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Modified Generalized Explicit Guidance for Midcourse with Near-Zero Lateral Acceleration in Terminal Phase

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Abstract: By appropriately modifying the generalized explicit guidance and developing it further, an optimal midcourse guidance law is proposed in this paper to ensure terminal impact angle constrained engagement of missiles with near-zero lateral acceleration in the terminal phase. This guidance law generates a desired intermediate point in space as well as the desired angles which needs to be achieved at the end of the midcourse. By reaching this intermediate point with the desired intermediate angles, it ensures that the missile will intercept the target with very less (near zero) acceleration demand in terminal phase with desired impact angles. When both the terminal impact angles are specified, the guidance law solves the constraint equations and find out the intermediate point along with the angles, whereas if only one terminal angle is specified (or both are left free), it generates the intermediate point with missing angle (both angles) which can lead to minimum control effort and then solves that problem.

Keywords: Explicit guidance, GENEX guidance, Midcourse guidance, Optimal guidance, Impact angle constrained guidance.

1. INTRODUCTION

Primary objective of Midcourse phase is to guide the missile properly so that it leads to a favourable initial condition for the terminal phase. Numerous midcourse missile guidance schemes have been proposed in the literature with optimal control theory Curtis and Cloutier (1998) for desired orientation of velocity vector (see Kumar and Bhattacharya (2006) for example). These guidance laws can be mainly grouped into two categories. In one approach, fast computational algorithms are employed (with good convergence behavior) and powerful processors are used to numerically solve the nonlinear trajectory optimization problem online. This concept (which is iterative in nature) is still evolving and is not mature enough to draw sufficient confidence to be used in onboard processors. As an alternative, however, closed form guidance laws are derived using 'linearized engagement model'. These are attractive as they are non-iterative and can be implemented with onboard processors easily. Moreover, by repeatedly computing the lateral acceleration command from the guidance law along the flight path, the effect of the associated errors due to the linearization process is minimized and hence the guidance law turns out to be quite useful in practice as well. Such an approach is followed in this paper.

Recently an innovative terminal angle constrained guidance law, called generalized explicit (GENEX) guidance, is proposed by Ohlmeyer and Phillips (2006). Lukacs and Yakimenko (2007) et al. have also solved a similar problem for ballistic missiles during the boost phase. A further development of this guidance law led to another approach, called 'kappa-guidance'Zarchan (2002), that exploits the curvature and torsion along the intercept trajectory, which is parameterized along the arc length or range to the predicted impact point Serakos and Lin (1994). Even though all these guidance laws solve the problem for a desired terminal angle(s), they do not ensure less lateral acceleration in the terminal phase.

In this paper, an optimal midcourse guidance law is proposed by appropriately exploiting the GENEX guidance (Ohlmeyer and Phillips (2006), Lukacs and Yakimenko (2007)) and developing it further. It not only ensures desired terminal impact angles at the end of the engagement, but also ensures near-zero lateral acceleration in the terminal phase. A key feature of this algorithm is that it generates a desired intermediate point in space to which the vehicle should be guided at the end of the midcourse phase along with the associated desired angles which needs to be achieved at that point. The guidance law is also generic in the sense that when both the terminal impact angles are specified, it solves that problem. However, if only one terminal angle is specified or both are left free, it generates the missing angle(s) which can lead to minimum control effort and then solves that problem. Note that closed form solutions have been obtained for all these cases. Effectiveness of this guidance has also been shown by considering different desired final conditions. Comparison with GENEX guidance has also been carried out to demonstrate the additional advantages one can get by using the proposed modified and extended GENEX guidance law.

2. MODIFIED GENEX GUIDANCE WITH ZERO ACCELERATION IN TERMINAL PHASE

In this section, first the GENEX guidance Ohlmeyer and Phillips (2006), Lukacs and Yakimenko (2007) is summarized in brief. After that, the modified GENEX guidance (MGG) has been discussed to ensure zero acceleration demand in the terminal phase.

2.1 GENEX Guidance (GG) and its Limitation

It is essential to define the coordinate frame in which all the variables and equations are to be derived. In figure-(1) the Earth fixed Vertical-East-North(VEN) frame is shown, which is used to derive the equations of motion. The flight path angle (γ) and heading angle (ψ) are defined as the missile velocity vector orientation in the local VEN frame. The proposed guidance law in this paper deals with achieving the desired terminal angles (γ_f) and (ψ_f).

