

Implicit Regulator Calculation for Regular MIMO-Systems with Predictive Functional Control Demonstrated at a Three Tank System

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Abstract: The concept of predictive functional control is a simplified method for model based predictive control, which allows an implicit regulator calculation in special cases. Mostly, the concept is used for SISO-systems or MIMO-systems with two input and two output variables. In this paper, the theory will be extended for regular MIMO-systems. The concept is demonstrated by a simulation case study of a three tank system, whereby the functionality of the controller and a suggested anti-wind-up method is analyzed.

1 INTRODUCTION

A typical task of control engineering is to control systems with multiple inputs and multiple outputs (MIMO). Therefore various control methods are established, whereby special issues for MIMO-control occur, which may easily be handled in the control of systems with single inputs and single outputs (SISO). For instance intuitive parameterization of the controller and avoidance of wind-up effects may be named.

In conventional MIMO-control theory, the design of diagonal controllers is very popular (see Zacher and Reuter 2009) like shown in fig. 1. Diagonal controllers compensate the influence of the proper process controllers. This approach works well for low-order systems; in high-order systems, the controller structure is not very obvious and the controller parameterization is not intuitive. Furthermore the integration of anti-wind-up strategies has to be done very carefully and at least in each diagonal controller. Another conventional method for the control of MIMO-systems is the design of state-space controllers (see Lunze, 2012) as shown in fig. 2. This concept is very often applied and anti-wind-up strategies are also suggested in several papers. A weakness of this concept is that the stationary exactness of the controlled variables is not given by designing only the state-space regulator. Also a pre-filter has to be designed to minimize the control error in the stationary case; if there is an error in the process model the pre-filter without any dynamical terms will not lead to stationary exactness of the controlled variables. Hence, an additional controller has to be designed (like the PI-controller in fig. 2).

The approach of predictive functional control (PFC) is a simplified method of model based predictive control, in which it is possible to calculate the regulator implicit in some cases. As shown in (Richalet and O'Donovan, 2009 or Luft, 2009), PFC becomes more and more accepted in industrial applications. Initially the PFC was designed for SISO-control or MIMO-control with two inputs and two outputs.

In this paper we present an extended approach of PFC, which allows the control of MIMO-systems that have the same number of inputs than process states; we name these systems regular MIMO-systems in the following. The aim of this paper is not to contest existing concepts for MIMO-control or compare the dynamical behavior of controlled MIMO-systems with the suggested approach; we just want to demonstrate, that it is very easy and intuitive to configure a PFC controller for regular MIMO-systems considering an anti-wind-up method.

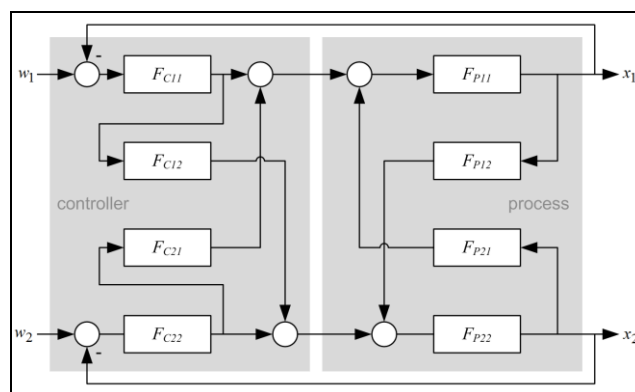


Fig. 1. Control of a MIMO-process with diagonal controllers (F_{C12} , F_{C21}) and process controllers (F_{C11} , F_{C22}).

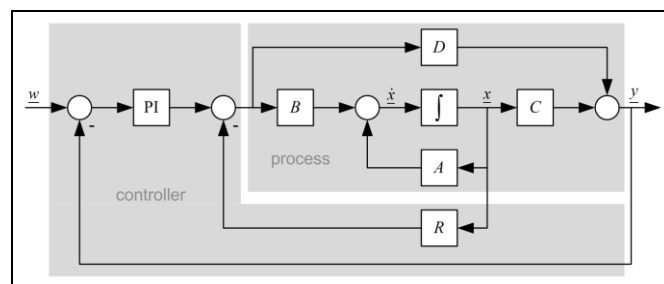


Fig. 2. Control of a MIMO-process with state-space controller and a preceding PI-controller.

