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Finite-Time Supervisory Stabilization for a Class of Nonholonomic Mobile Robots Under Input Disturbances

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Abstract: This paper provides a solution for the stabilization of a nonholonomic wheeled mobile robot which is affected by additive input disturbances. The solution is based on the supervisory control framework, finite-time stability and robust multi-output regulation. A supervisor and two controls are designed with the objective to stabilize the first output (in the input-to-output stability sense) while the second output has to be kept under a threshold. The results is then applied to the case of the unicycle mobile robot which has to reach a position (i.e. stabilization) avoiding eventual obstacle during the task (i.e. keeping the second output under a threshold). The effectiveness of the solution is proved mathematically, supported by simulation results and finally tested on a wheeled mobile robot in a real scenario.

1. INTRODUCTION

Various stabilization strategies have been proposed for wheeled mobile robots (WMRs) in the literature, like continuous time-varying feedback control, discontinuous feedback control laws, hybrid/switch control laws and optimal control laws to tackle the stabilization problem (D'Andréa-Novel et al. (1995), Oriolo and Vendittelli (2002), Ailon et al. (2011), Canudas de Wit and Sordalen (1992), Yamamoto and Watanabe (2010)) while one of the first work on the stabilization using a finite time technique can be found in Guldner and Utkin (1994).
Furthermore, for a realistic robotic application, stabilizing the system is not enough: a robot has to deal with real environments that means, in most of the cases, obstacles to be avoided. Collision Avoidance for WMR has been thoroughly investigated as well and various methods have been proposed. Among the earliest approaches in the literature we find the *bugs algorithms* (and their modifications) in Lumelsky and Stepanov (1987) and Magid (2004)). The *potential field method* firstly presented in Khatib (1986) drives the robot along a potential field whose minimum is the goal and in which the obstacle act as a additional force that repels the robots. Other approaches are based on the reflex behaviour, like the *Deformable Virtual Zone* (DVZ) approach by Lapierre et al. (2007) and Zapata and Lepinay (2004) in which the robot is surrounded by a time-varying risk zone that reacts to obstacles that enter it changing its shape and modifying the robot's behaviour by consequence. A variety of switching control strategies have been proposed addressing the stabilization and the collision avoidance for WMR by Efimov et al. (2009), Sanfelice and Prieur (2010), Tanner et al. (2001).
The presence of disturbances coming from different sources have to be taken into account. The analysis of the problem led to an application in which a mobile robot is asked to reach a point (i.e. stabilization of the origin) avoiding any obstacles, in addition, with a disturbance acting on the control input. Moreover it is not common to consider disturbances acting directly on the control inputs. Such a problem is addressed in the present work. It is required to robustly stabilize certain outputs of the system in the uniform output stability (uOS) sense.
The problem formulation for a nonholonomic WMR is presented in Section 2. The main result of this work is presented in Section 3 where the problem is formalized in a general way. Section 4 is devoted to the design of control laws for a unicycle WMR and finally Section 5 presents the results obtained in simulation and in reality for a WMR.

2. MOTIVATING APPLICATION

Let us consider a nonholonomic system like a unicycle WMR, in which the input is affected by an additive disturbance:

$$\begin{aligned} \dot{q}_x &= (1+d_1)v\cos(\theta), \\ \dot{q}_y &= (1+d_1)v\sin(\theta), \\ \dot{q}_\theta &= (1+d_2)\omega, \end{aligned} \tag{1}$$

where $q = [q_x\, q_y\, q_\theta]^T$ is the state space vector and $(q_x, q_y) \in \mathbb{R}^2$, define the Cartesian position of the robot, and $q_\theta \in [0, 2\pi)$ is the orientation of the robot with respect of the world reference frame, v and ω are the control inputs, the linear velocity and the angular velocity respectively. The additive disturbances on the inputs are unknown, but

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supposed to be bounded, $-1 < d_{min} \leq d_i \leq d_{max}$, $i = 1, 2$. The lower bound, $d_{min} > -1$, ensures that the disturbance does not induce a change of control sign (a constraint satisfied in practice). To achieve the tasks the robot has to be driven to the origin avoiding obstacles that it could, eventually, encounter during the path. As a solution, two independent controllers can be designed to reach the goals (i.e. stabilization in the origin and collision avoidance) with their posterior uniting (Efimov et al. (2009)). These controls can be designed in order to regulate two different outputs:

$$z_1(q_x, q_y) = \sqrt{q_x^2 + q_y^2}, \tag{2}$$

$$z_2(q_x, q_y) = \min\left[Y, \max_{1\leq i\leq N}\left(\sqrt{(q_x - x_{0_i})^2 + (q_y - y_{0_i})^2}\right)^{-1}\right] \tag{3}$$