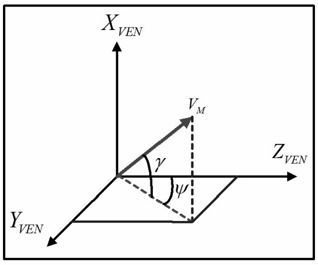

Fig. 1. Local Vertical East North(VEN) frame

GENEX guidance minimizes the control effort to reach final position with desired velocity. For this, the following state equation and boundary conditions are considered

$$\dot{X} = AX + Bu,\ X(t_0) = X_0,\ X(t_f) = X_f \tag{1}$$

where X is the state, u is the scaler control. The system of equation (1) is assumed to be fully controllable, with the control u unbounded.

The following cost function is minimized subject to (1).

$$J = \int_{T_0}^{0} \frac{u^2}{2T^n} dT \tag{2}$$

where $T = t_f - t$ is *time-to-go* and n is an integer ≥ 0. T_0 represents the *time-to-go* at the start of the guidance phase.

As a solution to this problem, the optimal control u^* is obtained as

$$u^* = -(MB)^T Q^{-1} M X T^n \tag{3}$$

where $M(T)$ is the fundamental matrix defined as

$$\frac{dM}{dT} = MA,\quad M(t_0) = I \tag{4}$$

and $Q(T)$ is defined as

$$Q(T) = \int_{0}^{T} (MB)(MB)^T T^n dT \tag{5}$$

The above result can be used for specification on final velocity vector. Let ZEM be zero effort miss with respect to a specified final position and let ZEM_V be zero effort velocity miss (difference between current velocity and final desired velocity). Define the state as

$$x_1 = ZEM = y_f - y_m - \dot{y}_m T \tag{6}$$

$$x_2 = ZEM_V = \dot{y}_f - \dot{y}_m \tag{7}$$

where y_f and y_m are the final desired position and current position of interceptor. And $\dot{y}_f$ and $\dot{y}_m$ are the final desired velocity and current velocity of interceptor. The dynamics of these states can be obtained as

$$\dot{x}_1 = \frac{d}{dt}(ZEM) = -uT \tag{8}$$

$$\dot{x}_2 = \frac{d}{dt}(ZEM_V) = -u \tag{9}$$

where u is the acceleration command of intercepter. The state equation can be written as

$$\dot{X} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} -T \\ -1 \end{bmatrix} u \tag{10}$$

For this system, u^* is obtained as

$$u^* = \frac{1}{T^2} \left[k_1(y_f - y_m - \dot{y}_m T) + k_2(\dot{y}_f - \dot{y}_m)T \right] \tag{11}$$

where,

$$k_1 = (n+3)(n+2), k_2 = (n+1)(n+2) \tag{12}$$

This u^* can be rewritten as

$$u^* = \frac{1}{T^2} \left[k_1 \cdot ZEM + k_2 \cdot ZEM_V \cdot T \right] \tag{13}$$

This result can be extended to three dimensional case, where ZEM and ZEM_V are defined as

$$ZEM = \begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} - \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \left[\begin{bmatrix} \dot{x}_f \\ \dot{y}_f \\ \dot{z}_f \end{bmatrix} - \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \right] t_{go} \tag{14}$$

and

$$ZEM_V = \begin{bmatrix} \dot{x}_f \\ \dot{y}_f \\ \dot{z}_f \end{bmatrix} - \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \tag{15}$$

For a three-dimensional case, the final desired velocity components in the inertial frame can be defined as

$$\begin{bmatrix} \dot{x}_f \\ \dot{y}_f \\ \dot{z}_f \end{bmatrix} = \begin{bmatrix} V_f \sin\gamma_f \\ V_f \cos\gamma_f \sin\psi_f \\ V_f \cos\gamma_f \cos\psi_f \end{bmatrix} \tag{16}$$

where $(\dot{x}_f, \dot{y}_f, \dot{z}_f)$ are the desired final velocity components in the inertial frame. V_f is the final velocity and γ_f and ψ_f are the final desired flight path angles. Here (x_f, y_f, z_f) are the desired final position in inertial frame and can be obtained by propagating the target position at time

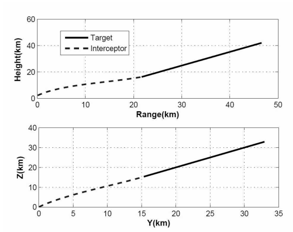

Fig. 2. Trajectory of missile and target

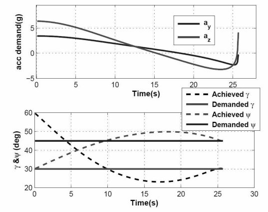

Fig. 3. Latax demand, demanded and achieved γ and ψ

of interception. This is also called as predictive intercept point (PIP).

