking performance. The development of the controller is based on a system identification method that is used for an update of a model of an underlying system. For the systems whose dynamics may change at an unknown time, this identification method uses only a relatively small amount of data recently collected from the underlying system. The signal generated by the controller is applied to the system and makes the system to produce input-output data such that models are more distinguishable and faults are more detectable. Since the optimization problem in the controller is nonconvex, a suboptimal solution via a semidefinite relaxation technique is pursued.

1. INTRODUCTION

One of the most important benefits of feedback control is that system behaviour is made less sensitive to changes in the plant dynamics. However, in some contexts this is also a problem: it can occur that feedback control successfully hides small changes in the system dynamics until the degradation is so severe that catastrophic failure occurs. In this paper, we consider a control problem with two competing objectives: firstly, to maintain sufficient tracking performance for the primary system task, and secondly, to generate sufficient information about the system dynamics to detect when a small change has occurred. Our design fits within the general structure of model predictive control (MPC) [Kwon and Han, 2005].

System identification at each time step of MPC has been studied in many ways. For example, in Tanaskovic et al. [2013], an adaptive MPC scheme is introduced where systems identification is performed at each time step by a set membership identification algorithm combined with a set of finite impulse response models. For most system identification methods, a persistent excitation of the system is necessary and a persistently exciting MPC (e.g. Shouche et al. [2002], Rathouský and Havlena [2012], and Marafioti et al. [2013]) produces a persistently exciting input signal. Further, an accuracy of system identification is taken into account in Larsson et al. [2013] where an MPC-based controller is augmented by a constraint on the Fisher information matrix.

The contribution in this paper parallels the approach in Larsson et al. [2013] in the sense that an MPC structure is modified for the purpose of producing an input signal maximizing a measure of an accuracy of system identification. However, our primary focus is on model discrimination and fault detection and isolation (FDI) (e.g. Cheong and Manchester [2014]). The underlying system may not be in a set of models but we search for the closest model among a finite number of selective models by developing a controller in the structure of the modified MPC to produce an input signal that guarantees the model discrimination. We investigate some cases where the guaranteed model discrimination leads to a feasibility issue. Then, we introduce alternative types of MPCs that avoid this issue.

Unlike other fault detection and isolation methods surveyed in Hwang et al. [2010], we address FDI problems by shaping an input signal generated by MPC. This shaping causes the optimization problem in the MPC to be in a similar form to the optimization problems in time-domain input signal designs (e.g. Manchester [2010] and Manchester [2012]), which are noncovex but comprised of inhomogeneous quadratic terms. Thus, we perform the homogenization and the semidefinite relaxation (SDR) techniques (e.g. Luo et al. [2007]) to obtain semidefinite programming (SDP) problems and, then, perform a random search procedure based on the solutions to the SDP problems.

In Section 2, we carefully formulate an output feedback MPC combined with a system identification method and a state estimator. Then, in Section 3, a condition for model discrimination is developed and is implemented into the optimization problem in the MPC. In Section 4, optimal solutions, if attainable, or suboptimal solutions to the optimization problems are sought and, in Section 5, an examle of fault detection and isolation is presented. A conclusion follows in Section 6.

The norm $\|\cdot\|_2$ denotes the Euclidean norm of a vector and its corresponding induced norm of a matrix. The norm $\|\cdot\|_\infty$ is the ∞-norm of a vector. The probability of an event and the expectation of a random variable is denoted by $P[\cdot]$ and $E[\cdot]$, respectively. The set of symmetric positive semidefinite matrices in $\mathbb{R}^{i\times i}$ is denoted by S^i_+. A vector

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e_j is the j-th standard basis vector and $\mathbf{1}$ is a vector whose elements are all 1's with an appropriate dimension. A square matrix I_i is an identity matrix in $\mathbb{R}^{i\times i}$ and the subscript i can be omitted when there is no confusion. Denote by $\mathbf{0}$ a zero vector or matrix with an appropriate dimension.

2. A SYSTEM IDENTIFICATION METHOD FOR MODEL PREDICTIVE CONTROL

Consider an uncertain single-input single-output (SISO) system $\mathcal{P}$ whose input signal u and output signal y are observed at time $t = 0, 1, \cdots$. For models of $\mathcal{P}$, we consider discrete-time, causal, linear time-invariant, and SISO models

$$\mathcal{P}_\theta : y_\theta(t) = \mathcal{F}_\theta x_{\theta,0} + \mathcal{G}_\theta u_\theta(t) + \mathcal{H}_\theta d_\theta(t),\ t = 0, 1, \cdots \quad (1)$$

that are parametrized in θ. The initial conditions of each model at time $t = 0$ are represented by $x_{\theta,0}$ and, thus, the operators $\mathcal{G}_\theta$ and $\mathcal{H}_\theta$ have zero initial conditions. The disturbance dynamics $\mathcal{H}_\theta$ is invertible. The signal $d_\theta(t), t = 0, 1, \cdots$, represents an unobserved disturbance signal. We assume that, for different parameters, the models are distinct in terms of the input and the output signals of the models. Note that the models may have different orders, which means that the dimensions of $x_{\theta,0}$'s may be different for different parameters.

