Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

H_∞ based decoupling tracking control of hypersonic vehicle

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Abstract: The H_∞ based decoupling tracking control is studied in this paper. A virtual system constituted by the controlled system and the no coupling reference model is firstly set up. The controlled system is driven to follow the reference model to realize the decoupling. And the tracking error can be formulated by the H_∞ norm of the virtual system. Then the controller is derived by minimizing the H_∞ norm, which can be described by Linear Matrix Inequalities (LMIs). The necessary and sufficient condition of existence of controller is derived based on the LMIs above. A flight control example is given to illustrate the effectiveness of the proposed method. The simulation results show that the proposed method is of better control performance than Linear Quadratic (LQ) tracking controller.

Keywords: decoupling control, reference model, H_∞, LMI, hypersonic vehicle

1. INTRODUCTION

Hypersonic vehicle is a offensive weapon, which is difficult to intercept. The harsh environment it faces and its highly integrated design make the hypersonic vehicle become a typical complex multiple-input multiple-output (MIMO) system with strong coupling and uncertainties. As the hypersonic vehicle influences the focus of future missile defense system (Dai et al. (2010)), it is urgent to overcome the bad effects of coupling of the vehicle. Currently, there are several ways of decoupling design: Firstly, design the controller individually for each channel by using classical control theory, and then introduce a cross-linking gain matrix to compensate the coupling effect. Secondly, use a variety of modern control theories to design the decoupling controller, including LQR/LTR, $GSLQ$, H_∞, variable structure adaptive control and so on. Thirdly, transform the coupled MIMO systems into a series of single-input single-output (SISO) systems by the multivariable frequency domain theory, and then design the controller using the classical frequency domain method respectively.

Historically, the flight controllers were always designed according to the characteristics of each single-channel without considering decoupling effect because of low flight speed and little coupling impact (Chiu et al. (1991), Canale et al. (2008), and Das et al. (2007)). However, with the large increase of the flight speed, the traditional methods were unable to be further used to achieve higher and multiple performance requirements. It is difficult to apply the single-channel design procedures directly to achieve the satisfactory performance in hypersonic vehicle manipulations.

It is commonly agreed that we should reduce the coupling effect first, and then design the advanced robust control strategies by the modern control theories. Here a brief survey of several typical decoupling approaches. Inverse Nyquist Array (INA) method and diagonally dominant ideas were presented by Rosenbrock (Rosenbrock (1969)) firstly for multi-variable coupling system. Then Hawkins (Hawkins (1972b)) proposed pseudo-diagonalization to obtain a pre-compensator to make the transfer function matrix of the compensated system approximately be a diagonal matrix. To cope with the problem that this method can only design controller for a single frequency, later Hawkins (Hawkins (1972a)) improved this method to be applied in several frequencies. The method based on the specific frequency only considers the characteristics of the selected frequency, ignoring the other frequency characteristic, so that the control effect is not very good. Generally, these traditional frequency-domain theories need to obtain the transfer function of vehicle model, but it is hard to obtain the accurate vehicle model because of aerodynamic uncertainties. Meanwhile, it is difficult to describe the uncertain factors in the frequency domain.

For this reason, scholars have adopted the norm of the system to design compensator for the system. Karimi-Ghartemani (Karimi-Ghartemani and Mobed (2008)) designed a state feedback controller and a pre-filter to decouple a class of multi-variable systems. On the other hand, H_∞ control theory has an advantage in solving the control problem of uncertain systems. And H_∞ norm can be calculated by using the LMI approach in a convex optimisation procedure. Furthermore, in recent years, there have been many LMI universal solvers, which make the time domain approach greatly developed and generally welcomed. On this basis, Chughtai (Chughtai et al. (2005)) designed a controller for the system, using the unit matrix as a reference model. Due to the amplitude attenuation and phase lag of system in high frequency, the actual characteristics of the system have a large difference with the unit matrix. Therefore, this method remains to be further improved.