Representative results of GENEX guidance are shown in Figures 2 and 3. It can be observed here that with this guidance very less miss has been achieved with desired γ and ψ. However the acceleration demand is very high in homing phase (range to go less than 10 km), which is undesirable.

2.2 Modified GENEX Guidance (MGG)

The key idea here is to obtain a meaningful 'intermediate point' in space (which can be considered as the end of the midcourse phase), such that once interceptor reaches intermediate point with some desired velocity vector angles, it will then fly towards the target only under the influence of gravity and drag for rest of the trajectory without the need of any additional lateral acceleration. This in fact is possible in a various ways as depicted in Figure 4 for the pitch plane. Different intermediate points with different values of flight path angles can satisfy the requirement of minimum miss distance with zero latax demand in the terminal phase.

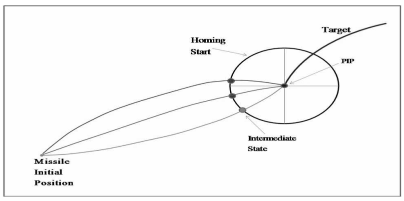

Fig. 4. Possible engagement scenarios in pitch plane

Multiple solutions are possible if either or both of the flight path angle and the heading angle at the time of interception are not specified. In such a scenario the optimal trajectory is chosen by solving an optimal control problem. The proposed guidance methodology finds out the minimum energy trajectory out of all possible trajectories. Otherwise if the terminal angles are fixed, it will find the unique intermediate point, which will satisfy the terminal conditions by solving the constraint equations.

So the problem boils down to obtain the set of values for $(x_i, y_i, z_i, \gamma_i, \psi_i)$ optimally, such that once the interceptor reaches the intermediate point (x_i, y_i, z_i) with the desired angles (γ_i, ψ_i), it will reach the final PIP point (x_f, y_f, z_f) with desired angle (γ_f, ψ_f) without any further lateral acceleration demand. From (11), u^* can be rewritten as

$$u^* = \frac{1}{T^2}\left[a + bT\right] \tag{17}$$

where

$$a = k_1(y_i - y_m); b = -k_1\dot{y}_m + k_2(\dot{y}_i - \dot{y}_m) \tag{18}$$

The optimal cost function can be calculated as

$$\begin{aligned} J^* &= \int_{T_0}^{0} \frac{u^{*2}}{2T^n} dT \\ &= \int_{T_0}^{0} \frac{(a+bT)^2}{2T^{n+4}} dT \\ &= \frac{1}{2}\left[a^2 \frac{T_0^{-(n+3)}}{n+3} + b^2 \frac{T_0^{-(n+1)}}{n+1} + 2ab\frac{T_0^{-(n+2)}}{n+2}\right] \end{aligned} \tag{19}$$

For the three dimensional case, cost function can be defined as

$$\begin{aligned} J^* &= J_x^* + J_y^* + J_z^* \\ &= \frac{1}{2}\begin{bmatrix} (a_x^2 + a_y^2 + a_z^2)\dfrac{T_0^{-(n+3)}}{n+3} \\ +(b_x^2 + b_y^2 + b_z^2)\dfrac{T_0^{-(n+1)}}{n+1} \\ +2(a_x b_x + a_y b_y + a_z b_z)\dfrac{T_0^{-(n+2)}}{n+2} \end{bmatrix} \end{aligned} \tag{20}$$

$$\begin{aligned} a_x &= k_1(x_i - x_m) \\ a_y &= k_1(y_i - y_m) \\ a_z &= k_1(z_i - z_m) \\ b_x &= -k_1\dot{x}_m + k_2(V_i \sin\gamma_i - \dot{x}_m) \\ b_y &= -k_1\dot{y}_m + k_2(V_i \cos\gamma_i \sin\psi_i - \dot{y}_m) \\ b_z &= -k_1\dot{z}_m + k_2(V_i \cos\gamma_i \cos\psi_i - \dot{z}_m) \end{aligned} \tag{21}$$