where z_1 is the distance from the origin and z_2 is the inverse of the distance from the closest obstacle represented by its Cartesian position $(x_{o_i}, y_{o_i})_{i,\ldots,N}$, with N is a finite number of obstacles, $Y > 0$ is a parameter ensuring global boundedness of z_2 and related with dimensions of the obstacles. Clearly, driving z_2 to a sufficiently small value means to move away from an obstacle avoiding it. Under the assumption that between the obstacles there were enough space we can consider one obstacle each time without loosing generality. We will also assume that $z_2(0, 0) > Y$, i.e.the origin is not occupied by an obstacle. Therefore, the problem we want to solve is to stabilize the system (1) regulating the output z_1 to reach the desired position, and z_2 to realize the collision avoidance.

3. THEORETICAL FORMULATION

Consider the following system (a nonholonomic WMR model):

$$\dot{x} = f(x, u, d),\ z_1 = h_1(x),\ z_2 = h_2(x), \tag{4}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $d \in \mathbb{R}^m$ is a disturbance, with $d \in \Omega = \{d \in \mathcal{L}_m^\infty : ||d|| \leq D\}$ for some $D \in \mathbb{R}_+$.

We want to regulate the outputs $z_1 \in \mathbb{R}^{p_1}$ and $z_2 \in \mathbb{R}^{p_2}$ assuming that the functions f, h_1 and h_2 are continuous and locally Lipschitz. It is needed to design a control $u : \mathbb{R}^n \to \mathbb{R}^m$ that will provide the uOS [1] property with respect to the output z_1, and will keep the second output z_2 in a predefined limit. In other words, to achieve the desired tasks it is needed that for all initial conditions $x_0 \in \mathbb{R}^n$, $d \in \Omega$ and $t \geq t_0 \geq 0$:

$$|z_1(t, x_0, d)| \leq \beta(|h_1(x_0)|, t - t_0), \tag{5}$$

$$|z_2(t, x_0, d)| \leq \sigma(\max(\Delta, |h_2(x_0)|)), \tag{6}$$

the value of Δ is given, β is a $\mathcal{KL}$ [2] function whereas σ is a function from class $\mathcal{K}$. It can be noted that (5) is exactly the definition of the uOS property. The second output must be smaller than $\sigma(\Delta)$. In the case $|h_2(x_0)| > \Delta$ the trajectory should converge to a subset where $|h_2(x)| \leq \sigma(\Delta)$. In addition, to solve the problem we need that the intersection between the sets $h_1(x) = 0$ and $|h_2(x)| \leq \sigma(\Delta)$ would be not empty, thus we assume the existence of a function ρ of class $\mathcal{K}$ and a scalar $0 < \rho_0 < \sigma(\Delta)$ such that:

$$|h_2(x)| \leq \rho(|h_1(x)|) + \rho_0. \tag{7}$$

3.1 Description of independent controls

Thus the problem consists in an output uniform stabilization under constraints imposed on another output. Following Efimov et al. (2009), assume that two right-continuous controls $u_i : \mathbb{R}^n \to \mathbb{R}^m$, $i \in \{1, 2\}$ are given guaranteeing an independent stabilization for the corresponding output z_i, i.e. the system

$$\dot{x} = f(x, u_i(x), d),\ z_i = h_i(x),$$

is forward complete and has continuous solutions $x(t, x_0, d)$, in addition the system is uOS with respect the output z_i and disturbance $d \in \Omega$. We also assume that during an activation of u_2 for all $t \geq 0$

$$|z_1(t, x_0, d)| \leq |h_1(x_0)|.$$

Next subsection is devoted to uniting of these controls in order to solve the posed problem.

3.2 Supervisory control

Under the assumption of having two controls which solve the output regulation for z_1 and z_2 independently, a *supervisor* is proposed to oversee the activation of the controls to achieve both required condition (5) and (6) simultaneously. The idea is that the controller u_2 is activated when $|z_2(x)|$ reaches a threshold Δ and remains active until the constraint $|z_2(x)| \leq \delta$ is satisfied, where $0 < \delta < \Delta$ is a given parameter. For this reason we define two sets

$$\mathbf{X}_1 = \{x \in \mathbb{R}^n : |h_2(x)| \leq \delta\}$$
$$\mathbf{X}_2 = \{x \in \mathbb{R}^n : |h_2(x)| \leq \Delta\}$$
$$\mathbf{X}_1 \subset \mathbf{X}_2.$$