In view of this, this paper proposes a new decoupling tracking control method, for the sake of combining the decoupling effect and the dynamic tracking performance, to deal with a kind of strong coupled MIMO vehicle systems. In this study, a reference model of diagonal transfer matrix based on the demand of the frequency domain characteristics is selected to reduce the differences between the actual characteristic and the reference model's properties. We describe the controller design problem as an H_∞ synthesis problem, which can be handled in the state space instead of obtaining the transfer function of the vehicle model. More importantly, the necessary and sufficient conditions for using the proposed method are derived and described by LMI. Finally, we use a flight control example based on the hypersonic vehicle to show the validity of the proposed method, and the simulation results turn out to be perfect.

The remaining parts of this paper are organized as follows: in Section 2, we describe the control problem and elaborate the design steps of the decoupling method. Section 3 is devoted to present the necessary and sufficient conditions for the existence of the H_∞ based decoupling tracking controller. On the basis of the former sections, Section 4 takes up a flight control simulation to achieve the proposed decoupling tracking control strategy and illustrate the effectiveness of the method by the comparison with the LQ method. Section 5 summarizes the study and points out the future work.

2. PROBLEM DISCRIPTION

The state-space form of control object G is (1).

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

Where $A \in R^{n\times n}$, $B \in R^{n\times p}$, $C \in R^{p\times n}$ are the system matrix, input matrix, and output matrix of the control object G, respectively, $x \in R^n$ is the state vector while $u \in R^p$ is the control input vector, and $y \in R^p$ is the output vector.

The control target is to design a controller for above linear MIMO system and make the closed-loop system decoupled with desired properties. A diagonal transfer matrix based on the demand of the frequency domain characteristics is selected as the reference model, and the controlled system is driven to follow the reference model to realize the decoupling by minimizing the H_∞ norm of the virtual system.

We solve this problem in the following three steps:

Step 1: Set a reference model R_{ref}, which has the desired properties.

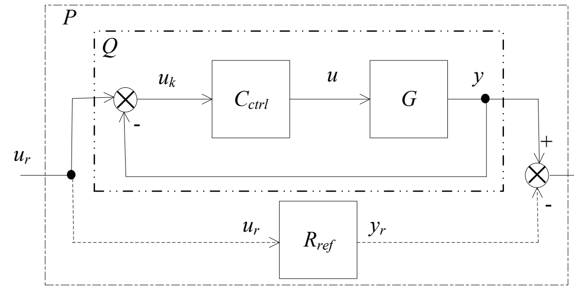

Fig. 1. Schematic diagram of Step 2

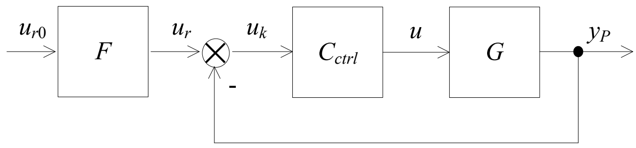

Fig. 2. Schematic diagram of Step 3

Step 2: As shown in Fig.1, the controller C_{ctrl} and the object G constitute a real closed-loop system Q. Assume that reference model R_{ref} is parallel with the closed-loop system Q. Then, R_{ref} and Q constitute a virtual system P. Our object is to design a controller C_{ctrl} to minimize the H_∞ norm of the virtual system P as (2).

$$\|P\|_\infty \triangleq \sup_\omega \sigma_{\max}(P(j\omega)) \tag{2}$$

If $\|P\|_\infty$ is small enough, u_r in Fig.1 will not affect the virtual output y_p. Then the characteristics of the closed-loop system Q will be similar with those of R_{ref}.

Step 3: Design a prefilter F, according to the characteristics of the closed-loop system in Fig.2, to make the characteristics of the overall system satisfy the required performance of the desired properties.