Our main interest in this paper lies on a finite number N of selective operating modes of $\mathcal{P}$, which are represented by parameters θ_n, $n = 1, \cdots, N$, and their vicinities defined by parameter sets Θ_n, $n = 1, \cdots, N$, e.g. $\Theta_n = \{\theta \in \Theta | \|\theta - \theta_n\|_2 \le \rho_\theta\}$, $n = 1, \cdots, N$, with a nonnegative constant ρ_θ. Usually, the parameter sets are constructed in a way that each set represents an operating mode of $\mathcal{P}$ and it is important to recognize which operating mode the system $\mathcal{P}$ is currently on, especially for fault detection and isolation.

The dynamics of the system $\mathcal{P}$ is suspected to be slowly time-varying due to aging or to have abrupt changes due to faults. Thus, each model in (1) is fitted to only recently collected input-output data of $\mathcal{P}$, i.e. $\mathbf{u}_b(t) := [u(t - T_b) \cdots u(t - 1)]'$ and $\mathbf{y}_b(t) := [y(t - T_b) \cdots y(t - 1)]'$ at time $t \ge T_b$ with a positive integer T_b. A quality of a model $\mathcal{P}_\theta$ for the fitting at time $t \ge T_b$ is quantified as

$$\begin{aligned} &V_M(\theta, \mathbf{u}_b(t), \mathbf{y}_b(t), t) \\ &= \min_{\tilde{x}_{\theta,0}\in\mathbb{R}^{T_\theta}, \tilde{\mathbf{d}}_\theta\in\mathbb{R}^{T_b}} \|\tilde{\mathbf{d}}_\theta\|_2 \\ &\qquad \text{s.t. } \mathbf{y}_b(t) = F_{b,\theta}\tilde{x}_{\theta,0} + G_{b,\theta}\mathbf{u}_b(t) + H_{b,\theta}\tilde{\mathbf{d}}_\theta \\ &\qquad\qquad \|\tilde{x}_{\theta,0}\|_2 \le \rho_x \end{aligned} \quad (2)$$

with a nonnegative constant ρ_x where $T_\theta = \dim x_{\theta,0}$ and the matrices $F_{b,\theta}$, $G_{b,\theta}$, and $H_{b,\theta}$ represent the model $\mathcal{P}_\theta$. For example, if $(A_\theta, B_\theta, C_\theta, D_\theta)$ is a state-space representation of $\mathcal{G}_\theta$, then we have

$$G_{b,\theta} = \begin{bmatrix} g_{\theta,0} & 0 & \cdots & 0 \\ g_{\theta,1} & g_{\theta,0} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_{\theta,T_b-1} & g_{\theta,T_b-2} & \cdots & g_{\theta,0} \end{bmatrix}$$

with $g_{\theta,0} = D_\theta$ and $g_{\theta,i} = C_\theta {A_\theta}^{i-1} B_\theta$ for $i = 1, 2, \cdots$. The matrices $F_{b,\theta}$ and $H_{b,\theta}$ are defined similarly to G_θ. Note that the matrices $G_{b,\theta}$ and $H_{b,\theta}$ are lower triangular due to the causality of the models and, in particular, the matrix $H_{b,\theta}$ is invertible.

Since $H_{b,\theta}$ is invertible, a solution to the optimization problem in (2) always exists. Denote by $({\tilde{x}_{\theta,0}}^*(t), {\tilde{\mathbf{d}}_\theta}^*(t))$ a solution to the optimization problem in (2). Then, this solution can be interpreted as a fictitious initial state and a fictitious disturbance signal for $\mathcal{P}_\theta$ since if the input signal $\mathbf{u}_b(t)$ is applied to the model $\mathcal{P}_\theta$ combined with an initial condition ${\tilde{x}_{\theta,0}}^*(t)$ and a disturbance signal is given as ${\tilde{\mathbf{d}}_\theta}^*(t)$, then the output signal $\mathbf{y}_b(t)$ is exactly reproduced.

The constant ρ_x in (2) represents a bound of a set of all possible initial conditions of $\mathcal{P}_\theta$ at time $t - T_b$ and is determined based on any prior knowledge of $\mathcal{P}$. Thus, the value $V_M(\theta, \mathbf{u}_b(t), \mathbf{y}_b(t), t)$ in (2) represents the model mismatch of $\mathcal{P}_\theta$ to the collected data $(\mathbf{u}_b(t), \mathbf{y}_b(t))$ at time t. When $t < T_b$, the optimization problem in (2) is modified to only take into account the data from time 0 to $t - 1$ unless the data from time $t - T_b$ to -1 are available.

Then, we pursue a parameter that minimizes the model mismatch so that an estimate of the parameter at time t is given as

$$\hat{\theta}(t) = \arg\min_{\theta\in\Theta} V_M(\theta, \mathbf{u}_b(t), \mathbf{y}_b(t), t) \quad (3)$$

where $\Theta = \cup_{n=1}^{N} \Theta_n$. Then, an estimate $\hat{x}_{\hat{\theta}(t)}(t)$ of the state of $\mathcal{P}_{\hat{\theta}(t)}$ at time t is determined by $\mathcal{P}_{\hat{\theta}(t)}$, $\mathbf{u}_b(t)$, ${\tilde{x}_{\hat{\theta}(t),0}}^*(t)$, and ${\tilde{\mathbf{d}}_\theta}^*(t)$. Note that if we fix ${\tilde{x}_{\theta,0}}^*(t) = \mathbf{0}$ in the optimization problem in (2) and solve it only for ${\tilde{\mathbf{d}}_\theta}^*(t)$, then the solution to the optimization problem in (2) is unique and reduces to ${\tilde{\mathbf{d}}_\theta}^*(t) = {H_{b,\theta}}^{-1}(\mathbf{y}_b(t) - G_{b,\theta}\mathbf{u}_b(t))$ and the fictitious disturbance signal for $\mathcal{P}_\theta$ can be generated as a filtered signal of u and y (Ljung [1999]).