Then the control

$$U(t) = u_{i(t)}(x(t)),\ i : \mathbb{R}_+ \to \{1, 2\} \tag{8}$$

is ruled by

$$t_0 = 0,\ i(t_0) = \begin{cases} 1 & \text{if } x(t_0) \in \mathbf{X}_2, \\ 2 & \text{otherwise,} \end{cases}$$

while $i(t) = i(t_j)$ for $t \in [t_j\ t_{j+1})$, and

$$i(t_{j+1}) = \begin{cases} 1 & \text{if } x(t_{j+1}) \in \mathbf{X}_1 \\ 2 & \text{if } x(t_{j+1}) \notin \mathbf{X}_2 \end{cases}, \tag{9}$$

where t_j is the generic switching instant defined as follows:

$$t_j = \begin{cases} \arg\inf\limits_{t\geq t_j}\ x(t) \notin \mathbf{X}_2 & \text{if } i(t_j) = 1 \\ \arg\inf\limits_{t\geq t_j}\ x(t) \in \mathbf{X}_1 & \text{if } i(t_j) = 2 \end{cases}.$$

A similar supervisor has been presented in Efimov et al. (2009), but in the present work a dwell time condition is not imposed. The control U has the u_1 part active if $|z_2| < \Delta$, which means that we are stabilizing the output z_1 according to condition (5). If $|z_2|$ becomes greater or equal than Δ, then u_2 will be activated driving z_2 to a value less than δ according to condition (6). Inside the set $H = \mathbf{X}_2 \backslash \mathbf{X}_1$ the control will not be switched, this set acts as a hysteresis zone being helpful to avoid a chattering

[1] A *forward complete* system $\dot{x} = f(x, u, d), y = h(x)$ is called uniformly Output-Stable (uOS) with respect to output y and input d, if for all $x_0 \in \mathbb{R}^n$ and $d \in \Omega$ there exists a function $\beta \in \mathcal{KL}$ such that $|y(t, x_0, d)| \leq \beta(|h(x_0)|, t - t_0)$ for all $t \geq t_0$.

[2] A continuous function $g : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{K}$ if it is strictly increasing and $g(0) = 0$; a continuous function $h : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{KL}$, if $h(\cdot, t) \in \mathcal{K}$ for any $t \in \mathbb{R}_+$, and $h(s, \cdot)$ is strictly decreasing to zero for any $s \in \mathbb{R}_+$ for $t \to \infty$.

phenomena of switching between u_1 and u_2.

Assumption 1. $\sup_{x\in H, d\in\Omega, i\in\{1,2\}} |f(x, u_i(x), d)| = F < +\infty$.

This assumption states that the system velocity on the set H is finite, then since $F < +\infty$ and $d \in \Omega$ there exists a dwell-time delay $\tau_D > 0$ between any two switches, i.e. $t_{j+1} - t_j \geq \tau_D$ for all $j \geq 0$. The conditions for solution of the posed problem using the supervisory control algorithm (8), (9) are described in the following theorem.

Theorem 1. Let Assumption 1 be satisfied and $\beta_1(s, \tau_D) = \lambda s$ for all $s \in \mathbb{R}_+$ and some $0 \leq \lambda < 1$. Then the system (4) with supervisor (9) and control (8) is forward complete and for all initial conditions $x_0 \in \mathbb{R}^n$, $d \in \Omega$ and $t \geq 0$:

$$|z_1(t, x_0, d)| \leq \beta_1(|h_1(x_0))|, 0),$$
$$|z_2(t, x_0, d)| \leq \sigma(\max\{\Delta, |h_2(x_0)|\}),$$
$$\lim_{t\to+\infty} |z_1(t, x_0, d)| = 0,$$

where $\sigma(s) = \beta_2(s, 0)$.

All proofs are omitted due to space limitation.

4. CONTROL TASKS

In this section two finite-time controllers (u_i, $i \in \{1, 2\}$) are designed for (1); the former one is designed to regulate the output z_1 in (2), for the stabilization part, and the second one is to regulate the output z_2 in (3), providing the collision avoidance. The main feature of these controls is that all control tasks are solved not asymptotically, but in a finite time. Such an advantage is quite important for some special applications.