where (x_m, y_m, z_m) are the current positions of the interceptor and $(\dot{x}_m, \dot{y}_m, \dot{z}_m)$ are the current velocities in inertial frame. So from above equations it is clear that optimal cost function J^* is a function of $(x_i, y_i, z_i, V_i, \gamma_i, \psi_i)$ and can be written as

$$J^* = f(x_i, y_i, z_i, V_i, \gamma_i, \psi_i) \tag{22}$$

The problem can be redefined as to find out the optimal values for $(x_i, y_i, z_i, V_i, \gamma_i, \psi_i)$, which minimize J^*. Here it can be observed that J^* (which is a scalar) is function of six variables. Hence, apart from this optimization, one can bring in additional constraint equations as well. This is exactly what has done in the paper to meet the objective of near-zero lateral acceleration in the terminal phase, which is discussed next.

2.3 Constraint Equations for Modified GENEX Guidance

As discussed above, from intermediate point to final point, interceptor has to reach without any demand under gravity and drag effect. As drag direction is always in the opposite to velocity, it will not effect the flight path angles. Apart from this the drag effect can always be neglected in terminal phase because its duration is small and the engagement is in higher altitude regime. So for the constraint equations, only gravity has been used as a external force. The problem can be written as follows. Interceptor has to reach from intermediate states $(x_i, y_i, z_i, V_i, \gamma_i, \psi_i)$ to the final states $(x_f, y_f, z_f, V_f, \gamma_f, \psi_f)$ under gravity without any lateral acceleration demand. The first constraint comes from the fact that the distance between the intermediate point and the final point is same as the homing distance (R).

$$(x_f - x_i)^2 + (y_f - y_i)^2 + (z_f - z_i)^2 = R^2 \tag{23}$$

From Figure-1 it is clear that gravity is acting along negative X-direction. Hence

$$\begin{aligned} \dot{V}_X &= -g \\ \dot{V}_Y &= 0 \\ \dot{V}_Z &= 0 \end{aligned} \tag{24}$$

Integration of these equations yield

$$\begin{aligned} V_{X_f} - V_{X_i} &= -g\ (t_f - t_i) \\ V_{Y_f} - V_{Y_i} &= 0 \\ V_{Z_f} - V_{Z_i} &= 0 \end{aligned} \tag{25}$$

Here the subscripts i and f represents the intermediate and final states. The resulting equation in X direction is

$$V_f \sin(\gamma_f) = V_i \sin(\gamma_i) - g\ T \tag{26}$$

and in Y and Z direction

$$\begin{aligned} V_f \cos(\gamma_f)\sin(\psi_f) &= V_i \cos(\gamma_i)\sin(\psi_i) \\ V_f \cos(\gamma_f)\cos(\psi_f) &= V_i \cos(\gamma_i)\cos(\psi_i) \end{aligned} \tag{27}$$

where T is time-to-go. By dividing the equations in (27),

$$\begin{aligned} tan(\psi_f) &= tan(\psi_i) \\ \psi_f &= \psi_i + k\pi,\ k = 0, 1, 2, ... \end{aligned} \tag{28}$$

if k is chosen to be 0

$$\psi_i = \psi_f \tag{29}$$

The above equality is valid when the missile ψ does not change by more than 180^0 in the homing phase. This is true as the homing duration is small and gravity does not affect the heading angle. Using equation(29) in (27)

$$V_i = \frac{V_f \cos(\gamma_f)}{\cos(\gamma_i)} \tag{30}$$

And time (T) can be obtained as

$$T = \frac{y_f - y_i}{V_i \cos(\gamma_i)\sin(\psi_i)} = \frac{z_f - z_i}{V_i \cos(\gamma_i)\cos(\psi_i)} \tag{31}$$

It can be further solved to obtain as

$$y_f - y_i = (z_f - z_i)\tan\psi_i \tag{32}$$

by putting the value of T in equation(26), we obtain

$$V_f \sin(\gamma_f) = V_i \sin(\gamma_i) - g\ \frac{y_f - y_i}{V_i \cos(\gamma_i)\sin(\psi_i)} \tag{33}$$

One more constraint equation can be obtained using the motion equation $(v^2 = {v_0}^2 + 2ah)$

$$(V_f \sin(\gamma_f))^2 = (V_i \sin(\gamma_i))^2 - 2g\ (x_f - x_i) \tag{34}$$

This completes the derivation of five constraint equations (23,29,30,33,34) associated with problem.