3. THE METHODOLOGY

Through the establishment of the virtual system P, we can use H_∞ norm of the virtual system to solve the decoupling problem, which can be described by LMI. In the following section, we will derive the necessary and sufficient conditions for the existence of the decoupling controller in detail.

Suppose that the state-space expression of controller C_{ctrl} is (3).

$$\begin{cases} \dot{x}_k = A_k x_k + B_k u_k \\ y_k = C_k x_k \end{cases} \tag{3}$$

Where $A_k \in R^{n_k\times n_k}$, $B_k \in R^{n_k\times p}$, $C_k \in R^{p\times n_k}$ are the system matrix, input matrix, and output matrix of the controller C_{ctrl}, respectively, $x_k \in R^{n_k}$ is the state vector, $u_k \in R^p$ is the input vector, and $y_k \in R^p$ is the output vector.

The state-space form of reference model R_{ref} is (4).

$$\begin{cases} \dot{x}_r = A_r x_r + B_r u_r \\ y_r = C_r x_r \end{cases} \tag{4}$$

Where $A_r \in R^{n_r \times n_r}$, $B_r \in R^{n_r \times p}$, $C_r \in R^{p \times n_r}$ are the system matrix, input matrix, and output matrix of the reference model R_{ref}, respectively, $x_r \in R^{n_r}$ is the state vector, $u_r \in R^p$ is the input vector, and $y_r \in R^p$ is the output vector. Moreover, the reference model R_{ref} is completely decoupled between the channels.

It can be obtained from Fig.1 that:

$$\begin{cases} u = y_k \\ u_k = u_r - y \end{cases} \tag{5}$$

Therefore, the state-space form of virtual system P can be described as (6).

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{x}_r \\ \dot{x}_k \end{bmatrix} = \begin{bmatrix} A & 0 & BC_k \\ 0 & A_r & 0 \\ -B_k C & 0 & A_k \end{bmatrix} \begin{bmatrix} x \\ x_r \\ x_k \end{bmatrix} + \begin{bmatrix} 0 \\ B_r \\ B_k \end{bmatrix} u_r \\ \qquad = A_a x_a + B_a u_r \\ y_p = \left[-C \ \ C_r \ \ 0 \right] \begin{bmatrix} x \\ x_r \\ x_k \end{bmatrix} = C_a x_a \end{cases} \tag{6}$$

Suppose that the dimension of the controlled object is n, the dimension of the reference model is n_r, the dimension of the input is p, and the dimension of the output is q.

Here we give the necessary and sufficient conditions for the H_∞ based decoupling tracking control method.

Theorem 1. There exists a (n+n_r)-dimensional controller such that $\|P\|_\infty < \gamma$ if and only if there exist $\hat{A} \in R^{(n+n_r)\times(n+n_r)}$, $\hat{B} \in R^{(n+n_r)\times q}$, $\hat{C} \in R^{p\times(n+n_r)}$, and symmetric matrixes $X, Y \in S^{(n+n_r)\times(n+n_r)}$ that satisfy (7)and (8).

$$\begin{bmatrix} \Phi_1 + \Phi_1^T & \hat{A}^T + \bar{\mathrm{A}} & \begin{bmatrix} 0 \\ B_r \end{bmatrix} & X \begin{bmatrix} -C^T \\ C_r^T \end{bmatrix} \\ * & \Phi_2 + \Phi_2^T & \Phi_3 & \begin{bmatrix} -C^T \\ C_r^T \end{bmatrix} \\ * & * & -\gamma I & 0 \\ * & * & * & -\gamma I \end{bmatrix} < 0 \tag{7}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \tag{8}$$

Where

$$\bar{\mathrm{A}} = \begin{bmatrix} A & 0 \\ 0 & A_r \end{bmatrix}, \Phi_1 = \bar{\mathrm{A}} X + \begin{bmatrix} B\hat{C} \\ 0 \end{bmatrix},$$

$$\Phi_2 = Y\bar{\mathrm{A}} + \left[-\hat{B}C \ \ 0 \right], \Phi_3 = Y \begin{bmatrix} 0 \\ B_r \end{bmatrix} + \hat{B}.$$