4.1 Stabilization

As mentioned above, the first control u_1 is designed in order to drive the robot to the desired point (the origin in this paper, thus stabilize z_1). Following control theory, let us consider the following Lyapunov function: $V_1 = 0.5z_1^2$. Its derivative has the form $\dot{V}_1 = \cos(\theta_0 - q_\theta)v(1 + d_1)z_1$ where $\theta_0 = \arctan(q_y/q_x)$. Define $\alpha = q_\theta - \theta_0 - \pi$, $\alpha \in [-\pi, \pi)$, which implies the deviation from the robot's current orientation q_θ to its desired final orientation $\theta_0+\pi$, then we have $\dot{V}_1 = -\cos(\alpha)v(1+d_1)z_1$. In order to ensure the negative definiteness (or semi-definiteness) of $\dot{V}_1$, the following control is proposed:

$$v = \begin{cases} k_1 z_1 & \text{if } |\alpha| \leq k\pi \\ 0 & \text{otherwise} \end{cases},$$

with k_1 positive and $0 < k \leq 0.5$, with which the semi-definiteness of $\dot{V}_1$ can be ensured.

Then let us consider the regulation of the robot's orientation to its desired one, i.e. stabilization of α to 0. Following the definition of α, its dynamics can be expressed as follows:

$$\dot{\alpha} = \omega(1 + d_2) + \sin(\alpha)z_1^{-1}(1 + d_1)v.$$

By choosing the following Lyapunov function $V_2 = 0.5\alpha^2$, we obtain its derivative as: $\dot{V}_2 = \omega\alpha(1+d_2)+\alpha\sin(\alpha)v(1+d_1)z_1^{-1}$. In order to guarantee the negative definiteness of $\dot{V}_2$, we propose the control ω in the following form:

$$\omega = -k_2\zeta(\alpha)\text{sign}(\alpha),\ k_2 \geq \frac{(1 + d_{max})k_1 + 2^{-3/4}\eta_1}{1 - d_{min}}$$
$$\zeta(\alpha) = \max\{|\alpha|^{0.5}, |\alpha|\},\ \eta_1 > 0. \quad (10)$$

Let us remark that the term k_2 allows us to compensate the disturbances. It can be stated that $\alpha(t)$ admits the following upper estimate:

$$|\alpha(t)| \leq \begin{cases} |\alpha_0|e^{-0.5\eta_1(t-t_0)} & \text{if } t \in [t_0, t_1], \\ \sqrt{2}[\min\{1, 2^{-1/4}\sqrt{|\alpha_0|}\} & \\ -2^{-2}\eta_1(t - t_1)]^2 & \text{if } t \in (t_1, t_2], \\ 0 & \text{if } t > t_2, \end{cases} \quad (11)$$
$$t_1 = t_0 + \max\{0, \eta_1^{-1}\ln(0.5\alpha_0^2)\},$$
$$t_2 = t_1 + 2^{-2}\eta_1^{-1}\min\{1, 2^{-1/4}\sqrt{|\alpha_0|}\},$$

where $t_0 \geq 0$ is the instant when this control has been activated and $\alpha_0 = \alpha(t_0) \in [-\pi, \pi)$ is the initial condition. Therefore there exists $0 \leq T_1(\alpha_0) < \infty$ for all $\alpha_0 \in [-\pi, \pi)$ such that $|\alpha(t)| < k\pi$ for all $t \geq t_0+T_1(\alpha_0)$. For simplicity, when $v = 0$, since $|\alpha_0| \leq \pi$ we obtain:

$$T_1(\pi) = \eta_1^{-1}\begin{cases} -2\ln(k) & \text{if } k\pi \geq 1, \\ \ln(0.5\pi^2) + 4(1 - 2^{-1/4}\sqrt{k\pi}) & \text{otherwise.} \end{cases}$$

It follows that:

$$z_1(t) \leq z_1(t_0)\min\{1, e^{-c_1(t-T_1(\pi)-t_0)}\}\ \forall t \geq t_0. \quad (12)$$

which implies that we can exponentially stabilize the first output z_1 by using the designed controls v and ω. Thus the first control u_1 can be summarized as follows:

$$u_1 = \begin{cases} v = \begin{cases} k_1 z_1 & \text{if } |\alpha| \leq k\pi \\ 0 & \text{otherwise} \end{cases}, \\ \omega = -k_2\zeta(\alpha)\text{sign}(\alpha). \end{cases} \quad (13)$$

The above arguments are equivalent to the following lemma.

Lemma 1. In the system (1) with control (13) the estimates (12) and (11) are satisfied (a uniform exponential stabilization for z_1 and a uniform finite-time stabilization for α).

Remark : It deserves to be precised that despite the fact that the convergence of the z_1 output is exponential, a zone around the origin can be always reached in a finite time. That allows to state that practically the controller can achieve the task in a finite time.

4.2 Collision Avoidance

As we can notice, that the first controller u_1 can drive the robot to the desired position, if no obstacle will be encountered during the navigation. This is however not the real case in practice. In order to take into account the obstacle, we need to construct another controller, named u_2, which needs to achieve the following two tasks:

- driving the robot away from the encountered obstacle (i.e. collision avoidance);
- keeping the distance z_1 between the robot and the desired final position not increasing (i.e. still approaching to the desired final position).