From the above equations the value of $(x_i, y_i, z_i, V_i, \psi_i)$ can be obtained in terms of $(x_f, y_f, z_f, V_f, \psi_f, \gamma_f, \gamma_i)$. By replacing this term in cost function, the final cost function can be obtained as a function of $(x_f, y_f, z_f, V_f, \psi_f, \gamma_f, \gamma_i)$. As (x_f, y_f, z_f) are fixed and there is no control on V_f, so final cost function can be obtained as a function of $(\psi_f, \gamma_f, \gamma_i)$ as

$$J^* = f(\psi_f, \gamma_f, \gamma_i) \tag{35}$$

and final constrained equation can be obtained by putting the value of $(x_i, y_i, z_i, V_i, \psi_i)$ in equation(23), as a function of $(\psi_f, \gamma_f, \gamma_i)$ as

$$\begin{aligned} G(\psi_f, \gamma_f, \gamma_i) &= 0.25(V_f^2 \cos^2\gamma_f \tan^2\gamma_i - V_f^2 \sin^2\gamma_f)^2 \\ &+ (V_f \tan\gamma_i \cos\gamma_f - V_f \sin\gamma_f)^2 V_f^2 \cos^2\gamma_f - R^2 g^2 \end{aligned} \tag{36}$$

The cost function J^* has to be minimized with respect to $(\psi_f, \gamma_f, \gamma_i)$ with the constrained equation given in (36). There exists four possibilities based on the (ψ_f, γ_f) condition as discussed below.

(1) **Case: 1** Both (ψ_f, γ_f) are fixed Here,there is no scope of optimization. Constraint equation (36) has to be solved to obtain the value of γ_i.

(2) **Case: 2** Both (ψ_f, γ_f) are free For this case, augmented cost function can be written as

$$\bar{J} = J^* + \lambda G \tag{37}$$

Apply optimal condition as

$$\frac{\partial \bar{J}}{\partial \gamma_f} = \frac{\partial J^*}{\partial \gamma_f} + \lambda \frac{\partial G}{\partial \gamma_f} = 0 \tag{38}$$

$$\frac{\partial \bar{J}}{\partial \psi_f} = \frac{\partial J^*}{\partial \psi_f} + \lambda \frac{\partial G}{\partial \psi_f} = 0 \tag{39}$$

$$\frac{\partial \bar{J}}{\partial \gamma_i} = \frac{\partial J^*}{\partial \gamma_i} + \lambda \frac{\partial G}{\partial \gamma_i} = 0 \tag{40}$$

$$\frac{\partial \bar{J}}{\partial \lambda} = G = 0 \tag{41}$$

These four system of nonlinear equations can be solved using numerical methods to get the optimal value of $(\gamma_f, \gamma_i$ and $\psi_f)$.

(3) **Case: 3** ψ_f is free and γ_f is fixed
As γ_f is fixed, optimization has to be done with respect to (ψ_f, γ_i). Equations (39, 40, 41) need to be solved here.

(4) **Case:4** ψ_f is fixed and γ_f is free
As ψ_f is fixed, optimization has to be done with respect to (γ_f, γ_i). The augmented cost function can be defined and subsequently be solved to find out the optimal values of flight path angles. Equations (38, 40, 41) need to be solved here.

Results of all four cases have been discussed in detail in the following section. Generalized nature of this guidance has also been shown by different initial condition and different final condition of missile. Comparison with GENEX guidance has also been done.

3. SIMULATION RESULTS

To show the performance of proposed guidance, the point mass model for both missile and target has been considered.

3.1 Point Mass Model of Interceptor and Target

In this section, the point mass model of missile and target used for simulation, have been given as

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \dot{V} \\ \dot{\gamma} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} V \sin \gamma \\ V \cos \gamma \sin \psi \\ V \cos \gamma \cos \psi \\ \dfrac{T - 0.5 \rho V^2 S C_D}{m} - g sin \gamma \\ \dfrac{-a_z - g cos \gamma}{V} \\ \dfrac{a_y}{V} \end{bmatrix} \tag{42}$$

where X, Y and Z are the positions in the launcher fixed inertial frame. V is the velocity and ψ and γ are the flight path angles. a_y and a_z are the commanded acceleration of the missile and for target the value is set to zero.