2 PREDICTIVE FUNCTIONAL CONTROL FOR REGULAR MIMO-PLANTS

Model based predictive control (MPC) is a very common method of advanced control strategies. The principle of MPC can be explained as follows: at first, the future process behavior without control is predicted based on a process model, which is initialized by measurements or observations of process states. The control sequence gets optimized in such a way, that an objective function of the future control deviation (difference between set point and controlled process variable) will be minimized. To calculate a control sequence for the control of complex processes (maybe with constrained process values or nonlinear processes) typically a solver with an (maybe iterative) optimization method is used (see Adamy, 2009 and Dittmar 2004). In case of dealing with simple problems, like processes that can be described by a linear state-space system without any constraints, it is possible to calculate an analytic solution. This method of predictive functional control (PFC) for SISO-systems was presented in (Richalet, 1978), even for simple processes with time delay and constraints in the manipulated variable.

Particularly the benefit of this concept is, that instead of solving a complex problem by an iterative optimization, simplified solving methods or even an analytical solution can be calculated, which allows also an easy implementation in practice. This may be the reason, why PFC is getting more and more accepted and applied in industrial applications (see Richalet and O'Donovan, 2009 or Luft, 2009). The PFC is described by two main characteristics to simplify the optimization problem, which are coincidence points and basis functions (Valencia-Palomo and Rossiter, 2012). The task is not to minimize a function of the control error in the complete prediction horizon (like the mean squared error) in this concept; only the control error at a coincidence point has to be minimized, so that the objective function gets simplified. Furthermore, basis functions of the manipulated variable are used for possible control actions in the prediction horizon, so that the solution space gets reduced by computing only a reduced sequence of manipulation variable.

We focus on a process, which may be formulated as a linear differential equation system with a state vector $\underline{x}(t)$ and the vector of the manipulated variables $\underline{u}(t)$ the systems matrix A_C and the input matrix B_C :

$$\dot{\underline{x}}(t) = A_C \cdot \underline{x}(t) + B_C \cdot \underline{u}(t) \quad (2.1)$$

To transform the continuous linear differential equation system into a discrete-time difference equation system with the matrices A_D and B_D

$$\underline{x}(k) = A_D \cdot \underline{x}(k-1) + B_D \cdot \underline{u}(k-1) \quad (2.2)$$

we apply:

$$\begin{aligned} A_D &= e^{A_C \cdot \Delta T} \\ B_D &= A_C^{-1} \cdot (e^{A_C \cdot \Delta T} - I) \cdot B_C \end{aligned} \quad (2.3)$$

The future process states will also be influenced by a disturbance on the process output:

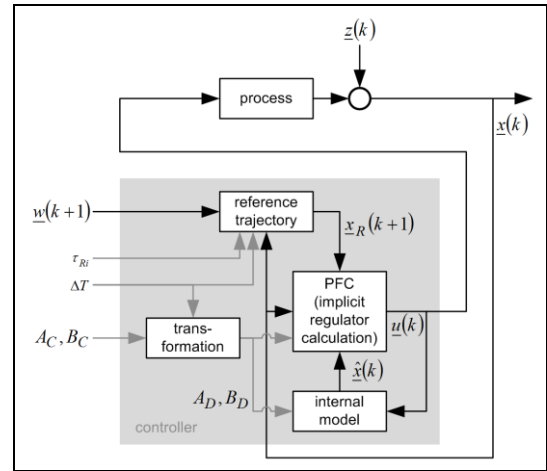


Fig. 3. Controller structure with calculation of the reference trajectory (2.5), the transformation of the systems equations (2.3), the internal model (2.2) and the control law (2.8).