With the estimates $\hat{\theta}(t)$ and $\hat{x}_{\hat{\theta}(t)}(t)$ above, we consider output feedback MPC. Specifically, the input signal of the system $\mathcal{P}$ at time t is given as

$$u(t) = {e_1}'{\mathbf{u}_f}^*(t)\ \forall t \in \{0, 1, \cdots\} \quad (4)$$

where ${\mathbf{u}_f}^*(t)$ is a solution to an optimization problem

$$\min_{\mathbf{u}_f\in\mathbb{R}^{T_f}} \|\mathbf{y}_f - \mathbf{r}_f(t)\|_2^2 + \rho_u\|\Delta\mathbf{u}_f\|_2^2$$
$$\text{s.t. } \mathbf{y}_f = F_{f,\hat{\theta}(t)}\hat{x}_{\hat{\theta}(t)}(t) + G_{f,\hat{\theta}(t)}\mathbf{u}_f$$
$$\underline{y} \le {e_i}'\mathbf{y}_f \le \overline{y}\ \forall i \in \{1, \cdots, T_f\} \quad (5)$$
$$\underline{u} \le {e_i}'\mathbf{u}_f \le \overline{u}\ \forall i \in \{1, \cdots, T_f\} \quad (6)$$
$$|{e_i}'\Delta\mathbf{u}_f| \le \overline{\Delta}_u\ \forall i \in \{1, \cdots, T_f\} \quad (7)$$

with given constants $\underline{y}$, $\overline{y}$, $\underline{u}$, $\overline{u}$, and $\overline{\Delta}_u$. The matrices $F_{f,\hat{\theta}(t)}$ and $G_{f,\hat{\theta}(t)}$ are defined similarly to $F_{b,\hat{\theta}(t)}$ and $G_{b,\hat{\theta}(t)}$ in (2) but possibly with different dimensions. A positive integer T_f represents a prediction horizon and $\mathbf{r}_f \in \mathbb{R}^{T_f}$ and $\Delta\mathbf{u}_f \in \mathbb{R}^{T_f}$ are defined by

$$\mathbf{r}_f(t) = [r(t) \cdots r(t + T_f - 1)]'$$

$$\Delta\mathbf{u}_f = \begin{bmatrix} {e_1}'\mathbf{u}_f - u(t-1) \\ (e_2 - e_1)'\mathbf{u}_f \\ \vdots \\ (e_{T_f} - e_{T_f-1})'\mathbf{u}_f \end{bmatrix} := \Xi\mathbf{u}_f - u(t-1)e_1$$

with a given reference signal r and a given value $u(-1)$. In the optimization problem above, the constraint in (5) is treated as a soft constraint but the constraints in (6) and (7) are treated as hard constraints. Since there is no data up until time $t = 0$, the estimates $\hat{\theta}(0)$ and $\hat{x}_{\hat{\theta}(0)}(0)$ are determined based on any prior knowledge of $\mathcal{P}$ or set to nominal values.

Depending on T_b, i.e. the amount of data used in the identification method in (3), there is a trade-off between sensitivity to a dynamics change and robustness to the disturbance signal. However, finding an appropriate T_b is out of the scope of the paper and we focus on implementation of a model discrimination property in MPC.

3. MODIFICATION OF MODEL PREDICTIVE CONTROL FOR MODEL DISCRIMINATION

In this section, identification of the operating mode of $\mathcal{P}$, which also can be called model discrimination, is performed via applying an appropriate input signal to $\mathcal{P}$. Thus, we modify the controller in (4) to pursue both the tracking performance and the model discrimination at the same time, which is called dual control.

From (2), it follows that $V_M(\theta_n, \mathbf{u}_b(t), \mathbf{y}_b(t), t) = \|\tilde{\mathbf{d}}_{\theta_n}{}^*(t)\|_2$ for any $n \in \{1, \cdots, N\}$. And, we have

$$\tilde{\mathbf{d}}_{\theta_{n_2}}{}^*(t) = \Gamma(\theta_{n_2}, \theta_{n_1})\mathbf{u}_b(t) + H_{b,\theta_{n_2}}{}^{-1} F_{b,\theta_{n_1}} \tilde{x}_{\theta_{n_1},0}{}^*(t) - H_{b,\theta_{n_2}}{}^{-1} F_{b,\theta_{n_2}} \tilde{x}_{\theta_{n_2},0}{}^*(t) + H_{b,\theta_{n_2}}{}^{-1} H_{b,\theta_{n_1}} \tilde{\mathbf{d}}_{\theta_{n_1}}{}^*(t)$$

for any $n_1, n_2 \in \{1, \cdots, N\}$ where

$$\Gamma(\theta_{n_2}, \theta_{n_1}) = H_{b,\theta_{n_2}}{}^{-1} \left(G_{b,\theta_{n_1}} - G_{b,\theta_{n_2}}\right). \tag{8}$$

Then, the lemma below guarantees model discrimination.