In order to design such a controller, we consider each obstacle as a point in the plane and then define an associated safe distance to be maintained. Each obstacle is an element of the set $O = \{(x_{o_i}, y_{o_i}, \rho_{i,min})\}_{i=1,\ldots,N}$, with N number of possible obstacles, $Y = 1/\min_{1\leq i\leq N}\{\rho_{i,min}\}$. It is assumed in this paper that each obstacle is entirely contained in the circle of radius $\rho_{i,min}$ which is a designed

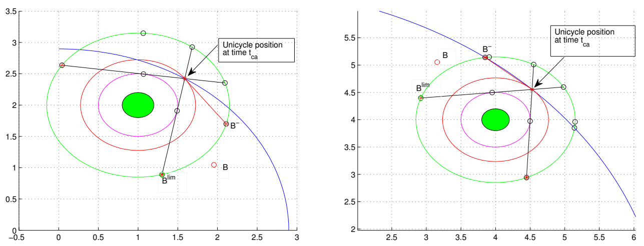

Fig. 1. Circles used in the definition of the B^{-},B^{lim} and then the B points with the "tangent" approach (left) and "circles" approach (right)

distance considering the radius of the obstacle itself and a distance equal to the radius of the circle in which the robot can be inscribed. Moreover, in order to augment the safety, the collision avoidance controller u_2 will be activated when the robot reaches a distance $\rho_i > \rho_{i,min}$, which adds an additional safety level to the collision avoidance manoeuvre. Then, the goal of the control u_2 is to ensure the avoidance by augmenting the distance from ρ_i to a predefined $R_i > \rho_i$. In terms of the output z_2, it is equivalent to decrease z_2 from $\Delta_i = \rho_i^{-1}$ to $\delta_i = R_i^{-1}$. Moreover, during this manoeuvre, it is required that the control u_2 will not make the output z_1 increasing.
Before stating collision avoidance controller u_2, for the sake of simplicities, let us make the following assumptions:

- Assume that $\max_{1\leq i\leq N} 1/\sqrt{x_{o_i}^2 + y_{o_i}^2} < \min_{1\leq i\leq N} \delta_i$, i.e. the origin is well separated from an obstacle.
- It is also assumed that $\Upsilon_i \cap \Upsilon_j = \emptyset$ for any $i \neq j \in \{1,\ldots,N\}$, where $\Upsilon_i = \{(q_x,q_y) \in \mathbb{R}^2 : (q_x - x_{o_i})^2 + (q_y - y_{o_i})^2 \leq R_i^2\} = \{(q_x,q_y) \in \mathbb{R}^2 : z_2(q_x,q_y) \geq \delta_i\}$, i.e. any two obstacles are separated and the collision avoidance problem can be addressed for an isolated obstacle.

In order to design the control u_2, we need to plan a strategy to move the robot from Δ_i to δ_i. For this, when the robot reaches a distance $\rho_i > \rho_{i,min}$, we define an intermediate point $B = (x_B, y_B)$, and the goal of u_2 is to control the robot moving from current position to this new point B such that $z_1(x_B,y_B) \leq z_1(q_x(t_{ca}),q_y(t_{ca}))$ and $z_2(x_B,y_B) \leq \delta_i$, where t_{ca} is the instant of time in which the control u_2 is switched on, i.e. $z_2(q_x(t_{ca}),q_y(t_{ca})) = \Delta_i$. The following details the algorithm for the choice of the point B.

Choice of point B Let us firstly define a preliminary point B^- as an intersection point of the circle centered in (x_{o_i}, y_{o_i}) of radius R_i and the tangent line to the circle centered at (x_{o_i}, y_{o_i}) of radius Δ_i (see the red one in Fig. 1 left). Although this approach is very efficient, under a special situation, it cannot provide the second requirement of the control, i.e. $\dot{z}_1(t) \leq 0$, that is the case when the obstacle center, the robot and the origin are on the same straight line (see Fig. 1 right). In this case, the preliminary B^- point will be in the intersection of two circles: the first centered in (x_{o_i}, y_{o_i}) of radius R_i (the green one in Fig. 1 right) right and the second circle centered at the origin of radius $|z_1(t_{ca})|$ (the blue one in Fig.1 right).