We select a non-singular matrix arbitrarily $M \in R^{(n+n_r)\times(n+n_r)}$, and set $N = (I - YX)M^{-T}$. Then we can obtain the controller by (9).

$$\begin{aligned} C_k &= \hat{C} M^{-T} \\ B_k &= N^{-1} \hat{B} \\ A_k &= N^{-1} \left(\hat{A} - \Psi \right) M^{-T} \end{aligned} \tag{9}$$

Where

$$\Psi = Y\bar{\mathrm{A}}X + N \left[-B_k C \ \ 0 \right] X + Y \begin{bmatrix} BC_k \\ 0 \end{bmatrix} M^T.$$

Proof. According to the Bounded Real Lemma, $\|P\|_\infty < \gamma$ means there exists a symmetric matrix W that satisfies (10) and (11).

$$\begin{bmatrix} A_a^T W + W A_a & W B_a & C_a^T \\ B_a^T W & -\gamma I & 0 \\ C_a & 0 & -\gamma I \end{bmatrix} < 0 \tag{10}$$

$$W > 0 \tag{11}$$

Suppose

$$\begin{aligned} W &= \begin{bmatrix} Y & N \\ N^T & \Theta_1 \end{bmatrix}, W^{-1} = \begin{bmatrix} X & M \\ M^T & \Theta_2 \end{bmatrix}, \\ \Pi_1 &= \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}, \Pi_2 = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix}. \end{aligned} \tag{12}$$

Where $X, Y \in S^{(n+n_r)\times(n+n_r)}$ are symmetric matrixes, $M, N \in R^{(n+n_r)\times(n+n_r)}$, $\Theta_1 = -N^T X M^{-T}$, $\Theta_2 = -N^{-1} Y M$, and $N = (I - YX)M^{-T}$.

Then

$$\Pi_2 = W \Pi_1. \tag{13}$$

According to this relationship, we obtain (14), (15), and (16).

$$\begin{aligned} {\Pi_1}^T W A_a \Pi_1 &= \Pi_2^T A_a \Pi_1 \\ &= \begin{bmatrix} I & 0 \\ Y & N \end{bmatrix} \begin{bmatrix} A & 0 & BC_k \\ 0 & A_r & 0 \\ -B_k C & 0 & A_k \end{bmatrix} \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix} \\ &= \begin{bmatrix} \bar{\mathrm{A}} X + \begin{bmatrix} BC_k \\ 0 \end{bmatrix} M^T & \bar{\mathrm{A}} \\ \Psi + N A_k M^T & Y\bar{\mathrm{A}} + N \left[-B_k C \ \ 0 \right] \end{bmatrix} \end{aligned} \tag{14}$$

$$\begin{aligned} {\Pi_1}^T W B_a &= {\Pi_2}^T B_a \\ &= \begin{bmatrix} I & 0 \\ Y & N \end{bmatrix} \begin{bmatrix} 0 \\ B_r \\ B_k \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ B_r \\ Y \begin{bmatrix} 0 \\ B_r \end{bmatrix} + N B_k \end{bmatrix} \end{aligned} \tag{15}$$

$$\begin{aligned} {\Pi_1}^T C_a^T &= \left(\left[-C \ \ C_r \ \ 0 \right] \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix} \right)^T \\ &= \begin{bmatrix} X \begin{bmatrix} -C^T \\ C_r^T \end{bmatrix} \\ -C^T \\ C_r^T \end{bmatrix} \end{aligned} \tag{16}$$

Where

$$\bar{\mathrm{A}} = \begin{bmatrix} A & 0 \\ 0 & A_r \end{bmatrix},$$

$$\Psi = Y\bar{\mathrm{A}}X + N \left[-B_k C \ \ 0 \right] X + Y \begin{bmatrix} BC_k \\ 0 \end{bmatrix} M^T.$$