Lemma 1. (Cheong and Manchester [2014]) Suppose that $P\left[V_M(\theta_{n_1}, \mathbf{u}_b(t), \mathbf{y}_b(t), t) < \delta\right] \geq \beta$ for the system $\mathcal{P}$ in Section 2 with a parameter θ_{n_1} and constants δ and β. For any given $n_2 \in \{1, \cdots, N\} \setminus \{n_1\}$, if $\|\Gamma(\varphi, \psi)\mathbf{u}_b(t)\|_2 \geq \gamma(\varphi, \psi)$ for ether $(\varphi, \psi) = (\theta_{n_2}, \theta_{n_1})$ or $(\theta_{n_1}, \theta_{n_2})$ where

$$\gamma(\varphi, \psi) = \rho_x \left(\|H_{b,\varphi}{}^{-1} F_{b,\psi}\|_2 + \|H_{b,\varphi}{}^{-1} F_{b,\varphi}\|_2\right) + \left(1 + \|H_{b,\varphi}{}^{-1} H_{b,\psi}\|_2\right)\delta$$

with ρ_x in (2), then $P\left[V_M(\theta_{n_2}, \mathbf{u}_b(t), \mathbf{y}_b(t), t) > \delta\right] \geq \beta$.

Based on this lemma, we design an input signal such that

$$\|\Gamma(\theta_{n_2}, \theta_{n_1})\mathbf{u}_b(t)\|_2 \geq \bar{\gamma}_{n_2,n_1} \tag{9}$$

for all $n_1, n_2 \in \{1, \cdots, N\}$ satisfying $n_1 < n_2$ where $\bar{\gamma}_{n_2,n_1} = \max\{\gamma(\theta_{n_2}, \theta_{n_1}), \gamma(\theta_{n_1}, \theta_{n_2})\}$. Then, Lemma 1 guarantees that the number of models with the cost value of model mismatch less than δ at time t is at most 1 with probability at least $100\beta\%$, provided that the condition is satisfied. The choice of δ relies on any prior knowledge of the system $\mathcal{P}$ and can be supported by placing lots of selective models in Θ.

Note that, for any given unordered pair (n_1, n_2) or any given two models, there is only one condition imposed by (9). Thus, the total number of conditions in (9) is $M = \frac{N(N-1)}{2}$, which is the total number of the unordered pairs of the models $\mathcal{P}_{\theta_n}$'s. Then, using (8), we can rewrite the condition in (9) as

$$V_D(\mathbf{u}_b(t)) := \min_{m \in \{1, \cdots, M\}} \frac{1}{\bar{\gamma}_m{}^2} \left\|\bar{\Gamma}_m \mathbf{u}_b(t)\right\|_2^2 \geq 1 \tag{10}$$

where $\Gamma(\theta_{n_2}, \theta_{n_1})$ and $\bar{\gamma}_{n_2,n_1}$ are represented by $\bar{\Gamma}_m$ and $\bar{\gamma}_m$, respectively. Alternatively, we can consider a weighted average version $V_D(\mathbf{u}_b(t)) := \frac{1}{M}\sum_{m=1}^{M} \frac{w_m}{\bar{\gamma}_m{}^2} \left\|\bar{\Gamma}_m \mathbf{u}_b(t)\right\|_2^2$ where w_m's are the weights. The latter may be appropriate if, based on prior knowledge, certain models are highly likely and should be favored for discrimination.

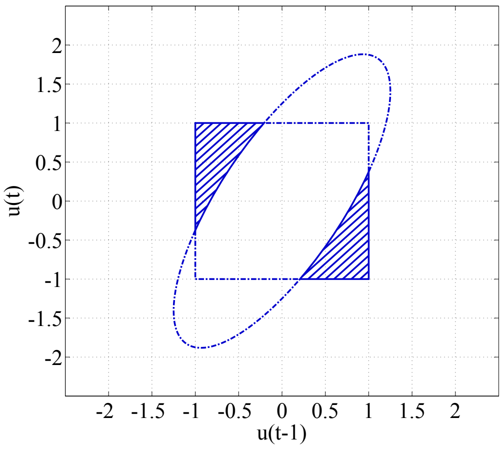

Fig. 1. The feasible values (the hatched areas) of $u(t) = e_1{}'\mathbf{u}_f$ depending on $u(t-1)$ in the first example.

In order to guarantee the condition in (10), the input signal $u(t)$ is generated by the MPC-based controller in (4) with an additional hard constraint

$$V_D(\mathbf{u}_{D,\min\{T_b,t+1\}}) \geq 1 \tag{11}$$

where $\mathbf{u}_{D,i} = \Psi_i \mathbf{u}_f + \psi_i$ for $i = 1, \cdots, T_b$ with $\Psi_1 = [I_{T_b} \;\; \mathbf{0}] \in \mathbb{R}^{T_b \times T_f}$, $\psi_1 = \mathbf{0} \in \mathbb{R}^{T_b}$,

$$\Psi_i = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ I_{T_b-i+1} & \mathbf{0} \end{bmatrix} \in \mathbb{R}^{T_b \times T_f}, \;\; \psi_i = \begin{bmatrix} u(t-i+1) \\ \vdots \\ u(t-1) \\ \mathbf{0} \end{bmatrix} \in \mathbb{R}^{T_b}$$

for $i = 2, \cdots, \min\{T_b, t+1\}$. In other words, we have

$$\mathbf{u}_{D,\min\{T_b,t+1\}} = \begin{cases} \left[e_1{}'\mathbf{u}_f \; \cdots \; e_{T_b-t}{}'\mathbf{u}_f\right]' & \text{for } t = 0 \\ \left[u(0) \; \cdots \; u(t-1) \; e_1{}'\mathbf{u}_f \; \cdots \; e_{T_b-t}{}'\mathbf{u}_f\right]' & \text{for } 0 < t < T_b - 1 \\ \left[u(t-T_b+1) \; \cdots \; u(t-1) \; e_1{}'\mathbf{u}_f\right]' & \text{for } t \geq T_b - 1 \end{cases}.$$