$$\underline{x}(k+1) = A_D \cdot \underline{x}(k) + B_D \cdot \underline{u}(k) + \underline{z}(k+1) \quad (2.4)$$

The future process state has not to be predicted; for control, the future process states should be reached. To reduce the variations in manipulated variables, a dynamic behavior of the process states should be defined. Therefore we introduce a reference trajectory with the time constant τ_{Ri} for each process state, which describes, in which way the control error between the set point w_i and the current process state x_i has to be minimized in future:

$$x_{Ri}(k+1) = e^{-\frac{\Delta T}{\tau_{Ri}}} \cdot x_i(k) + \left(1 - e^{-\frac{\Delta T}{\tau_{Ri}}}\right) \cdot w_i(k+1) \quad (2.5)$$

Hence, a vector of the expected values of the process states in the next sample is:

$$\underline{x}_R(k+1) = [x_{R1}(k+1) \quad x_{R2}(k+1) \quad \dots \quad x_{Rn}(k+1)]^T \quad (2.6)$$

In the simplified case, we assume that the disturbance of the next sample will be the same as for the current one. The current disturbance is estimated by the difference of the process states and the output of an internal process model:

$$\underline{z}(k+1) \approx \underline{z}(k) = \underline{x}(k) - \hat{\underline{x}}(k) \quad (2.7)$$

Finally the values of the manipulated variables are calculable, if the inverse of B_D exists; therefore we use equation (2.7) in (2.4) and solve $u(k)$:

$$\underline{u}(k) = [B_D]^{-1} \cdot [\underline{x}_R(k+1) + \hat{\underline{x}}(k) - (A_D + I) \cdot \underline{x}(k)] \quad (2.8)$$

In doing so, the values of the controlled variables may be implicit calculable, if the observability and the controllability of the system is given, the number of the process states is equal to the number of the manipulated variables and the manipulated variables are not constrained. The structure of the controller is shown in fig. 3.

Constraints in the manipulated variables lead to suboptimal solutions of the closed loop behavior. For handling these

constraints, we have to distinguish two demands. On the one hand, the constraints should be considered in the regulator calculation to get an optimal solution for the control task. On the other hand, wind-up effects in the controller should be avoided.

If the constraints have to be considered explicit in the regulator calculation, an objective function has to be defined and the resulting optimization problem has to be solved with an optimization solver. The focus of this paper lies on an implicit regulator calculation, so that this task will not be considered here. Hence, we just consider the anti-wind-up criterion and accept suboptimal solutions for control tasks for constrained manipulated variables.

Richalet suggest avoiding wind-up effects in predictive functional control by using the constrained manipulated variable instead of the prior calculated manipulated variable (shown in fig. 4) (Richalet, 2009). In the following simulation case study, we demonstrate the functionality of the implicit regulator calculation by predictive functional control for MIMO-systems. Furthermore we analyze the effect of the anti-wind-up method described above.

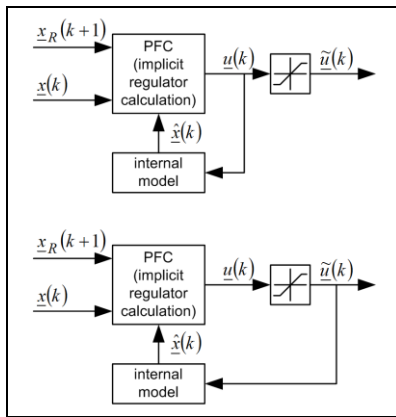


Fig. 4. Neglecting constraints (above) and simplified anti-wind-up method in predictive functional control (below).

3 PROCESS OF A THREE TANK SYSTEM

In the simulation case study, we will focus on a version of a three tank system (fig. 5), which is a common benchmark example. The floor area for each tank i is defined by A_{Fi} . The task is to control the level h_i in each tank by adjusting

- the water supply Q_{IN} to tank 1,
- the cross sectional area A_{12} in the connection pipe from tank 1 to tank 2 (via valve position) and
- the cross sectional area A_{23} in the connection pipe from tank 2 to tank 3 (via valve position).