Note that

$$\begin{aligned} \hat{A} &= \Psi + N A_k M^T, \\ \hat{B} &= N B_k, \\ \hat{C} &= C_k M^T. \end{aligned} \tag{17}$$

Multiply (10) by $diag(\left[\Pi_1^T \ I \ I \right])$ and $diag(\left[\Pi_1 \ I \ I \right])$ on the left and right, respectively. Then we get (18).

$$\begin{bmatrix} \Pi_1^T A_a^T W \Pi_1 + \Pi_1^T W A_a \Pi_1 & \Pi_1^T W B_a & \Pi_1^T C_a^T \\ B_a^T W \Pi_1 & -\gamma I & 0 \\ C_a \Pi_1 & 0 & -\gamma I \end{bmatrix} =$$

$$\begin{bmatrix} \Phi_1 + \Phi_1^T & \hat{A}^T + \bar{\mathrm{A}} & \begin{bmatrix} 0 \\ B_r \end{bmatrix} & X \begin{bmatrix} -C^T \\ C_r^T \end{bmatrix} \\ * & \Phi_2 + \Phi_2^T & \Phi_3 & \begin{bmatrix} -C^T \\ C_r^T \end{bmatrix} \\ * & * & -\gamma I & 0 \\ * & * & * & -\gamma I \end{bmatrix} \quad (18)$$

Where

$$\bar{\mathrm{A}} = \begin{bmatrix} A & 0 \\ 0 & A_r \end{bmatrix}, \Phi_1 = \bar{\mathrm{A}} X + \begin{bmatrix} B\hat{C} \\ 0 \end{bmatrix},$$

$$\Phi_2 = Y\bar{\mathrm{A}} + \begin{bmatrix} -\hat{B}C & 0 \end{bmatrix}, \Phi_3 = Y \begin{bmatrix} 0 \\ B_r \end{bmatrix} + \hat{B}.$$

Therefore, (10) is equivalent to (7).

Since $W > 0$, we can get (19).

$$\begin{aligned} 0 < {\Pi_1}^T W \Pi_1 &= {\Pi_2}^T \Pi_1 \\ &= \begin{bmatrix} I & 0 \\ Y & N \end{bmatrix} \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix} \\ &= \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \end{aligned} \quad (19)$$

Hence, (11) is equivalent to (8).

Thus, $\|P\|_\infty < \gamma$ means that there exist symmetric matrices $X, Y \in S^{(n+n_r)\times(n+n_r)}$ and arbitrary ones $\hat{A}, \hat{B}, \hat{C} \in R^{(n+n_r)\times(n+n_r)}$ that satisfy (7) and (8).

Then, according to (12) and (13), we can get (9).

This completes the proof of the theorem.

For a given model, we can get an optimal controller by solving the following optimization problem.

$$\min \gamma$$
$$s.t. \quad (7), (8).$$

Remark 1: In order to avoid ill-conditioned matrix NM^T, usually we add the constraint (20).

$$\begin{bmatrix} X & \varepsilon_0 I \\ \varepsilon_0 I & Y \end{bmatrix} > 0 \quad (20)$$

Where, ε_0 is a constant, and $\varepsilon_0 > 1$.

Remark 2: In order to avoid the extreme situations of the numerical solution, the condition (21) is added to the objective function, by which the elements in X and Y will not be extremely too large.

$$\min \varepsilon_1 \gamma + \varepsilon_2 \left(Trace(X) + Trace(Y) \right) \quad (21)$$

Where ε_1 and ε_2 are constants, satisfying $\varepsilon_1 > \varepsilon_2$.

Remark 3: The designed controller satisfying Theorem 1 can make the characteristics of the closed-loop system closed to the one of the desired reference model. However, it can not guarantee that the performance of the system in the main frequency band is consistent with the desired one fully. Therefore, a prefilter F is introduced to cover this deficiency.