In order to determine the final coordinates of B for both cases, let us define the distance $\rho_{i,min}$ as a limit not to be crossed, represented in Fig. 1 (right) by the purple circle. Then we can determine the point B^{lim}, which is an intersection of a straight line initiated at the robot position and tangent to the circle centered at (x_{o_i}, y_{o_i}) with radius $\rho_{i,min}$. Thus we can freely choose a point B' on the circle of radius R_i between the points B^{lim} and B^- taking a safe distance from them proportional to d_{max} (in order to avoid the risk of being steered backward due to a disturbance). Finally, the point $B = (x_B, y_B)$ can be selected on the line passing the current robot position and the point B' with the condition that $z_2(x_B, y_B) < \delta$ (outside the set Υ_i, green circle in both Fig. 1 left and right). With such a selection of the point B, it is possible to achieve the avoidance and to keep, in addition, the condition $\dot{z}_1(t) \leq 0$. Once the point B is defined, we can then design a control u_2, which should drive the robot from current position to this point, which will be detailled in the next section.

Collision avoidance controller The collision avoidance problem can be solved by using a similar approach as the stabilization problem in the previous section, which needs only to replace the origin in the stabilization problem by the chosen point B. For this, let us define the distance from the robot to the point B as

$$y_B(x,y) = \sqrt{(q_x - x_B)^2 + (q_y - y_B)^2},$$

in such a formulation the imposed restriction $z_2(x_B, y_B) < \delta$ becomes crucial and the point B will not be reached during the collision avoidance manoeuvre. It is important, since in a stabilized point the robot loses controllability (in our case this corresponds to division on the distance y_B in the equation (14) below). Define $\vartheta = \inf_{(q_x,q_y)\in\Upsilon_i} y_B(q_x,q_y)$ the distance from the point B to the set Υ_i.
Since

$$\dot{y}_B = -\cos(\gamma)v(1 + d_1),$$

in order to stabilize y_B, we propose the following controller for v:

$$v = \begin{cases} k_3 y_B & \text{if } \cos(\alpha) \geq 0 \text{ and } |\gamma| \leq \epsilon\pi \\ 0 & \text{otherwise} \end{cases},$$

where $k_3 > 0$ and $0 < \epsilon < 0.5$. Since this control has to be applied into the set Υ_i only, then $v \geq k_3\vartheta$. Using a Lyapunov approach it can be shown that v stabilizes y_B. Following the same argument used in the stabilization section, let us define the angle of desired orientation of the robot towards the point B as $\theta_g = \tan^{-1}\left(\dfrac{q_y - y_B}{q_x - x_B}\right)$, and define the deviance from the desired angle for the collision avoidance control as $\gamma = \theta_g - q_\theta$. Then γ has the following dynamics:

$$\dot{\gamma} = -\omega(1 + d_2) + \frac{\sin\gamma}{y_B}v(1 + d_1). \tag{14}$$

Setting $\zeta(\gamma) = \max\{|\gamma|^{0.5}, |\gamma|\}$, the proposed expression for the control ω has the form:

$$\begin{aligned} \omega &= k_d\dot{\gamma} + \frac{\sin\gamma}{y_B}v + k_{ca}\zeta(\gamma)sign(\gamma),\ k_d > 0, \\ k_{ca} &\geq k_3\frac{\sqrt{\pi}(d_{max} - d_{min})[1 + k_d(1 + d_{max})]}{(1 - d_{min})[1 + k_d(1 - d_{min})]} \\ &\quad + 2^{-3/4}\eta_2\frac{1 + k_d(1 + d_{max})}{1 - d_{min}},\ \eta_2 > 0. \end{aligned} \tag{15}$$

Using the Lyapunov function $W_2 = 0.5\gamma^2$, a straightforward calculation shows that $\dot{W}_2 \le -\eta_2 \max\{W_2, W_2^{3/4}\}$, then

$$|\gamma(t)| \le \begin{cases} |\gamma_0| e^{-0.5\eta_2(t-t_{ca})} & \text{if } t \in [t_{ca}, t_3], \\ \sqrt{2}[\min\{1, 2^{-1/4}\sqrt{|\gamma_0|}\} \\ -2^{-2}\eta_2(t-t_3)]^2 & \text{if } t \in (t_3, t_4], \\ 0 & \text{if } t > t_4, \end{cases} \quad (16)$$

$$t_3 = t_{ca} + \max\{0, \eta_2^{-1}\ln(0.5\gamma_0^2)\},$$

$$t_4 = t_3 + 2^{-2}\eta_2^{-1}\min\{1, 2^{-1/4}\sqrt{|\gamma_0|}\},$$

Being $t_{ca} \ge 0$ the instant of activation of u_2, the control steers the robot in a finite time to the desired orientation, indeed there exists $0 < T_2 < \infty$ such that $\gamma(t_{ca}+T_2) < \epsilon\pi$ for all $\gamma \in [-\pi, \pi)$:

$$T_2 = \eta_1^{-1} \begin{cases} -2\ln(\epsilon) & \text{if } \epsilon\pi \ge 1, \\ \ln(0.5\pi^2) + 4(1 - 2^{-1/4}\sqrt{\epsilon\pi}) & \text{otherwise.} \end{cases}$$

As we can note, the controls (10) and (15) used for regulation of α and γ respectively are rather similar and have analogous stability properties, so a kind of control (15) can be used for stabilization of α, and *vice versa*.