Every tank level may be disturbed by a leakage of A_{Zi} . In this section we will describe at first the physical process model of the system. After that, the process model will be linearized for using it in the controller as described above.

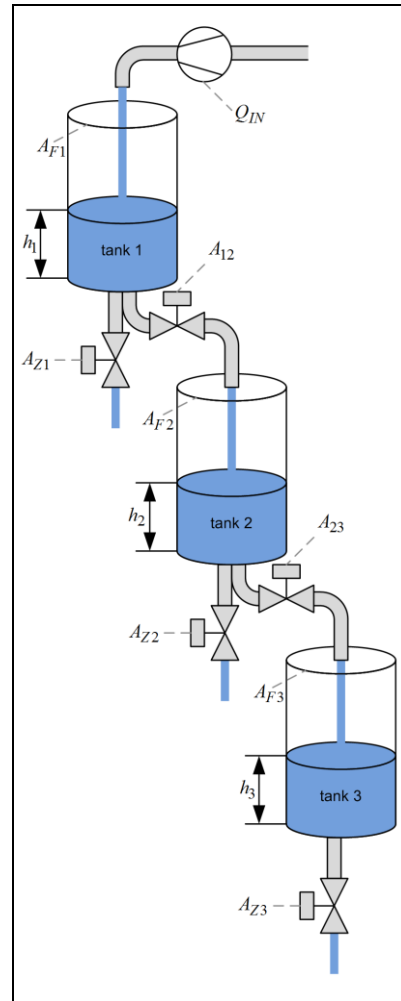


Fig. 5. Schematic illustration of the process with tank floor areas A_{Fi} , the controlled variables h_i , the disturbances A_{Zi} the manipulated variables Q_{IN} , A_{12} and A_{23} .

3.1 Process Model

The change of volume V inside a tank may be calculated by the balance of mass flows into Q_{in} and out Q_{out} of a tank:

$$V(t) = A_F \cdot h(t) = \int (\sum Q_{in}(t) - Q_{out}(t)) \cdot dt + V \quad (3.1)$$

Hence, the derivative of the tank level \dot{h} is:

$$\frac{dh(t)}{dt} = \dot{h}(t) = \frac{1}{A_F} \sum Q_{in}(t) - Q_{out}(t) \quad (3.2)$$

The complete model of the three tank system is defined as

$$\begin{aligned} \dot{h}_1(t) &= \frac{Q_{IN}(t) - Q_{12}(t) - Q_{Z1}(t)}{A_{F1}} \\ \dot{h}_2(t) &= \frac{Q_{12}(t) - Q_{23}(t) - Q_{Z2}(t)}{A_{F2}} \\ \dot{h}_3(t) &= \frac{Q_{23}(t) - Q_{Z3}(t)}{A_{F3}} \end{aligned} \quad (3.3)$$

Whereby the mass flow rate Q_{Zi} is the disturbance and the mass flow Q_{ij} rate is the mass flow from tank i to tank j . These mass flow rates are defined by Torricelli's law with the gravity g :

$$\begin{aligned} Q_{Zi}(t) &= A_{Zi} \cdot \sqrt{2 \cdot g \cdot h_i(t)} \\ Q_{ij}(t) &= A_{ij} \cdot \sqrt{2 \cdot g \cdot h_i(t)} \end{aligned} \quad (3.4)$$