4. FLIGHT CONTROL EXAMPLE

During the flight of aircraft, it is influenced strongly by the aerodynamic coupling, the inertial coupling and the kinematic coupling, synchronously. In such cases, some scholars(such as Deng et al. (2010), Zhou (1995), and Enns et al. (1988)) have already conducted researches on the decoupling control problems of aircraft. To investigate the effectiveness of the proposed control method, we research a flight control example based on the hypersonic vehicle.

4.1 Flight Model

The nominal linear model, with speed of about Mach 10, during 350-400s is given by (22).

$$\begin{bmatrix} \Delta\dot{\alpha} \\ \Delta\dot{\omega}_{z1} \\ \Delta\dot{\beta} \\ \Delta\dot{\omega}_{y1} \\ \Delta\dot{\gamma} \\ \Delta\dot{\omega}_{x1} \end{bmatrix} = \mathrm{A} \begin{bmatrix} \Delta\alpha \\ \Delta\omega_{z1} \\ \Delta\beta \\ \Delta\omega_{y1} \\ \Delta\gamma \\ \Delta\omega_{x1} \end{bmatrix} + \mathrm{B} \begin{bmatrix} \Delta\delta_\varphi \\ \Delta\delta_\psi \\ \Delta\delta_\gamma \end{bmatrix} \quad (22)$$

Where

$$\mathrm{A} = \begin{bmatrix} -0.01862 & 1 & 0 & 0 & 0 & 0 \\ -5.91559 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.00969 & 0.96196 & 0 & 0.27319 \\ 0 & 0 & -5.40318 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -28.41634 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathrm{B} = \begin{bmatrix} -0.00052 & 0 & 0 \\ -2.60529 & 0.0011 & 0 \\ 0 & -0.00007 & -0.00016 \\ 0 & -0.31205 & -1.40788 \\ 0 & 0 & 0 \\ 0 & -1.2735 & -54.51722 \end{bmatrix}.$$

$\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$, are small deviations of angle of attack, sideslip, and roll, respectively. $\Delta\omega_{z1}$, $\Delta\omega_{y1}$, and $\Delta\omega_{x1}$ are small deviations of pitch rate, yaw rate, and roll rate. $\Delta\delta_\varphi$, $\Delta\delta_\psi$, and $\Delta\delta_\gamma$ are small deviations of angle of pitch, yaw, and roll. $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$ are the outputs of the system. Then (23) is the output matrix of the system.

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (23)$$

Here, the $\{\Delta\delta_\varphi, \Delta\omega_{z1}, \Delta\alpha\}$ constitutes the pitch channel of the system, while $\{\Delta\delta_\psi, \Delta\omega_{y1}, \Delta\beta\}$ and $\{\Delta\delta_\gamma, \Delta\omega_{x1}, \Delta\gamma\}$ constitute the yaw and roll channels.

To represent the coupling of the system, we use its direct Nyquist array (DNA) within the frequency band from 0.01rad/s to 2rad/s, which is shown in Fig. 3. Obviously, the system is seriously coupled between the yaw and roll channels.

4.2 Control Design

Practically, according to the frequency analysis of control instruction of the aircraft, we select 1.884 rad/s (about 0.3Hz)as the bandwidth frequency of the vehicle in pitch channel, while 1.256 rad/s (about 0.2Hz)in yaw channel,