However, the condition in (11) may cause the optimization problem in the MPC to be infeasible. To see this, consider a simple case of the MPC in (4) with the constraint in (11), $T_b = T_f = 2$, $\underline{u} = -1$, and $\overline{u} = 1$ but without the constraints in (5) and (7). Then, at a certain time $t \geq 1$, we have $\mathbf{u}_{D,\min\{T_b,t+1\}} = \left[u(t-1) \;\; e_1{}'\mathbf{u}_f\right]'$. If the constraint in (11) is imposed with $V_D(\mathbf{u}_{D,\min\{T_b,t+1\}}) = \left\|\begin{bmatrix} 0.8 & 0 \\ -0.9 & 0.8 \end{bmatrix} \mathbf{u}_{D,\min\{T_b,t+1\}}\right\|_2^2$, then the feasible values of $e_1{}'\mathbf{u}_f$, i.e. $u(t)$, depending on $u(t-1)$ are described as the hatched areas in Fig. 1. Notice that since $u(t-1)$ is produced by the MPC at time $t-1$, we have $-1 \leq u(t-1) \leq 1$. From the figure, it is clear that if the MPC produces a value $u(t)$ satisfying $|u(t)| < 0.2069$ at time t, the optimization problem in the MPC does not have a feasible solution at time $t+1$.

In order to overcome this feasibility problem, we consider a stronger condition

$$\bar{V}_D(\mathbf{u}_f) := \min_{i \in \{1, \cdots, \min\{T_b,t+1\}\}} V_D(\mathbf{u}_{D,i}) \geq 1, \tag{12}$$

which produces, in the example above, an additional constraint $\left\|\begin{bmatrix} 0.8 & 0 \\ -0.9 & 0.8 \end{bmatrix} \mathbf{u}_f\right\|_2^2 \geq 1$. Thus, the MPC selects a value $u(t) = e_1{}'\mathbf{u}_f$ satisfying $|u(t)| \geq 0.2069$ and has

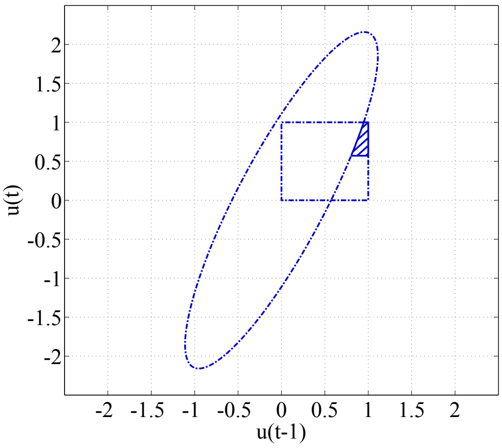

Fig. 2. The feasible values (the hatched area) of $u(t) = e_1{}'\mathbf{u}_f$ depending on $u(t-1)$ in the second example.

a feasible solution at time $t+1$. Nevertheless, even with the condition in (12), the feasibility of the optimization problem in the MPC may not be guaranteed. For example, consider the simple case above except that $\underline{u} = 0$, the constraint in (12), and $V_D(\mathbf{u}_{D,i}) = \left\|\begin{bmatrix} 0.9 & 0 \\ -1.5 & 0.9 \end{bmatrix} \mathbf{u}_{D,i}\right\|_2^2$ for $i = 1, 2$. Then the feasible values of $e_1{}'\mathbf{u}_f$, i.e. $u(t)$, depending on $u(t-1)$ are described as the hatched area in Fig. 2. Note that $u(t) \geq 0.5717$ is imposed at time t because $V_D(\mathbf{u}_{D,1}) \geq 1$. However, if $u(t) = 0.5717$ is chosen by the MPC at time t, it is clear, from the figure, that the MPC does not have a feasible solution at time $t+1$.

Therefore, in order to avoid a potential problem of feasibility, the condition in (12) is implemented as a soft constraint. Alternatively, we may employ another optimization problem

$$\begin{aligned} &\min_{\mathbf{u}_f \in \mathbb{R}^{T_f}} J(\mathbf{u}_f) - \rho_J \bar{V}_D(\mathbf{u}_f) \\ &\text{s.t. } \max\{Z_O(\mathbf{u}_f), Z_I(\mathbf{u}_f), Z_{DI}(\mathbf{u}_f)\} \leq 1 \end{aligned} \tag{13}$$

with a constant ρ_J where $J(\mathbf{u}_f) = \mathbf{u}_f{}' Q_J \mathbf{u}_f + 2 q_J{}' \mathbf{u}_f$, $Z_O(\mathbf{u}_f) = \|Q_O \mathbf{u}_f + q_O\|_\infty$, $Z_I(\mathbf{u}_f) = \|Q_I \mathbf{u}_f + q_I\|_\infty$, $Z_{DI}(\mathbf{u}_f) = \|Q_{DI}\mathbf{u}_f + q_{DI}\|_\infty$, and