Following the geometric construction of the point B, the inequality $\cos(\alpha)|_{\gamma=0} > 0$ is verified, then there is a time instant $t_{ca} \le \bar{t} \le T_2$ such that the conditions $\cos(\alpha(t)) \ge 0$ and $|\gamma(t)| \le \epsilon\pi$ (involved in the control v activation) are satisfied for $t \ge \bar{t}$. Starting from the instant $\bar{t}$ the robot starts to move without an interruption since $v \ge k_3\vartheta$. Therefore, with the decreasing properties of γ, the distance y_B is decreasing and admits an estimate:

$$y_B(t) \le y_B(t_0)e^{-c_2(t-T_2-t_{ca})} \; \forall t \ge t_{ca}.$$

Since the point B is located outside the set Υ_i, then there exists a finite time $T_{ca} > t_{ca}$ such that $z_2(T_{ca}) = \delta$, hence the collision avoiding is accomplished. It is worth to stress that it is possible to have a local increment of the regulated output z_2 due to the geometric construction of the point B. On the other hand, after a certain amount of time the output z_2 decreases with the controller v. In addition, as it has been shown above, it is not possible to steer the robot toward the obstacle, and the robot itself will not enter the circle of radius $\rho_{i,min}$. The output z_1 does not increase during the collision avoiding manoeuvre since the constraint $\cos(\alpha) \ge 0$ has been introduced in the control v (and v is positive).

The controller u_2 for the two control inputs v and ω pushes the robot in a finite time toward a point far from the obstacle, while keeping the distance z_1, and it can be summarized as follows:

$$u_2 = \begin{cases} v = \begin{cases} k_3 y_B & \text{if } \cos(\alpha) \ge 0 \text{ and } |\gamma| \le \epsilon\pi \\ 0 & \text{otherwise} \end{cases}, \\ \omega = k_d\dot{\gamma} + \dfrac{\sin\gamma}{y_B}v + k_{ca}\zeta(\gamma)\mathrm{sign}(\gamma). \end{cases} \quad (17)$$

The following properties have been substantiated.

Lemma 2. The system (1) with control (17) has the properties for $t_{ca} \ge 0$:

1. Uniform finite-time stability with respect to the variable $\gamma(t)$ (the estimate (16)).
2. There exists $T_{ca} > t_{ca}$ such that $\delta_i \le z_2(t) < \rho_{i,min}^{-1}$ for all $t \in [t_{ca}, T_{ca}]$ and $y_2(T_{ca}) = \delta$.
3. $\dot{V}_1(t) \le 0$ for all $t \in [t_{ca}, T_{ca}]$.

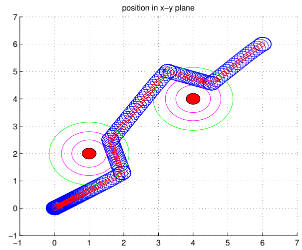

Fig. 2. Stabilization of the unicycle with two obstacles

Following the presented strategy for the choice of point B when encountering an obstacle, the item 1 of the above lemma implies that the control u_2 can orient the robot in the direction of the chosen point B in finite time. The second and the third item of the above lemma state that this control u_2 can drive the robot away from the obstacle, at the same time it will not increase the distance between the robot and the desired final point.

4.3 Supervision

The supervisor (9) presented in Section 3 can now be applied giving an interpretation for the sets $\mathbf{X_1} : \{(q_x, q_y) \in \mathbb{R}^2 : \mathbb{R}^2 \setminus \cup_{j=1}^N \Upsilon_j\}$ and $\mathbf{X_2} : \{(q_x, q_y) \in \mathbb{R}^2 : \mathbb{R}^2 \setminus \cup_{j=1}^N \Xi_j\}$, $\Xi_j = \{(x, y) \in \mathbb{R}^2 : (q_x - x_{o_j})^2 + (q_y - y_{o_j})^2 \le \rho_i^2\}$. Thus the control u_1 is applied if $z_2 < \delta_j$ and the control u_2 has to be activated if $z_2 = \Delta_j$ for some $j \in \{1, \ldots, N\}$. The stability properties of the WMR (1) with the controls (13) and (17) and the supervisor (9) can be then achieved.