Applying (3.4) in equations (3.3) will result in the following systems model, which is used as process model in simulation:

$$\begin{aligned} \dot{h}_1(t) &= \frac{Q_{IN}(t) - A_{12} \cdot \sqrt{2 \cdot g \cdot h_1(t)} - A_{Z1} \cdot \sqrt{2 \cdot g \cdot h_1(t)}}{A_{F1}} \\ \dot{h}_2(t) &= \frac{A_{12} \cdot \sqrt{2 \cdot g \cdot h_1(t)} - (A_{23} + A_{Z2}) \cdot \sqrt{2 \cdot g \cdot h_2(t)}}{A_{F2}} \\ \dot{h}_3(t) &= \frac{A_{23} \cdot \sqrt{2 \cdot g \cdot h_2(t)} - A_{Z3} \cdot \sqrt{2 \cdot g \cdot h_3(t)}}{A_{F3}} \end{aligned} \quad (3.5)$$

3.2 Stationary Values of the manipulated variables

For given initialization states of the tank levels h_{i0} and the disturbances A_{Zi0} , we have to calculate the initialization values of the manipulated variables Q_{IN0} , A_{120} and A_{230} for stationary initialization ($\dot{h}_i = 0$). These will be:

$$\begin{aligned} \begin{bmatrix} Q_{IN0} \\ A_{120} \\ A_{230} \end{bmatrix} &= \\ \begin{bmatrix} 1 & -\sqrt{2 \cdot g \cdot h_{10}} & 0 \\ 0 & \sqrt{2 \cdot g \cdot h_{10}} & -\sqrt{2 \cdot g \cdot h_{20}} \\ 0 & 0 & \sqrt{2 \cdot g \cdot h_{20}} \end{bmatrix}^{-1} \cdot \begin{bmatrix} A_{Z10} \cdot \sqrt{2 \cdot g \cdot h_{10}} \\ A_{Z20} \cdot \sqrt{2 \cdot g \cdot h_{20}} \\ A_{Z30} \cdot \sqrt{2 \cdot g \cdot h_{30}} \end{bmatrix} \end{aligned} \quad (3.6)$$

3.3 Linearization of the process model

In order to linearize the nonlinear systems equations, we use the Taylor Series expansion of functions. Therefore we define operating points and the variation around these manipulation variables and the process states:

$$\begin{aligned} Q_{IN}(t) &= Q_{IN0} + \Delta Q_{IN}(t) \\ A_{12}(t) &= A_{120} + \Delta A_{12}(t) \\ A_{23}(t) &= A_{230} + \Delta A_{23}(t) \\ h_i(t) &= h_{i0} + \Delta h_i(t) \\ \dot{h}_i(t) &= \dot{h}_{i0} + \Delta \dot{h}_i(t) \end{aligned} \quad (3.7)$$

Hence, the linearized model in state-space notation is

$$\begin{bmatrix} \Delta \dot{h}_1(t) \\ \Delta \dot{h}_2(t) \\ \Delta \dot{h}_3(t) \end{bmatrix} = A_C \cdot \begin{bmatrix} \Delta h_1(t) \\ \Delta h_2(t) \\ \Delta h_3(t) \end{bmatrix} + B_C \cdot \begin{bmatrix} \Delta Q_{IN}(t) \\ \Delta A_{12}(t) \\ \Delta A_{23}(t) \end{bmatrix} \quad (3.8)$$

For $A_{Z10} = A_{Z20} = 0$ and $A_{Z30} \neq 0$ the system matrices are:

$$\begin{aligned} A_C &= \begin{bmatrix} \frac{-A_{120}}{A_{F1}} \cdot \sqrt{\frac{g}{2 \cdot h_{10}}} & 0 & 0 \\ \frac{A_{120}}{A_{F2}} \cdot \sqrt{\frac{g}{2 \cdot h_{10}}} & \frac{-A_{230}}{A_{F2}} \cdot \sqrt{\frac{g}{2 \cdot h_{20}}} & 0 \\ 0 & \frac{A_{230}}{A_{F3}} \cdot \sqrt{\frac{g}{2 \cdot h_{20}}} & \frac{-A_{Z30}}{A_{F3}} \cdot \sqrt{\frac{g}{2 \cdot h_{30}}} \end{bmatrix} \\ B_C &= \begin{bmatrix} \frac{1}{A_{F1}} & \frac{-\sqrt{2 \cdot g \cdot h_{10}}}{A_{F1}} & 0 \\ 0 & \frac{\sqrt{2 \cdot g \cdot h_{10}}}{A_{F2}} & \frac{-\sqrt{2 \cdot g \cdot h_{20}}}{A_{F2}} \\ 0 & 0 & \frac{\sqrt{2 \cdot g \cdot h_{20}}}{A_{F3}} \end{bmatrix} \end{aligned} \quad (3.9)$$