Here, the objective is to intercept the target with specified terminal angle constraint. All four possibilities are discussed here. For all these four cases, the intermediate position vector has been calculated to meet the objective of the proposed guidance.

3.2 Case 1: Both angles in terminal constraint are fixed

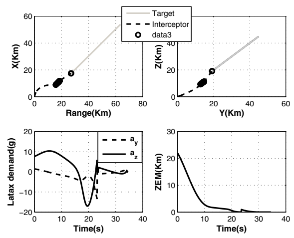

Fig. 5. Case 1: Engagement scenario, acceleration demand and ZEM

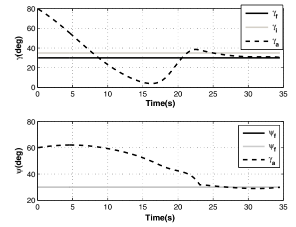

Fig. 6. Case 1: Demanded and achieved flight path angles

In this case, simulation has been done with both γ_f and ψ_f as fixed. Desired angles $\gamma_f = 30^0$ and $\psi_f = 30^0$ are supplied to the optimal control formulation. As discussed above, here there is no scope of optimality. (36) has to be solved to find out the value of γ_i. Here the value of γ_i has come as 35^0. For these value of angles, the intermediate virtual target (x_i, y_i, z_i) has been calculated by using the constraint equation as discussed above.

Figure 5 shows the engagement scenario, demanded acceleration and ZEM. It can be observed here that desired performance has been achieved here with very less demand in terminal phase. Figure 6 shows the demanded and achieved flight path angles.

3.3 Case 2: Both angles in terminal constraint are free

In this case, simulation has been done with desired angle γ_f and ψ_f as free. The optimal value of (γ_f, γ_i and ψ_f) have come as ($6.0^0, 12.5^0$ and 45^0) after solving the four equations as given in (38-41). For these value of angles, the intermediate virtual target (x_i, y_i, z_i) has been calculated by using the constraint equation as discussed above. The interceptor has to reach these intermediate point with intermediate angle (γ_i, ψ_i) before the start of homing phase.

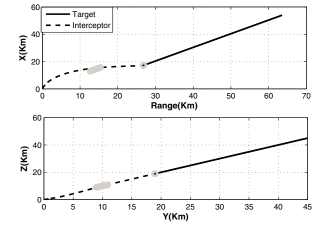

Fig. 7. Case 2: Engagement scenario

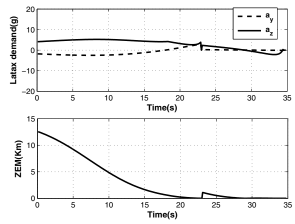

Fig. 8. Case 2: Latax demand and ZEM

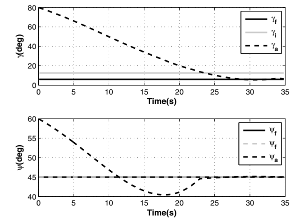

Fig. 9. Case 2: Demanded and achieved flight path angles

The figures show that the missile is achieving the intermediate conditions perfectly, and subsequently the guidance demand is very less in the homing phase and the terminal angles are satisfied at the interception point as well as final intercept point.

In Figure 7, the blue dots represents the intermediate target positions. Till homing, interceptor is try to reach the intermediate point with intermediate angles. Once the missile reaches the intermediate position, target position changes to the actual PIP, which is dynamically calculated. It can be observed here that the acceleration demand is very less in terminal phase. Some small acceleration has been observed in terminal phase. It is because of continuous update of PIP and drag effect of interceptor. However it is much less than GENEX guidance (comparison has been done in later section). The same figure also shows the zero effort miss (ZEM), which shows that very less miss distance has been achieved.

Demanded latax and the ZEM profiles are shown in Figure 8. Figure 9 shows the intermediate and final optimal flight path angles $(\gamma_f, \gamma_i, \psi_f, \psi_i)$. The same plot also shows the achieved flight path angles (γ_a, ψ_a).