4.4 Application to unicycle

We will now provide the main result for the unicycle considering the supervisory control described in this section.

Corollary 1. Consider the system (1) with the supervisor (9) and control (13) and (17), then:

$$z_1(t) \le z_1(0) \quad \forall t \ge 0,$$

$$\lim_{t\to\infty} z_1(t) = 0,$$

$$z_2(t) \le \sigma(\max\{\Delta, z_2(0)\}) \quad \forall t \ge 0,$$

where $\Delta = \max_{1\le i\le N} \Delta_i$ and $\sigma(s) = s/(\Delta Y)$.

5. SIMULATION AND PRACTICAL RESULTS

For simulation purpose the number of obstacles is $N = 2$, the sample time used is $t_s = 0.1$, the disturbances have form $d_i = \chi\sin(t) + 0.1 * rand$ where $rand$ is a pseudo-random values drawn from the standard uniform distribution on the open interval $(0, 1)$ with $i \in \{1, 2\}$ and $|\chi| \le 0.5$. For the collision avoidance part the distances were defined as follows: let r be the generic obstacle radius, $\rho_{i,min} = r + 0.3$, $\rho_i = \rho_{i,min} + 0.3$ $R_i = \rho_i + 0.35$. The ϵ and k values are equal to $1/30$. The values of control gains used in the simulations are the following: $k_1 = 0.5$, $\eta = 0.5$, $k_3 = 1.0$, $k_p = 1.5$, $k_d = 0.5$, $k_{ca} = 0.1$. As it can be seen in Fig. 2 each time the robot enters the zone where $z_2 \ge \Delta$, it starts the manoeuvre to reach the point B making collision avoidance. Once it enters the zone $z_2 \le \delta$ it continues to move toward the origin. The center of mass of the robot, red circle in Fig. 2, never enters the circle of radius $\rho_{i,min}$ preserving the robot to collide. This explains

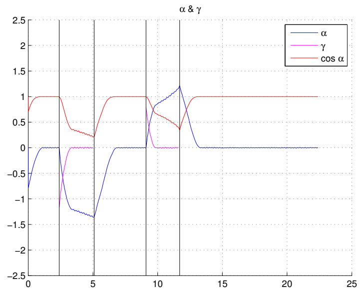

Fig. 3. Evolution of angles α and γ

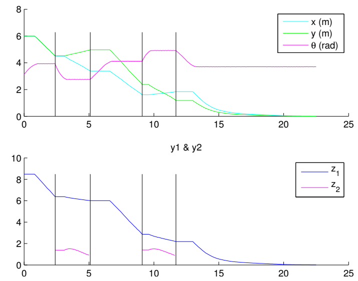

Fig. 4. a) State evolution b) Outputs

us better why in Section 4 the radius of the robot has been considered as a design parameter $\rho_{i,min}$; indeed the two figures showed in 4.2 (Fig. 1) display the behaviour of the algorithm to choose the B point in the two activations of the controller. In Fig. 3 and Fig. 4 the vertical black lines represent the switching instants and it is shown that all controlled variables behave as wanted. In particular in Fig. 3 it is shown how the variables α and γ are stabilized both in finite time by the controllers to steer the robot facing the desired point. The angle γ appears in the plot only when the controller u_2 is active. In the same plot the value of $\cos\alpha$ is shown (in order to demonstrate that the condition for the v part of the controller under the collision avoidance manoeuvre is always kept). It is also shown the behaviour of the two outputs, z_2 indeed increases the value during the collision avoidance manoeuvre in all the activations, but before the successive switch the value is always less than the starting one. We would like to remark that z_1 never increases. The presented control has been implemented on a Wifibot V2 (`www.wifibot.com`) WMR (Fig. 5) equipped with a Hokuyo® (`http://www.hokuyo-aut.jp`) UTM-30LX LIDAR device. Robotic Operating System (ROS) (www.ros.org) allowed us to easily implement our controller and in Fig.5 is shown how the robot behaves in a real environment with several obstacles.

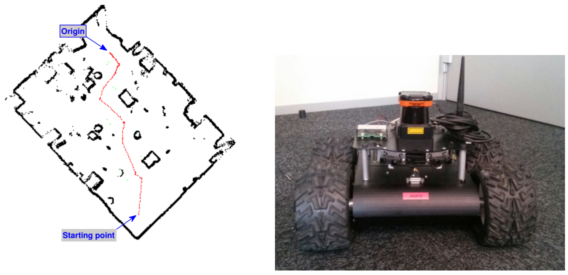

Fig. 5. The path followed (in red) by the Wifibot WMR using the proposed algorithm

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