A discrete formulation of the system is given by:

$$\begin{bmatrix} \Delta h_1(k+1) \\ \Delta h_2(k+1) \\ \Delta h_3(k+1) \end{bmatrix} = A_D \cdot \begin{bmatrix} \Delta h_1(k) \\ \Delta h_2(k) \\ \Delta h_3(k) \end{bmatrix} + B_D \cdot \begin{bmatrix} \Delta Q_{IN}(k) \\ \Delta A_{12}(k) \\ \Delta A_{23}(k) \end{bmatrix} \quad (3.10)$$

To transform the linear differential equation system into a discrete difference equation system, we apply equation (2.3).

4 SIMULATION CASE STUDY

For the case study a simulation scenario is defined. Therefore the parameters for configuration and initialization are defined as shown in table 1. At first, we define a step for the set points for each tank level simultaneously. Then we admit various disturbances as shown below in fig. 6. And finally we define new set points for each tank level again. To simplify the configuration of the controllers, all reference trajectories are defined with the same time constant. Initially we want to compare the systems reaction with SISO-controllers, whereby the coupling effects are neglected. The controller architecture is shown in fig. 7.

4.1 Simulation of an unconstrained process

Initially, we simulate unconstrained manipulated variables. Fig. 9 illustrates the simulation results. It is obvious, that the controlled variables following the desired behavior better by using the MIMO-controller as using the SISO-controller. The coupling effects of the manipulation variables and process states are considered in the MIMO-controller, what is neglected in the SISO-controllers. Hence, it is easy to indicate, that the consideration of the coupling effects works in the suggested MIMO-controller. Furthermore the dynamic behavior during changing set points and compensate disturbances of all tank levels is similar, as desired with the equal time constant for all reference trajectories.

4.2 Simulation of a constrained process

In the simulation of the control with constrained manipulated variables, we focus four control approaches:

- SISO-controllers without considering constraints
- SISO-controllers considering constraints
- MIMO-controller without considering constraints
- MIMO-controller considering the constraints

The constraints are listed in table 2. Fig. 10 shows the tank levels in the simulations scenario for the various controllers. At first, it is not easy to evaluate the work of each controller. Hence, we compare the simulations results by the sum of the control errors:

$$e(t) = \sum_{i=1}^3 h_{i,W}(t) - h_{i,X}(t) \quad (4.1)$$

and also by the cumulative sum of the absolute control error

$$\tilde{e}(t) = \sum_t \left| \sum_{i=1}^3 h_{i,W}(t) - h_{i,X}(t) \right| \quad (4.2)$$

The results are shown in fig. 8. Especially by looking at the cumulative sum of the absolute control error, it is obvious, that the wind-up effects are avoided in the suggested way. The results of the MIMO-controller neglecting coupling effects are already better than the SISO-controllers considering them. Finally it may be noted, that the anti-wind-up method for the MIMO-controller works in the desired way and improves the dynamic behavior of controlled variables.

5 CONCLUSIONS

We presented an approach for the implicit regulator calculation for MIMO-systems applying model predictive control. The functionality of the suggested methods was demonstrated at a three tank system with coupled process states. A weakness of this concept is, that constraints may not be considered explicit in the regulator calculation (but this is also not possible in conventional approaches without using any solvers). Furthermore the approach is just useable in processes where the number of the controlled process states is equal to the number of manipulated variables. Nevertheless, the proposed approach seems to be a very transparent and easy method to design a MIMO-controller. Hence we close the paper with a statement of Richalet: to predictive functional control: "it is easy to understand, to implement, to tune".