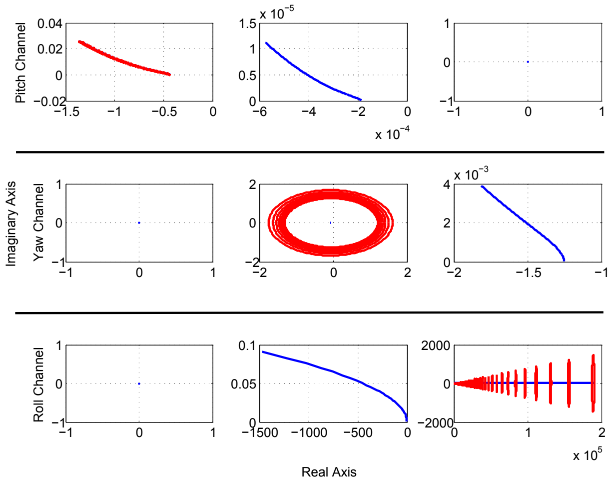

Fig. 3. The curve of DNA of the original system

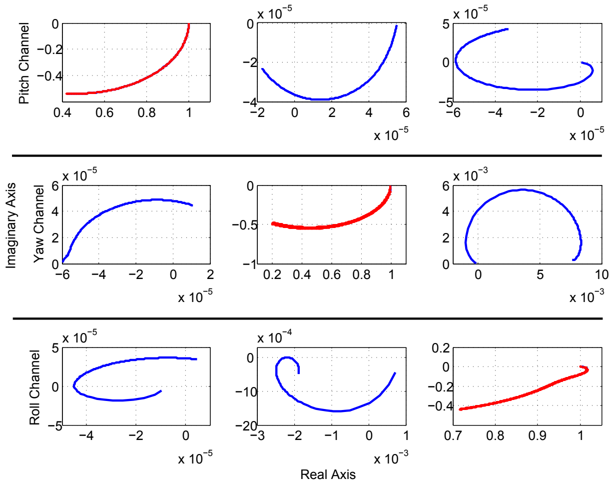

Fig. 4. The curve of DNA of the closed-loop system

and 3.768 rad/s (about 0.6Hz)in roll channel, respectively. Therefore, the reference model is selected as (24) based on the requirement of the bandwidth.

$$R_{ref} = \begin{bmatrix} \frac{1}{0.5308s+1} & 0 & 0 \\ 0 & \frac{1}{0.7962s+1} & 0 \\ 0 & 0 & \frac{1}{0.2654s+1} \end{bmatrix} \quad (24)$$

Firstly, some necessary constrains in Remark 1 and Remark 2 are given: $\varepsilon_0 = 3$, $\varepsilon_1 = 10$, $\varepsilon_2 = 10^{-4}$. Secondly, we use the free toolbox "YALMIP" (Lofberg (2004)) for matlab and its most commonly used solver "SeDuMi" (Sturm (1999)) to design the controller by Theorem 1, which is a typical LMI problem. Then, the curve of DNA of the new closed-loop system is shown in Fig. 4.

Compared with the original system, obviously, the interact intensity of the new one among the channels is greatly reduced.

Additionally, to guarantee the tracking performance, the preposition controller F is designed as (25).

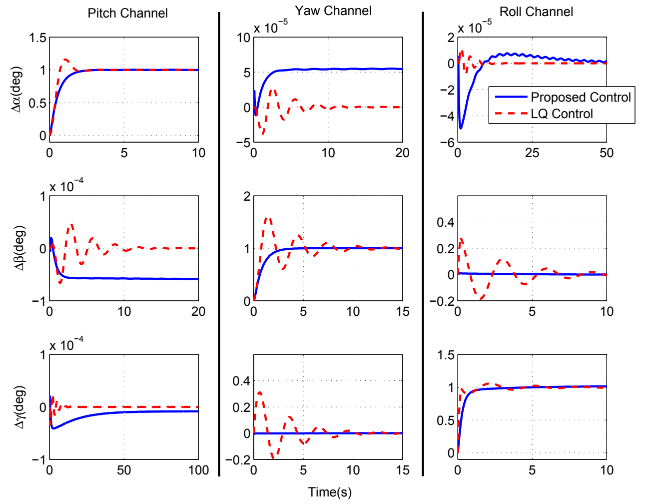

Fig. 5. The step response of the closed-loop system

$$F = \begin{bmatrix} 1.0823 & 0 & 0 \\ 0 & 1.0497 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (25)$$

The simulation results are shown in the following subsection.