$$\bar{V}_D(\mathbf{u}_f) = \min_{i \in \{1,\cdots,T_b\}} \min_{m \in \{1,\cdots,M\}} \frac{1}{\overline{\gamma}_m{}^2} \left\|\overline{\Gamma}_m (\Psi_i \mathbf{u}_f + \psi_i)\right\|_2^2$$

with matrices and vectors

$$\begin{aligned} Q_J &= G_{f,\hat{\theta}(t)}{}' G_{f,\hat{\theta}(t)} + \rho_u \Xi' \Xi \\ q_J &= G_{f,\hat{\theta}(t)}{}' (F_{f,\hat{\theta}(t)} \hat{x}_{\hat{\theta}(t)}(t) - \mathbf{r}_f(t)) - \rho_u u(t-1) \Xi' e_1 \\ Q_O &= \frac{2}{\overline{y} - \underline{y}} G_{f,\hat{\theta}(t)}, \; q_O = \frac{2}{\overline{y} - \underline{y}} F_{f,\hat{\theta}(t)} \hat{x}_{\hat{\theta}(t)}(t) - \frac{\overline{y} + \underline{y}}{\overline{y} - \underline{y}} \mathbf{1} \\ Q_I &= \frac{2}{\overline{u} - \underline{u}} I, \; q_I = -\frac{\overline{u} + \underline{u}}{\overline{u} - \underline{u}} \mathbf{1} \\ Q_{DI} &= \frac{1}{\overline{\Delta}_u} \Xi, \; q_{DI} = -\frac{u(t-1)}{\overline{\Delta}_u} e_1. \end{aligned}$$

The optimization problem in (13) is a quadratic optimization problem with convex quadratic constraints and a nonconvex objective function so that the computation becomes demanding as either of M and T_f increases. Thus, we pursue, in the next section, semidefinite relaxation techniques (e.g. Luo et al. [2007]) to obtain an SDP and, then, perform a random search procedure based on a solution to the SDP problem.

4. COMPUTATIONAL ASPECTS

For a large M or T_f, the optimization problem in (13) may become intractable within a given time. In this case, it may be better to compromise on the optimality and to pursue a suboptimal solution instead.

Following the SDR in Luo et al. [2007], we can obtain, from the optimization problem in (13), an SDP

$$\begin{aligned} &\min_{U \in S_+^{T_f+1}} \hat{J}(U) - \rho_J \hat{V}_D(U) \\ &\text{s.t. } \max\{\hat{Z}_O(U), \hat{Z}_I(U), \hat{Z}_{DI}(U)\} \leq 1 \\ &\quad \operatorname{Tr}\left(e_{T_f+1} e_{T_f+1}{}' U\right) = 1 \end{aligned} \tag{14}$$

with appropriate functions $\hat{J}$, $\hat{V}_D$, $\hat{Z}_O$, $\hat{Z}_I$, and $\hat{Z}_{DI}$, e.g.

$$\begin{aligned} \hat{J}(U) &= \operatorname{Tr}\left(\begin{bmatrix} Q_J & q_J \\ q_J{}' & 0 \end{bmatrix} U\right) \\ \hat{V}_D(U) &= \min_{i \in \{1,\cdots,T_b\}} \min_{m \in \{1,\cdots,M\}} \operatorname{Tr}\left(\frac{1}{\overline{\gamma}_m{}^2} \begin{bmatrix} \Psi_i{}' \\ \psi_i{}' \end{bmatrix} \overline{\Gamma}_m{}' \overline{\Gamma}_m [\Psi_i \; \psi_i] U\right) \\ \hat{Z}_O(U) &= \max_{i \in \{1,\cdots,T_f\}} \operatorname{Tr}\left(\begin{bmatrix} Q_O{}' \\ q_O{}' \end{bmatrix} e_i e_i{}' [Q_O \; q_O] U\right). \end{aligned}$$

Even though the SDP in (14) is different from the optimization problem in (13), an advantage is that the SDP can efficiently be solved using freely available solvers such as Sedumi (Sturm [1999]) and interfaces such as Yalmip (Löfberg [2004]) and CVX (Grant and Boyd [2012]).

If an optimal solution $U^*(t)$ to the SDP in (14) satisfies $\operatorname{rank}(U^*(t)) = 1$, then the solution can be decomposed into $U^*(t) = \left[\mathbf{u}_f{}^*(t)' \; 1\right]' \left[\mathbf{u}_f{}^*(t)' \; 1\right]$ and, then, $\mathbf{u}_f{}^*(t)$ is an optimal solution to the original optimization problem. Otherwise, we employ a randomization scheme in the following for good suboptimal solutions.

Algorithm 1. Denote an optimal solution to the SDP in (14) by $U^*(t) = \begin{bmatrix} \Phi^*\Phi^{*\prime} + \phi^*\phi^{*\prime} & \phi^* \\ \phi^{*\prime} & 1 \end{bmatrix}$.

Step 1 : Generate a realization $\xi \in \mathbb{R}^{T_f}$ of a standard normal distribution.

Step 2 : Search for a constant a^* such that, in the original optimization problem in (13), a vector $a^*\Phi^*\xi + \phi^*$ (i) is a feasible solution and (ii) produces a smaller objective value than $a\Phi^*\xi + \phi^*$ with any other constant a. If such a constant does not exist, go to Step 1.