Table 1. Model parameters

	tank 1	tank 2	tank 3
A_{Fi}	1m	2m	1m
A_{Zi}	0m ²	0m ²	0.1m ²
h_{i0}	0.5m	0.5m	0.5m
u_0	$Q_{IN0}=312\text{l/sec}$	$A_{120}=0.1\text{m}^2$	$A_{230}=0.1\text{m}^2$

Table 2. Constraints of the manipulated variables

	tank 1	tank 2	tank 3
A_{Fi}	412l/sec	∞	∞
h_{i0}	0	0.075m ²	0.075m ²

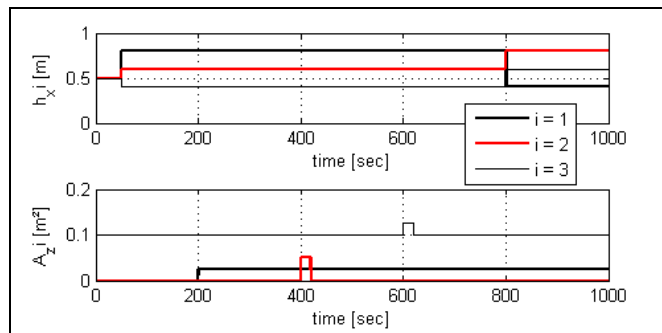


Fig. 6. Defined simulation scenario with set points (above) and disturbances (below).

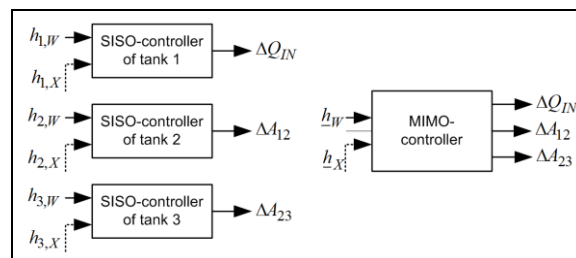


Fig. 7. Structure of control approach with SISO-controllers (right) and the MIMO-controller (left).

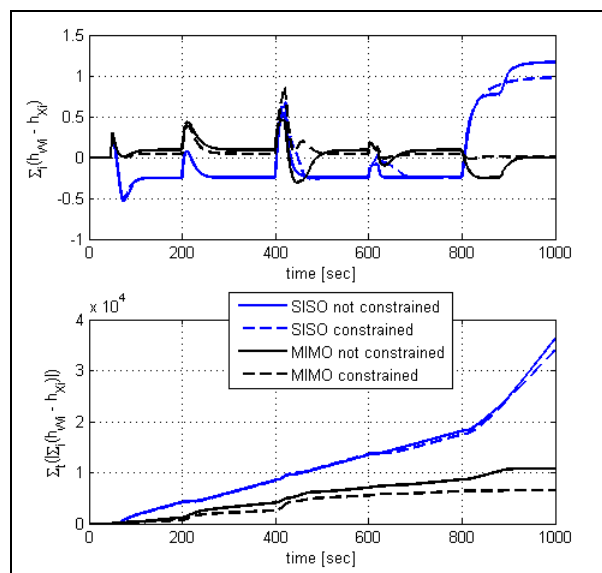


Fig. 8. Comparing the simulations with constrained manipulated variables, sum of control error (above) and cumulative control errors (below) of all tanks.