4.3 Simulation and Comparison with Linear Quadratic (LQ) Controller

In order to illustrate the effectiveness of the proposed control method, we design an LQ tracking controller for comparison as below.

Firstly, a state feedback matrix K is designed using the linear quadratic regulator (LQR) algorithm based on the performance indexes Q and R.

$$K = \begin{bmatrix} -2.3057 & -0.1331 & 0.0002 & 0.0005 & -0.0001 & 0 \\ -0.0025 & -0.0013 & -0.4278 & -1.3702 & 0.1210 & 0.0288 \\ 0.0001 & -0.0001 & -0.2069 & 1.0230 & -6.3234 & -0.5306 \end{bmatrix} \quad (26)$$

$$\begin{aligned} Q &= diag(40, 0.01, 15, 0.02, 40, 0.02) \\ R &= diag(2.5, 1, 1) \end{aligned} \quad (27)$$

Then, we can obtain transfer function $G_{LQR}(s)$ of the system with the state feedback matrix K. Further, based on the amplitude matrix of the system on zero frequency, the pre-compensation matrix is designed as (28) by $G_{LQR}(j0)$ and the controller is achieved.

$$L = G_{LQR}^{-1}(j0) = \begin{bmatrix} -4.5995 & 0.0072 & -0.0001 \\ -0.0025 & -17.1662 & 0.1210 \\ 0.0001 & -0.3284 & -6.3234 \end{bmatrix} \quad (28)$$

After that, we test both systems by adding a unit step signal to each channel respectively. Responses of the systems by the proposed control method and LQ control method are shown in Fig 5.

In the unit step response of the pitch channel, compared with the LQ control method, the performance of $\Delta\alpha$ by the proposed control method has the similar setting time and no overshoot. Then, studying the performance of $\Delta\beta$

and $\Delta\gamma$ in this channel, the static error is existed by the proposed control method, of which the order of magnitude is as small as 10^{-4}, and can be ignored.

In the yaw channel, by the proposed control method, $\Delta\alpha$ has static error as well, and it also can be ignored since the magnitude is 10^{-5}. However, the LQ control method has greater overshoot in this channel, and it produces some shakes on $\Delta\gamma$ with the amplitude of about 0.25 degrees.

Moreover, in the roll channel, $\Delta\alpha$ appears a negligible overshoot with the magnitude of 10^{-5} by using both methods. But the LQ method produces some shakes on $\Delta\beta$ as it does on $\Delta\gamma$ in the yaw channel, while the proposed control method has little effect on $\Delta\beta$ and the regulation is much smoother than LQ method on $\Delta\gamma$.

Altogether, the proposed control method has much better performance on dynamic properties of controller. On the other hand, it has small static errors in certain channels which are negligible. So, the proposed method is better performed on the system with strong coupling than the LQ method.

5. CONCLUSIONS AND FUTURE WORK

In this article, we conduct a series of studies about the reference model based decoupling control issues with the dynamic compensator. The coupling characteristics of the hypersonic vehicle are analysed by the DNA. Our contribution is to propose a necessary and sufficient theory of decoupling controller design by solving a LMI problem. To solve it conveniently, the free toolbox "YALMIP" for matlab with the solver "SeDuMi" is utilized. Lastly, a comparison between the proposed method and LQ method is obtained by simulation, which illustrates that the proposed control method has better effects on the system with strong coupling.

However, some further researches are necessary to improve this method. The high order of the controller by the proposed decoupling method will make it difficult to realize in practical systems. To address this problem, in the future work, we will try to reduce the order of the resulting controller consciously, or to design the lower order controller directly to make it of great significance in application.

ACKNOWLEDGEMENTS

The authors would like to thank the supports from the National Nature Science Foundation of China, No. 61203081 and 61174079, Doctoral Fund of Ministry of Education of China NO. 20120142120091, Precision manufacturing technology and equipment for metal parts NO. 2012DFG70640.

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