Step 3 : Update $\hat{\mathbf{u}} = a^*\Phi^*\xi + \phi^*$ if this vector $a^*\Phi^*\xi + \phi^*$ produces the best objective value so far through this algorithm. If the number of the generations of ξ is less than a certain positive number, then go to Step 1. Otherwise, terminate the algorithm with $\mathbf{u}_f{}^*(t) = \hat{\mathbf{u}}$.

5. AN EXAMPLE

A pitch angle y of a blade of a wind turbine is the angle between the rotor plane and the blade chord line and, thus, a pitch angle $y = 0°$ means that the blade is aligned in parallel with the rotor plane. The blade is rotated by a hydraulic system and a popular model of this actuator is a closed-loop transfer function $\frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$ between the pitch angle y and a reference angle u where ζ and ω are the damping ratio and the natural frequency, respectively. See,

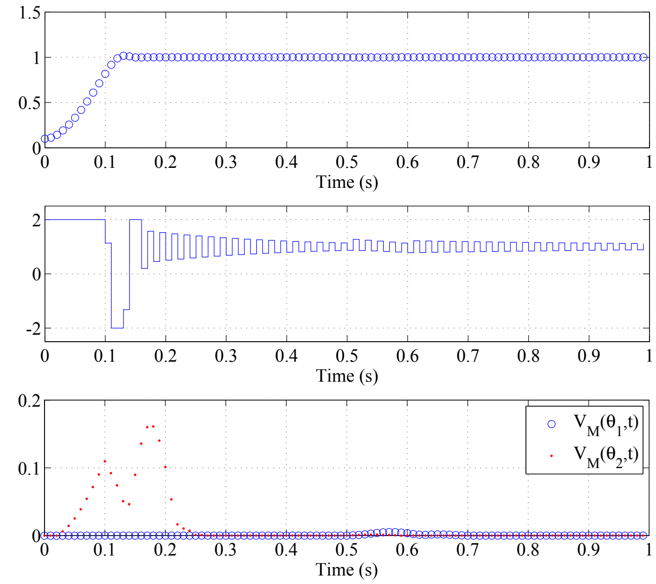

Fig. 3. The result of the MPC without model discrimination. The output signal (Top), the input signal (Middle), and the cost values of the models (Bottom).

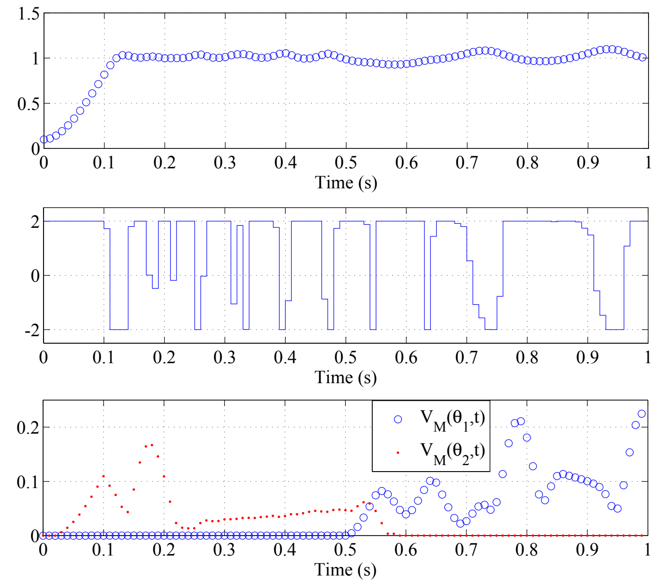

Fig. 4. The result of the MPC with model discrimination. The output signal (Top), the input signal (Middle), and the cost values of the models (Bottom).

for example, Odgaard and Johnson [2013] for the details. In a normal condition, the values are approximately $\zeta_1 = 0.6$ and $\omega_1 = 11.11$.

We consider a fault in the pitch angle control that is caused by an abrupt drop of the hydraulic pressure. In this case, the parameters change to around $\zeta_2 = 0.45$ and $\omega_2 = 5.73$. We discretize these two models of the actuators, i.e. the normal and the fault dynamics, to obtain $\mathcal{G}_{\theta_1}$ and $\mathcal{G}_{\theta_2}$ with $\theta_1 = [\zeta_1\ \omega_1]'$ and $\theta_2 = [\zeta_2\ \omega_2]'$, respectively. The discretization is performed with a sampling time $T_s = 0.01s$ and a zero-order hold.

Based on these models, the system is driven by the MPC with a sampling time $T_s = 0.01s$ and a zero-order hold. In order to complete the model structures as in (1), we use identity operators for both $\mathcal{H}_{\theta_1}$ and $\mathcal{H}_{\theta_2}$ and let θ_1 and θ_2 represent two models $(\mathcal{G}_1, \mathcal{H}_1)$ and $(\mathcal{G}_2, \mathcal{H}_2)$, respectively. The other parameters are set to $T_b = T_f = 10$, $\rho_x = 50$, $\rho_u = 0$, and $r(t) = 1$, $t = 0, 1, \cdots$. For simplicity, the MPC contains only two models, i.e. $\Theta = \{\theta_1, \theta_2\}$, and the amplitude constraint in (6) with $\underline{u} = -2$ and $\overline{u} = 2$.