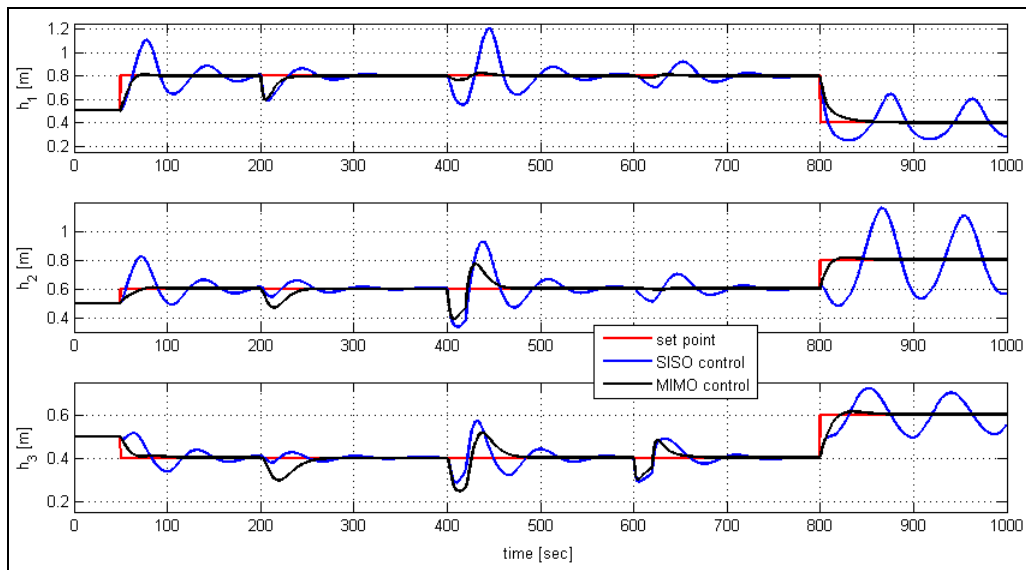


Fig. 9. Simulations results of the unconstrained scenario for each tank level. In contrast to the SISO-controllers, the MIMO-controller considers the coupling effects, what leads to a better dynamic behaviour of the controlled variables.

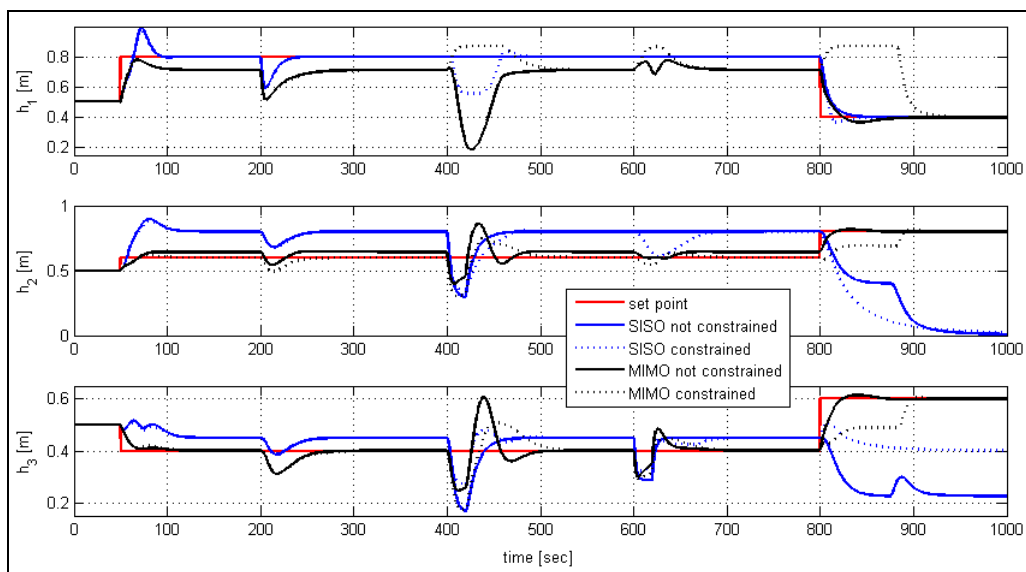


Fig. 10. Simulations results of the constrained scenario for each tank level. Caused on the constraints it is not possible to reach a stationary exactness in all cases. To compare the simulations results in a more objective way, fig. 8 should be focused.

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