In the azimuth plane as expected, ψ_f of 45^0 has come, which is correspond to shortest path between missile and target. It is also observed that the value of cost function achieved here is 1.54×10^7. To show that any other terminal condition will demand more cost function, many other runs have been taken with different terminal conditions and results have been tabulated in Table 1. It can be observed here that cost function is minimum for an optimal case.

3.4 Case 3: Azimuth flight angle is free and elevation flight path angle is fixed

In this case, simulation has been done with desired angle γ_f as fixed and ψ_f as free. Desired angles $\gamma_f = 30^0$ is

Table 1. Comparison with other terminal condition with constraint free flight path angles

Sl No	Case	γ_f(deg)	ψ_f(deg)	Cost function
1	Optimal value	6.0	45.0	1.54×10^7
2	γ_f less and ψ_f more than optimal	0.0	50.0	1.64×10^7
3	γ_f more and ψ_f less than optimal	10.0	40.0	1.71×10^7

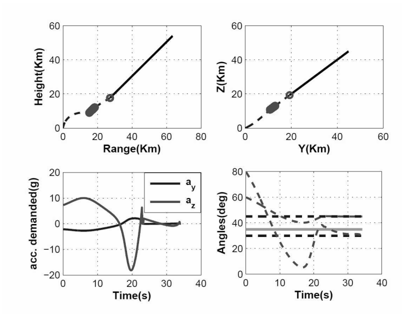

Fig. 10. Case 3: Engagement scenario, Lateral acceleration demand and Angles

supplied to the optimal control formulation. The optimal value of (γ_i and ψ_f) have come as (34.9^0 and 45^0), after solving the three equations mentioned earlier.

Figure 10 shows the engagement scenario, demanded acceleration and ZEM. It can be observed here that desired performance has been achieved here with very less demand in terminal phase. This figure also shows the demanded and achieved flight path angles.

3.5 *Generalized nature of proposed guidance*

Figure 11 and 12 shows that with this guidance, the same final condition can be achieved with very less demand in terminal phase even with different initial condition.

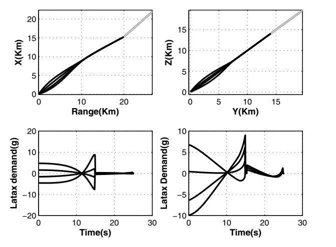

Fig. 11. Engagement scenario, Latax demand and ZEM for different initial condition

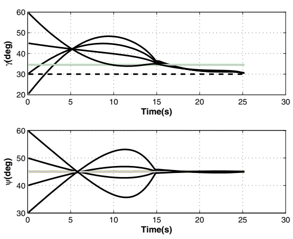

Fig. 12. Flight path angles for different initial condition

3.6 *Comparison between proposed guidance with GENEX Guidance*

In this section, a comparison study of modified GENEX guidance (MGG) has been done with GENEX guidance (GG).

It is evident form the figure 13 and 14 that the proposed guidance successfully engages the target, by driving the missile towards the target through an intermediate point,

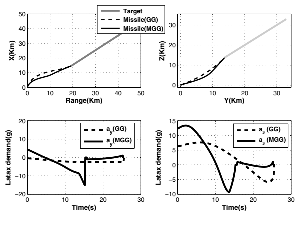

Fig. 13. Comparison of engagement scenario, Lateral acceleration demand

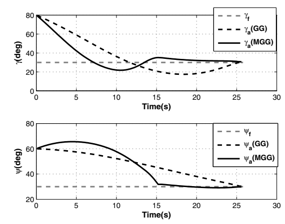

Fig. 14. Comparison of demanded and achieved flight path angles

after which a very small guidance demand is enough for interception.

4. CONCLUSION

By appropriately modifying the GENEX guidance, an optimal midcourse guidance law is proposed in this paper which ensures the desired terminal performance, including impact angle constraints, with very less lateral acceleration demand in the terminal phase. The guidance law is also generic in the sense that when both the terminal impact angles are specified, it solves that problem. However, if only one terminal angle is specified or both are left free, it generates the missing angle(s) which can lead to minimum control effort and then solves that problem. A key feature of this guidance law is to generate an intermediate point in the space to which the vehicle must be guided at the end of the midcourse. It also generates the necessary angles at that point so that the vehicle can be flown with near-zero lateral acceleration in the terminal phase and still achieve the desired objective.

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