In the first MATLAB simulation, the system is operated in a normal condition with $\zeta = 0.6$, $\omega = 11.11$, and an initial condition $[0.1\ 0]'$ up to time $0.5s$ and, then, is operated in a fault condition with $\zeta = 0.45$ and $\omega = 5.73$. The input signal of the system is provided by the MPC in (4) and Fig. 3 shows the result. As desired, the output signal y of the system converges to 1 quickly and the amplitude of the input signal u lies between -2 and 2. However, the cost values $V_M(\theta_1, t)$ and $V_M(\theta_2, t)$ stay close after time $0.25s$ so that it is not easy to detect the occurrence of the fault at time $0.5s$.

In the second simulation, in order for fault detection and isolation, the system is driven by the MPC associated with the optimization problem in (13) and $\rho_J = \overline{\gamma}_1{}^2$ with the same other conditions. The result is shown in Fig. 4. Noticeably, the cost values $V_M(\theta_1, t)$ and $V_M(\theta_2, t)$ stay apart so that it is easy to decide that the system is in a normal and a faulty condition before time $0.5s$ and after time $0.6s$, respectively. We achieve this model discrimination in the expenses of output tracking and input power.

In the third and the forth simulations, we repeat the above simulations except that the system is operated with $\zeta = 0.54$ and $\omega = 10$ from time $0s$ to $0.49s$ and these parameters change to and stay at $\zeta = 0.5$ and $\omega = 6.3$ from time $0.50s$. Thus, the true parameters are not in the model set Θ. Fig. 5 and Fig. 6 show the results of the simulations with the MPC in (4) and the MPC associated with the optimization problem in (13), respectively.

Even with the model mismatch, the MPC in (4) shows a good tracking performance in Fig. 5, which indicates a robust property against the model mismatch. And, the MPC associated with the optimization problem in (13) shows an ability of the fault detection and isolation in addition to the robust tracking performance.

6. CONCLUSION

In this paper, we propose a modification of MPC for the purpose of model discrimination and fault detection and isolation based on a small amount of recent data. This modification is developed for the situation where it is not known if and when dynamics of an underlying system changes so that old data are not reliable. Since the amount of data for system identification is small, we employ a modified version of the prediction error method to accommodate the effect of an initial condition. We analyze a potential feasibility problem of the optimization problem in the modified MPC and we propose an alternative optimization problem to circumvent it. And, we provide a simulation example that illustrates the model discrimination performed by the modified MPC.

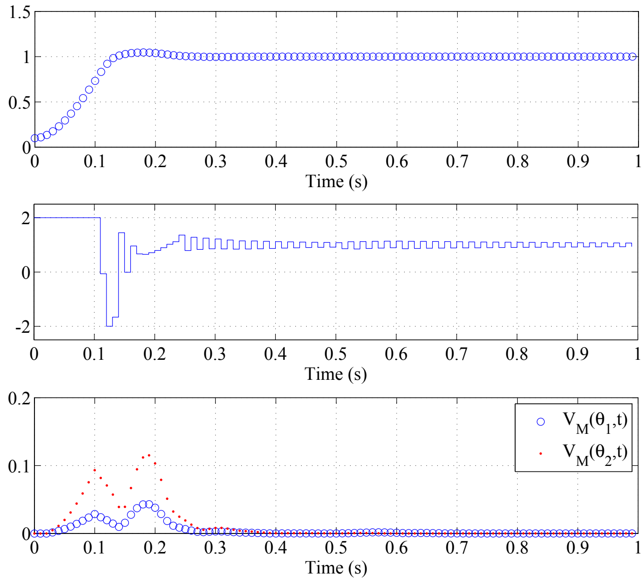

Fig. 5. The result of the MPC with model mismatch and without model discrimination. The output signal (Top), the input signal (Middle), and the cost values of the models (Bottom).

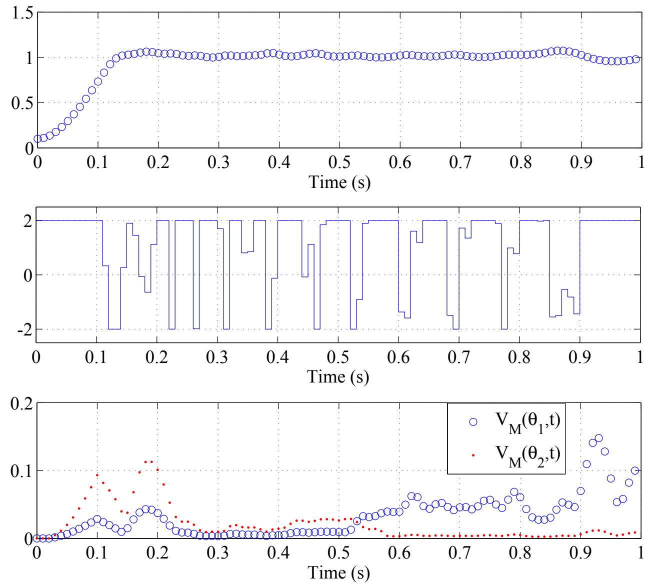

Fig. 6. The result of the MPC with model mismatch and model discrimination. The output signal (Top), the input signal (Middle), and the cost values of the models (Bottom).

The true system may not belong to a set of models but we are interested in the closest model among a finite number of selective models. These selective models represent operating modes of the underlying system so that the MPC uses a model around the closest operating mode. If the closest operating mode represents a fault dynamics, a fault is detected and should be fixed.

The optimization problem in the modified MPC may become intractable as either the number of models or the length of the receding horizon increases so that we apply SDR techniques to obtain an SDP and search for a suboptimal solution via a randomization procedure.

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